

# Altair Activate<sup>®</sup>

Altair Activate<sup>®</sup> 2020.1  
Extended Definitions

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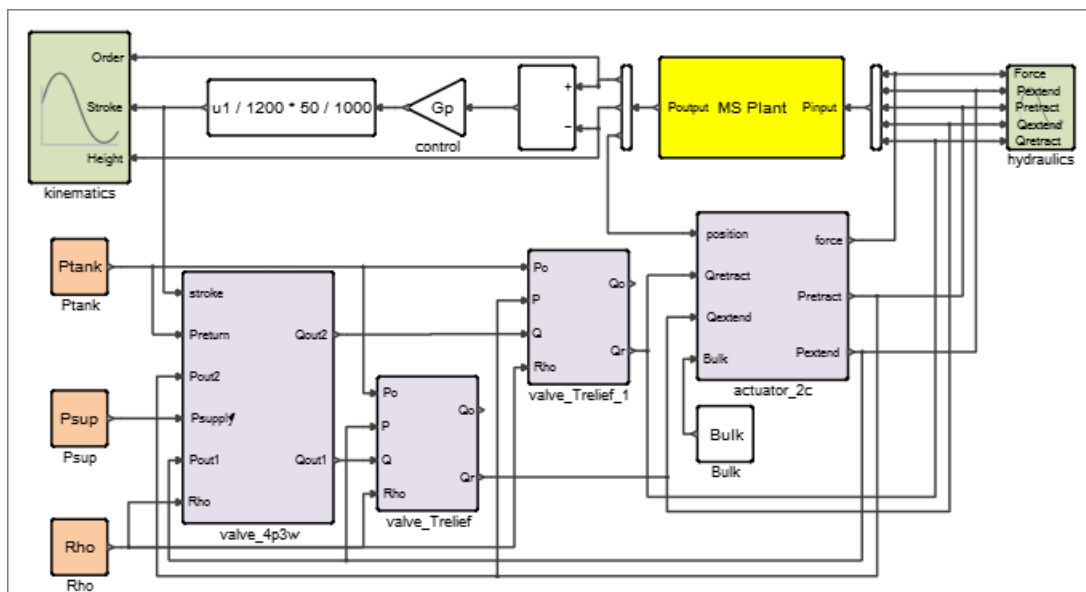
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# Introduction to Altair Activate

The chapters of this *Extended Definitions* book provide background and theoretical information on advanced features and other concepts used in **Activate**. The book is an extension of the User Guide available directly from within the product.

This book describes the features available in the *Professional Edition* of **Activate**. Some of the topics (for example, chapters 13, 14 and the section 12.6) are not applicable to the *Basic Edition* of **Activate**.



**Activate** is a multi-discipline system simulation tool with powerful Modeling and Simulation capabilities. It includes, amongst other features,

- Handling of Continuous and discrete models through powerful hybrid simulator,
- Mixing both Signal-based as well as physical models using Modelica and Spice,
- Co-Simulation with different domains such as multibody dynamics, electromagnetics, electronic circuits,
- Functional Mock-up Interface for Import and Export,

- Library Management,
- Code generation capabilities,
- Strong scripting language integration

Note - All models used in this book can be found in the installation directory under:  
/tutorial\_models/Extended\_Book\_Models/.

# Chapter 1

## Activation signals in Altair Activate

### 1.1 Introduction

Activation signals in **Activate** control the execution of block functions. These signals can be explicitly manipulated providing powerful modeling capabilities not available in other simulation environments. Here we provide an introduction to the role that Activation signals play in the **Activate** environment and present, through simple examples, their usage in modeling certain types of systems.

Associated with red links connected to ports usually placed on top and at the bottom of blocks, Activation signals are present in most **Activate** diagrams. The most common usage is the activation of blocks at a fixed frequency with a signal generated by a **SampleClock** block. This block generates a series of isolated activations, called events, regularly spaced in time.

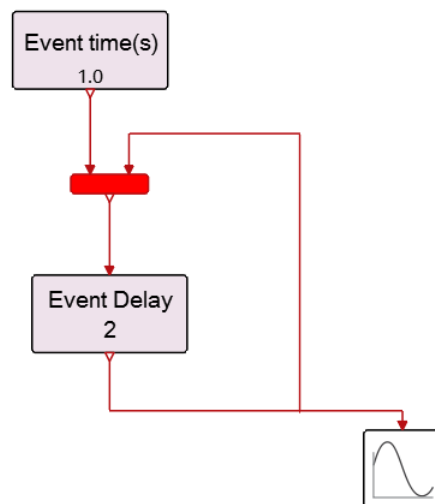


Figure 1.1: Event Delay model (Eventdelay.scm)

Events can be explicitly operated on in **Activate**: they can be conditionally subsampled, and the union and intersection of events can be constructed. Blocks can generate delayed events, allowing for example the implementation of operations such as event delaying. In the EventDelay model shown above,

the output activation port of an event delay block is fed back to its input activation port, creating a sequence of events where the time spacing with successive events corresponds to the value of the delay.

A first event generated by the block **EventGenerate** initiates the cycle, which is its first (and only) event at 1.0 seconds. The union of this Activation signal and the Activation signal, fed back from the **EventDelay** block, is generated by the red "Event Union" block and activates the **EventDelay** block at 1.0 seconds. At this point, the **EventDelay** block will create an event that is delayed by 2 seconds, which means the next event will be generated at 3.0 seconds. Since the **EventDelay** block's Activation output activates itself, it will continue to create events every two seconds thereafter for the remainder of the simulation. This combination of blocks provides the same behavior as an **EventClock** block, and indeed the **EventClock** and **SampleClock** blocks are constructed in the same spirit.

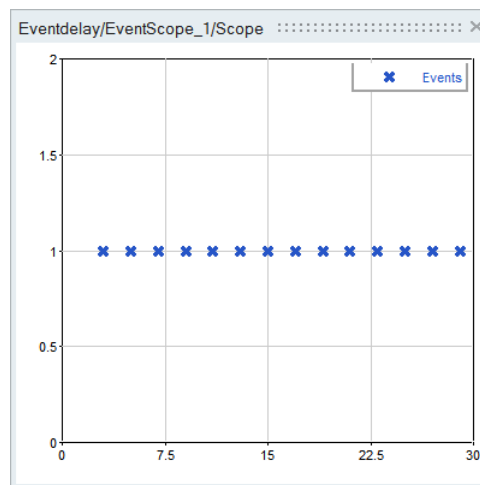


Figure 1.2: Event Scope results (Eventdelay.scm)

Note that the **SampleClock** block provides a similar behavior, but have a significant difference in how its activation output is synchronized with other blocks. More details on this can be found in Sections [1.2.5](#) and [1.4.2](#).

A random sequence of events (times between events following a random law) may be constructed using the **EventVariableDelay** block. This block delays events but the amount of delay is variable and its value is given by the value of the signal at its regular input port. The following diagram illustrates how such a random event process may be generated:

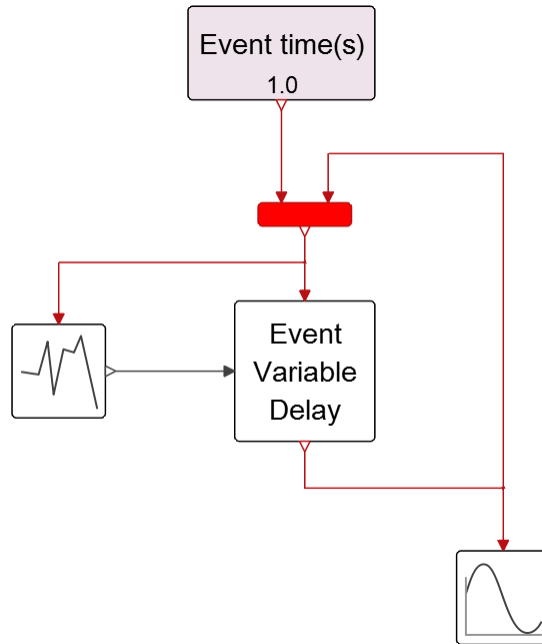


Figure 1.3: Random Event Delay model (RandEventdelay.scm)

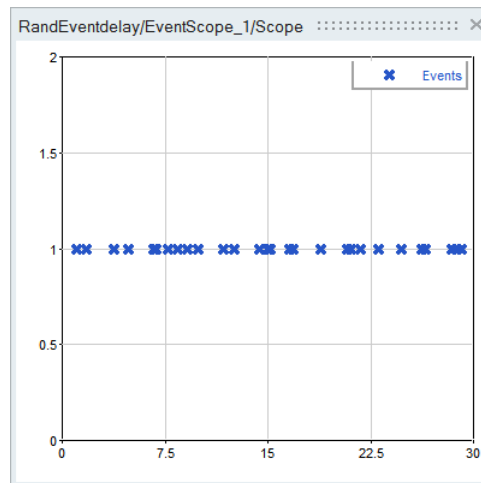


Figure 1.4: Event Scope results (Eventdelay.scm)

If the random law representing the inter-event times follows the exponential law then the random event process is a Poisson process. The Poisson process is often used in the modeling of queuing systems, for example to study the waiting time and queue size at Supermarket checkout lines.

### 1.1.1 Simple queue

A very simple model for a supermarket with a single checkout counter can be constructed as shown in figure 1.5

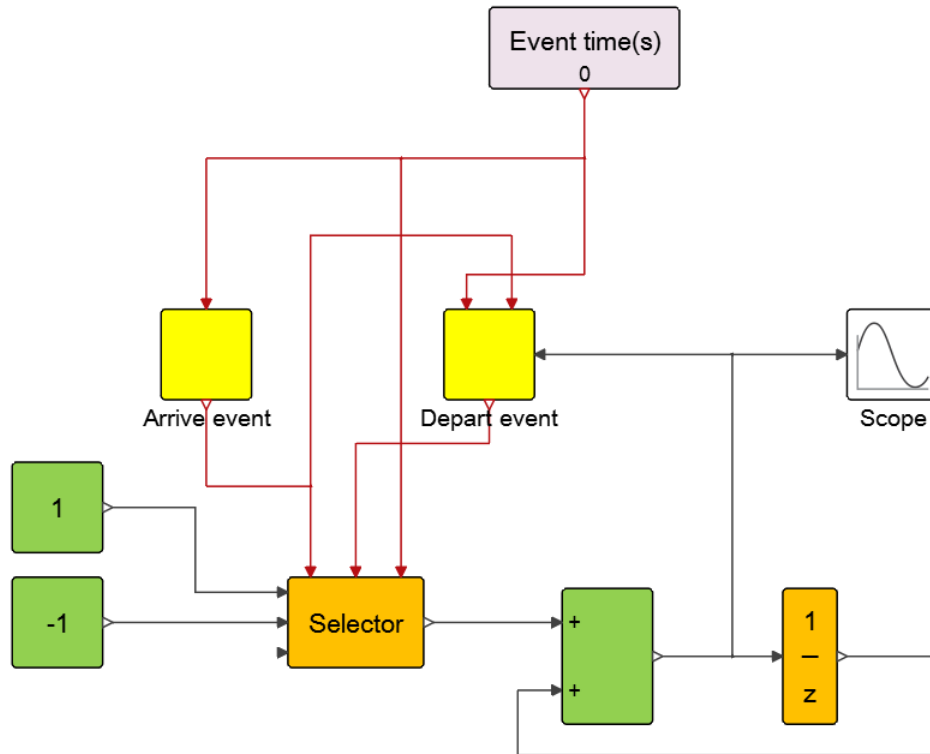


Figure 1.5: Simple Queuing model (Queue.scm)

The arrivals here are modeled as shown previously but using an exponential law (see figure 1.6).

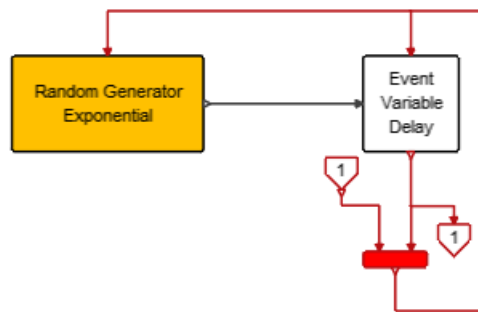


Figure 1.6: Arrival super block in Simple Queuing model

The initial event starts the generation of arrival events.

But departures are slightly more complicated to model, even if exponential service time is considered. The reason is that unlike arrivals, departures may be halted, in particular when the queue is empty. Thus the state of the queue is required for modeling the departure process. As long as the queue is not empty, the departure process operates similarly to the arrival process: the feedback loop generates successive events. But when the queue is emptied, the feedback loop should be broken; no event should be generated in such a situation. When the queue is empty, a departure occurs only following an arrival. So the generation of the departure process depends also on the arrivals, in particular when the



queue contains a single element (it has been empty and an arrival has just occurred). The departure diagram contains, in addition to the Activation input used to initialize the process (as in the case of arrivals) a second input receiving the arrival process. This Activation signal is filtered by an **IfThenElse** block. The Activation signal from the “Else” branch, corresponding to the case where the state of queue is 1, is used for generating departure events. Another **IfThenElse** block is used to filter out events if the queue is empty (figure 1.7).

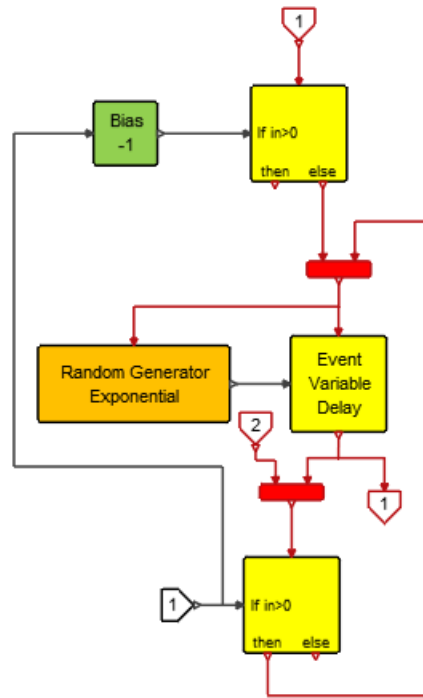


Figure 1.7: Departure super block in Simple Queuing model

At each event activation, the state of the buffer is updated displayed by the **Scope** block. It is obtained as the sum of the output of the **SelectInput** block and the previous state of the buffer (stored in the **DiscreteDelay** block). The **SelectInput** block copies one of its inputs to its output depending on the way by which it is activated. In particular if it is activated by an arrival event, the value of +1 is copied to the output. If it is activated by a departure event, the value of -1 is copied. The initial event copies the value 0 (an unconnected input is assumed to have value 0 in **Activate**).

The simulation result shown in figure 1.8 gives the evolution of the state of the queue for the case of exponential service time.

It is of course simple to replace the exponential random delay with a constant delay or a more complex expression such as the sum of a constant with a random value. Indeed a realistic service time at a counter includes a time for payment which a priori does not depend on the number of purchased items and thus may be considered constant.

A useful block for modeling event sequences is the **EventDelayedChannel**. The **EventDelay** and **EventVariableDelay** blocks cannot be used to delay a train of events unless the delay is smaller than the time between two incoming events. The reason is that if an event is received by any of these block before the event programmed on its output is fired, the output event is reprogrammed and the previous event is lost. This has not been a limitation in the models considered previously but in some applications

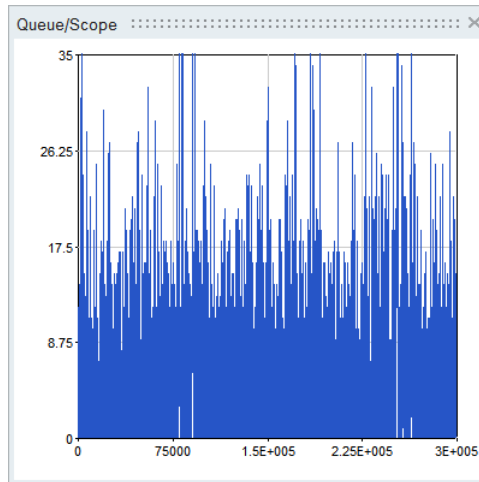


Figure 1.8: Queue state evolution vs. time

the **EventDelayedChannel** block must be used to delay Activation signals.

### 1.1.2 Traffic simulation

Consider modeling the traffic, as for example in figure 1.9. Each traffic light on a road may be modeled as a queuing system with each event corresponding to a vehicle.

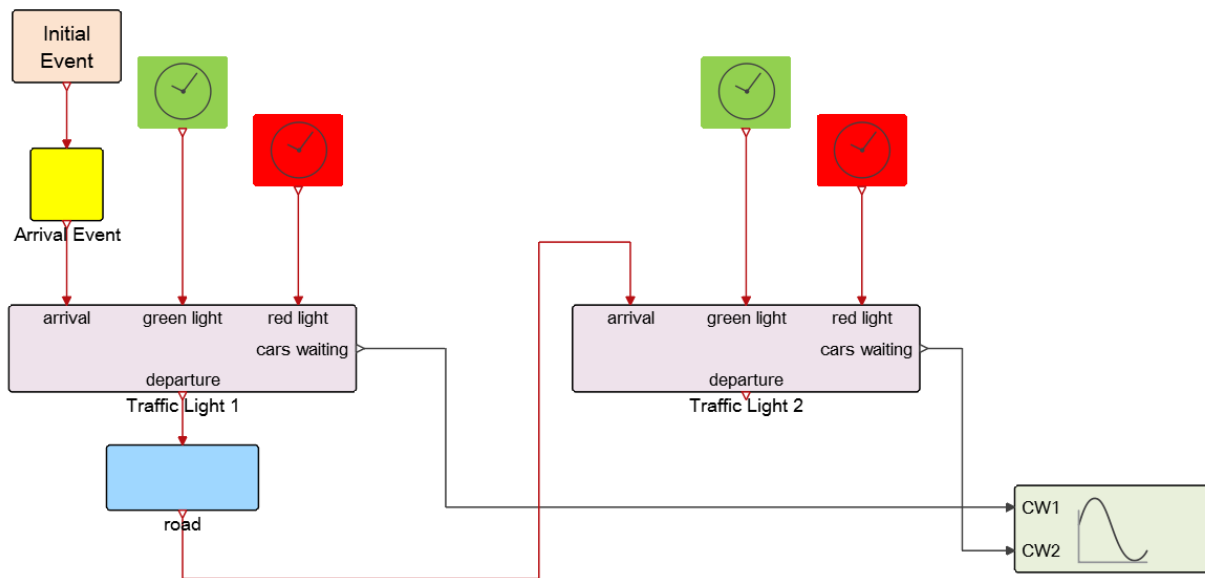


Figure 1.9: Circulation simulation model (Circul.scm)

**Traffic light** Super Blocks model the arrival and departure of cars (events). The regular output indicates the number of cars waiting behind the light. The road from one light to the next can be modeled using an **EventDelayedChannel** block where a travel time may be associated with each vehicle on the road.

Unlike **EventDelay** blocks, **EventDelayedChannel** block contains an internal buffer that can be used to store the state of individual vehicles on the road between the two traffic lights. Note that cars on the road can travel at different speeds and pass each other. The **road** Super Block models the road assuming that the travel time of each car is an independent random number (see figure 1.10).

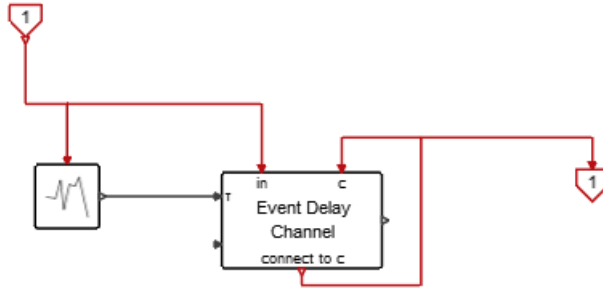


Figure 1.10: road super block in Circulation model

The numbers of vehicles, as a function of time, waiting behind the two lights are displayed by the scope and are illustrated below in figure 1.11

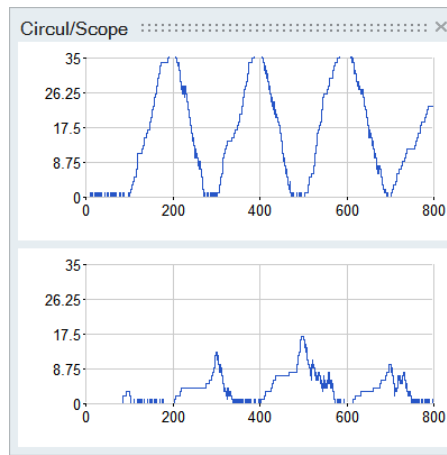


Figure 1.11: Number of vehicles behind the traffic lights

This paradigm may be used to model more complex queuing systems such as multiple dependent queues, finite capacity queue buffers, and complex timing models. Such models may be used to study stochastic properties of complex queuing systems and optimize control strategies via for example Monte Carlo methods.

Events may also be used in models of systems usually seen as finite state machines. Consider for example a simple thermostat-based cooling system that functions as follows: when the temperature goes above certain threshold value, it waits for a fixed amount of time and then turns on the cooling system unless in the meantime the temperature has dropped below the threshold value. For that, the block **EventVariableDelay**, and in particular its reprogramming capability may be used (see figure 1.12).

In this "Plant" diagram, when the output of the **Sum** block becomes positive, i.e., the temperature goes beyond the threshold temperature, one of the **EdgeTrigger** block generates an event which is programmed for `deltaT` unit of time later, at the output of the **EventVariableDelay** block. If the output

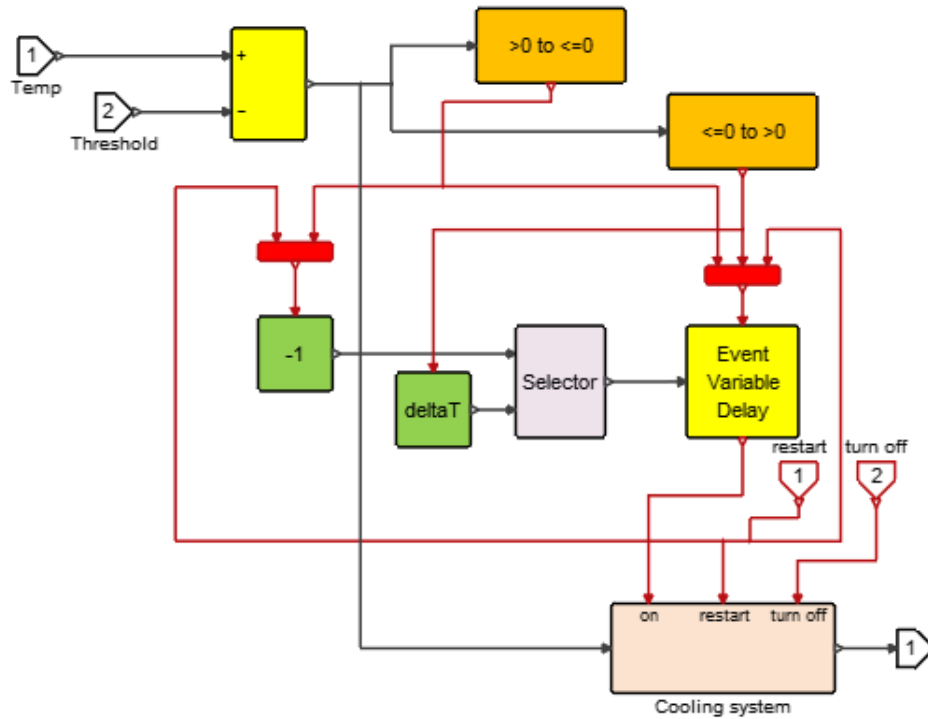


Figure 1.12: Plant super block in Cooling model (Cooling.scm)

of the **Sum** block becomes negative before this event is fired, or a “restart” event is received, the output event of the **EventVariableDelay** block is reprogrammed, in this case de-programmed since programming with a negative delay corresponds to not programming any event.

The Super Block “Cooling System” at the bottom of the diagram generates the on/off command that is actually sent to the device. The strategy is as follows: when the “on” or “restart” event is received, the cooling system is turned on for a fix period  $C_{Per}$  of time. At the end of this period, if the temperature is still above the threshold temperature then the cooling system is turned off, otherwise it stays on for another  $C_{Per}$  unit of time.

Note that the  $C_{Per}$  period where the output is on may be repeated multiple times before the temperature goes below the threshold value.

There are of course specialized tools for modeling these types of systems, what makes event modeling in **Activate** particularly powerful is its ability to combine events, and continuous and discrete-time dynamical systems. Events may be used to create discontinuities in the dynamics of the system, and events may be generated by the dynamics. For example the thermostat model discussed previously is usually combined with a continuous-time model describing the action of the cooling system and the evolution of the temperature. A simple example is shown in figure 1.14.

The output of the integrator represents the room temperature. The simulation result is displayed in figure 1.15.

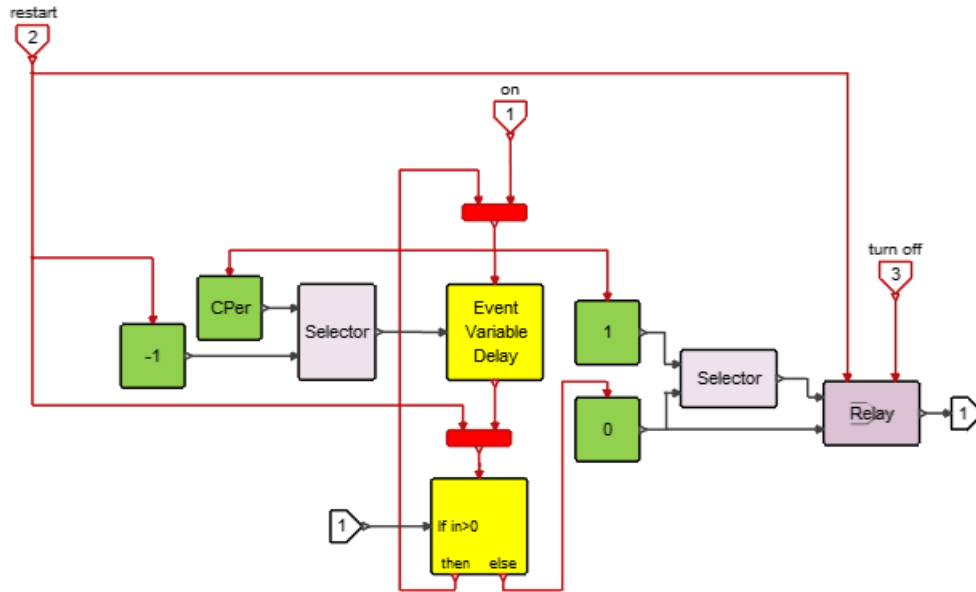


Figure 1.13: Cooling System super block (inside Plant) in Cooling model

### 1.1.3 Re-initialization of continuous-time state

The following example shows how an event can be used to re-initialize a continuous-time state. This example corresponds to a model of a hormone-releasing hormone pulse generator proposed in [1]. A **Activate** model has been presented in [2].

The following diagram (figure 1.16) models a Super Block, named Pulse Generator, which provides both the event and the value of the jump corresponding to the event.

The outputs of this block are used to create a jump in the state of the system.

The complete model including the Pulse Generator Super Block is shown in figure 1.17.

The nonlinear dynamics of the system modeled by **MathExpression** blocks are as follows

$$\begin{aligned}\dot{v} &= s(-v(v-c)(v-1) - k_1g + k_2a), \\ \dot{g} &= b(v)(k_3a + k_4v - k_5g), \\ \dot{z} &= p(v) - k_6z,\end{aligned}$$

where

$$b(v) = b_1 - \frac{b_2}{1 + \exp(-b_3(v - b_4))}$$

and

$$p(v) = \frac{p_1}{1 + \exp(-p_2(h(v) - p_3))}$$

with  $h(v) = v$  if  $v > 0$  and  $h(v) = 0$  otherwise. The  $s$ ,  $a$ ,  $k_i$ ,  $p_i$  and,  $b_i$  are model parameters.

The **JumpStateSpace** block is used to store the system state and model its re-initialization. In the absence of activation on its input activation port, this block realizes a standard continuous-time linear system (in this case a simple integrator) with system input corresponding to the first input of the block. When activated through its activation port, the the state jumps to the value specified on the second input. In this case the state is  $x = (v, g, z)$ . The simulation results are given in figure 1.18.

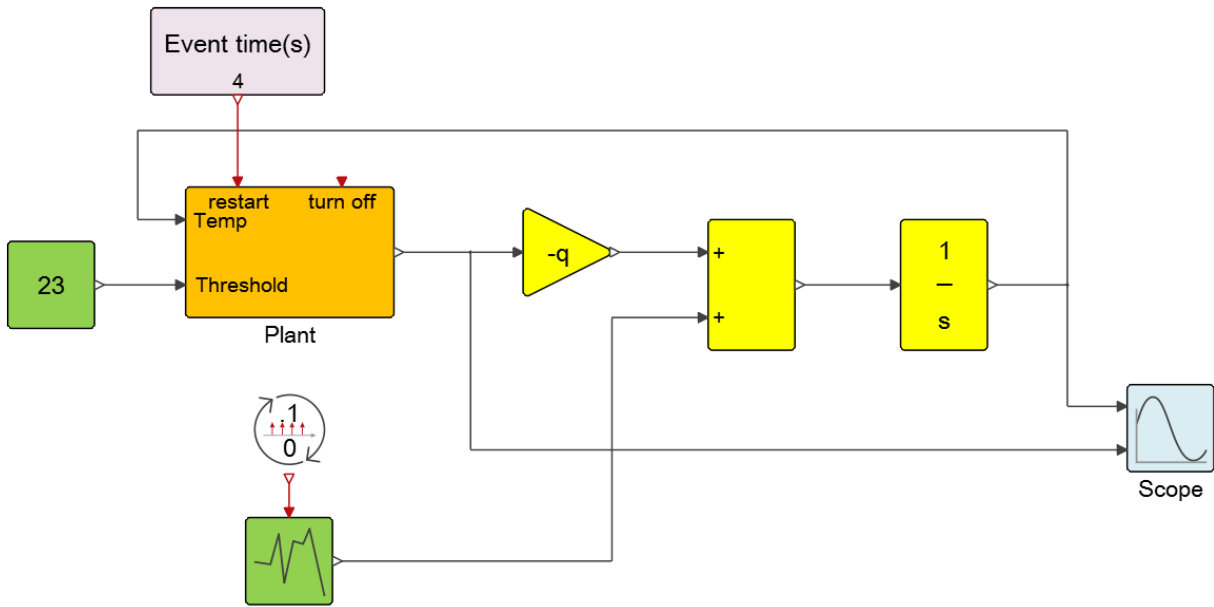


Figure 1.14: Cooling System model (Cooling.scm)

### 1.1.4 Communication delay

The ability to explicitly operate on Activation signals may also be used for studying the effects of communication delays in for example control systems. Consider a classical inverted pendulum controller with state feedback control (figure 1.19). The feedback is sampled and held over periods of 0.15 unit of time.

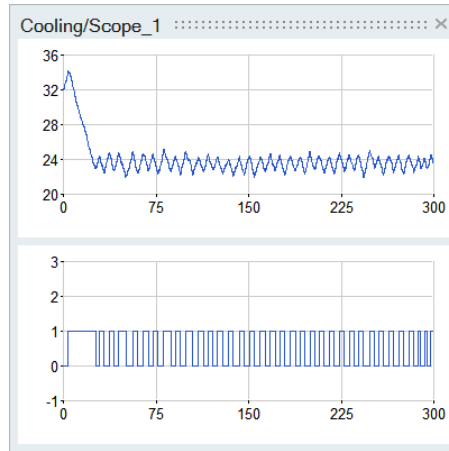


Figure 1.15: Room Temperature vs. Time

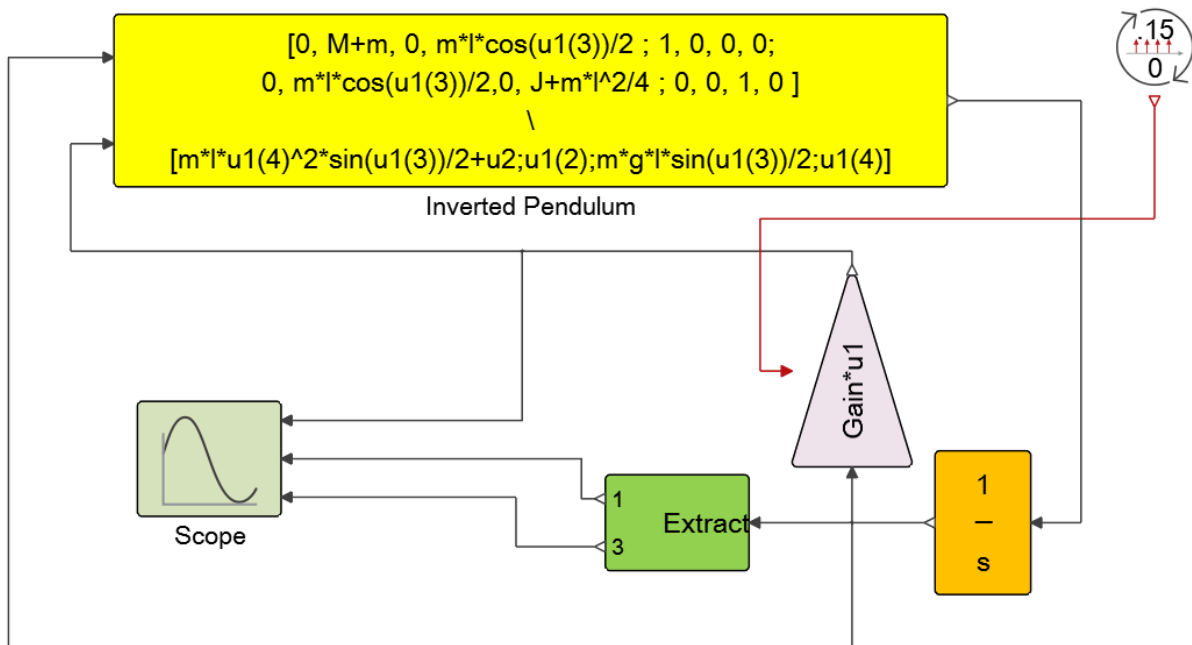


Figure 1.19: Inverse pendulum with delay model (PendInvcdelay.scm)

If the initial state is not far from the equilibrium state, then this controller is capable of stabilizing the system:

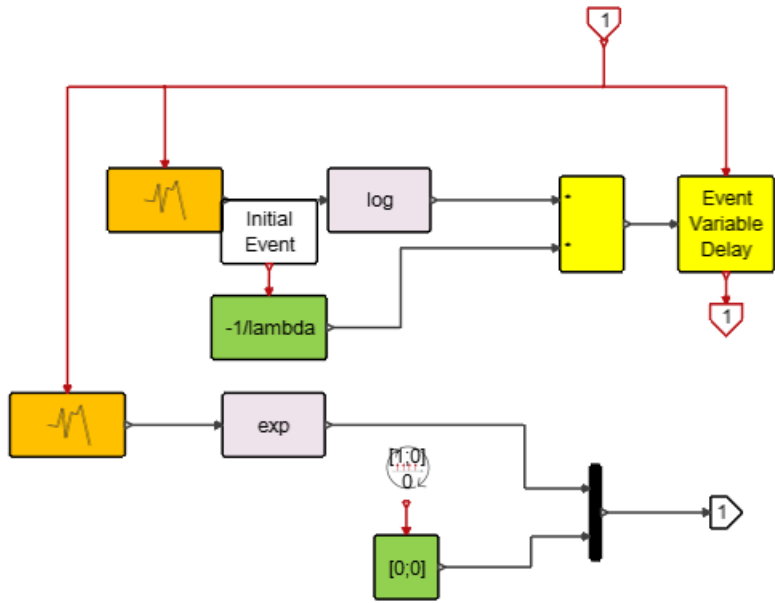


Figure 1.16: Pulse Generator superblock in lut.scm model

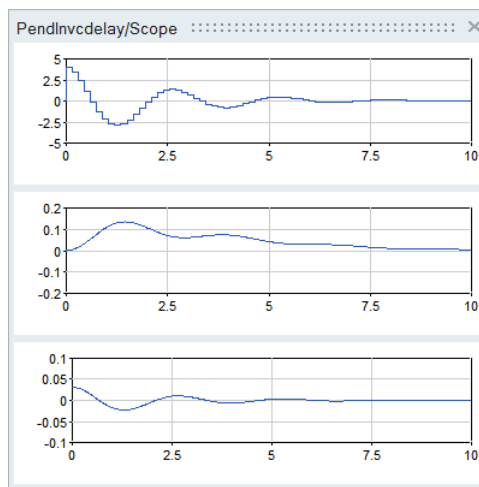


Figure 1.20: Results of Inverse pendulum with delay model (PendInvcdelay.scm)

To study the effects of communication delay in the application of the control signal, the model can be modified by applying a constant delay to the control signal. Since the control signal is a discrete signal, this can be done with a simple **EventDelay** block or a **EventVariableDelay** block with a constant input, as long as the amount of delay is less than the sampling period. Recall from Section 1.1.1, that if an event is received by this block before the event programmed on its output is fired, the output event is reprogrammed and the previous event is lost.



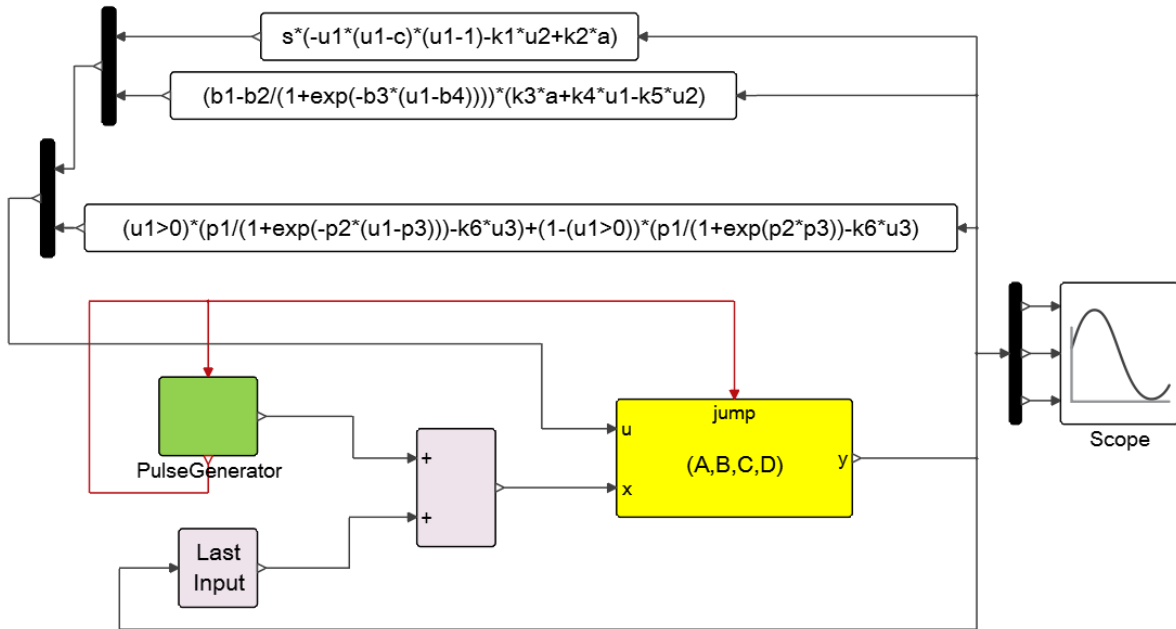
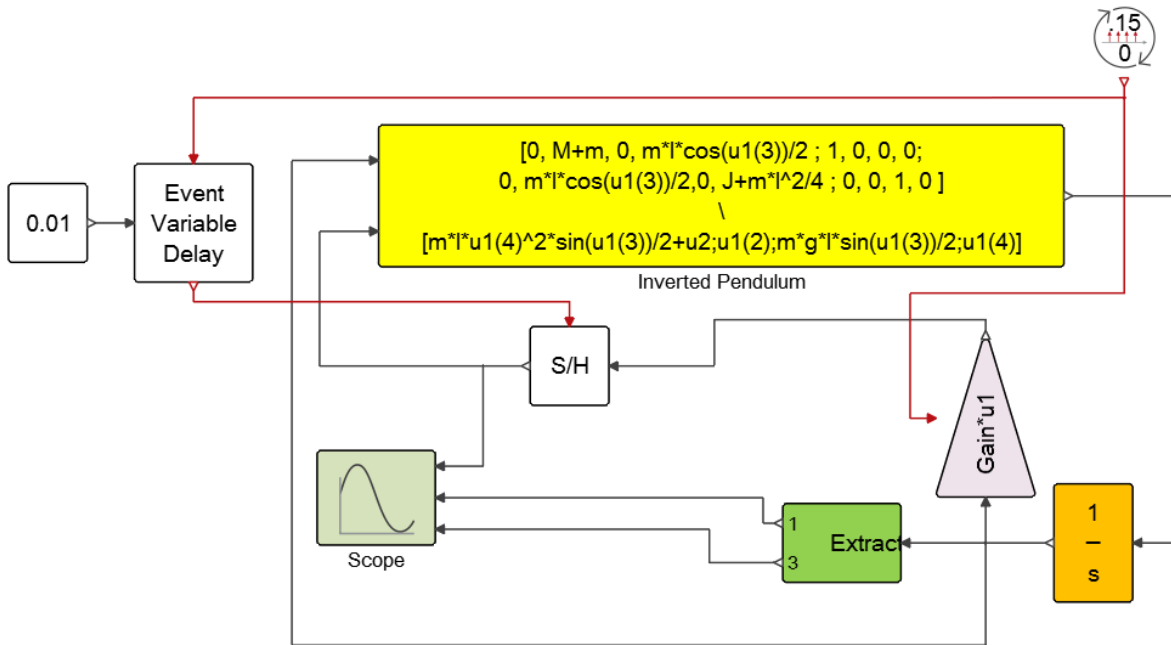


Figure 1.17: Hormone releasing model (lut.scm)



PendInvdelay01.scm

Figure 1.21: Inverse pendulum with delay model, variant 1 (PendInvdelay01.scm)

With a delay of 0.01, the system remains stable but the convergence to the steady-state position takes more time:

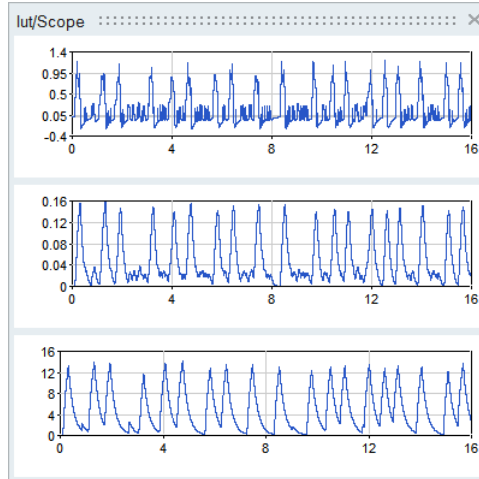


Figure 1.18: Results for the Hormone releasing model

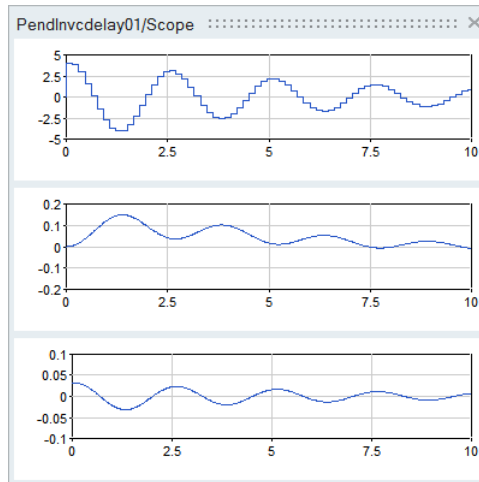


Figure 1.22: Results of Inverse pendulum with delay model, variant 1 (PendInvcdelay01.scm)

Increasing the delay to 0.02 completely destabilizes the system:

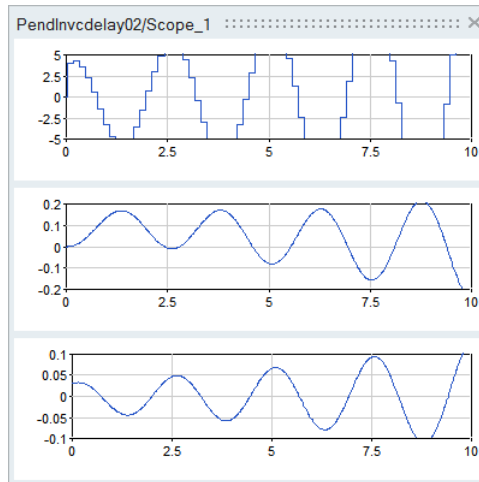


Figure 1.23: Results of Inverse pendulum with delay model, variant 2 (PendInvcdelay02.scm)

The model can be modified to study the effect of jitter (random delay) as shown in figure 1.24:

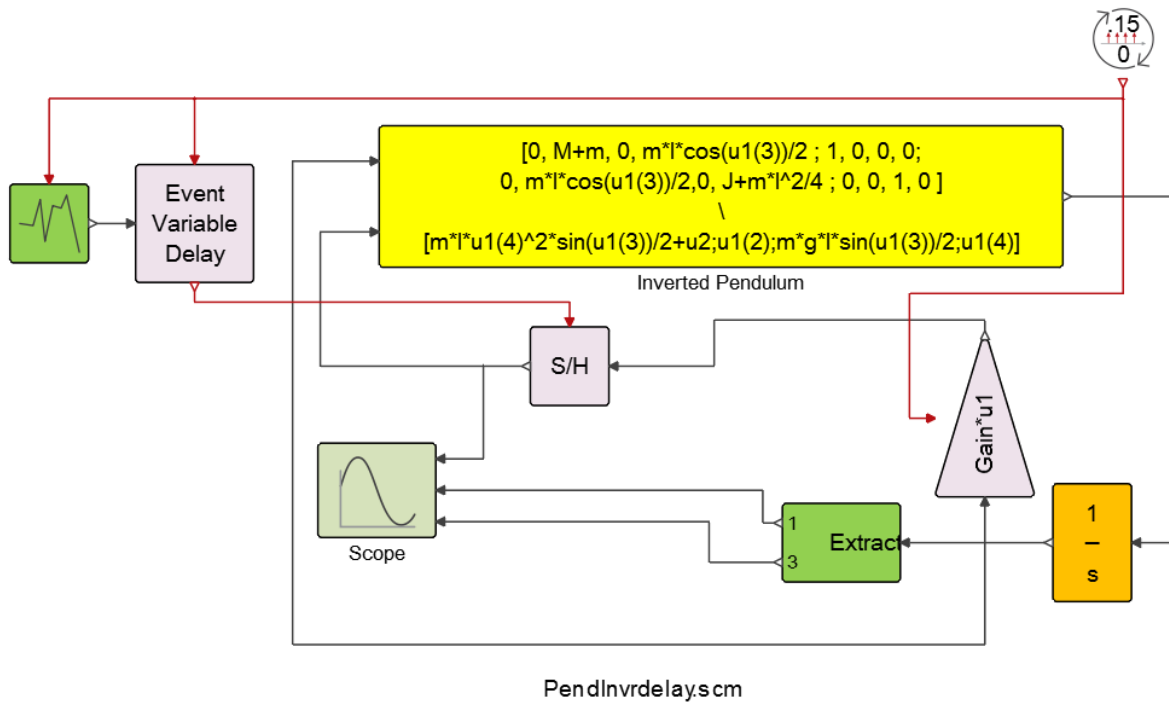


Figure 1.24: Inverse pendulum with random delay model (PendInvrdelay.scm)

This is done by feeding the **EventVariableDelay** block with a random signal instead of a constant.

If the amount of delay is larger than the sampling period, the **EventDelayedChannel** block must be used to model the behavior of the system.

## 1.2 Types of activation signals

Activation signals are used to specify activation times of the blocks they are connected to. **Activate** supports different types of activation signals.

### 1.2.1 Programmed events

Activation signals seen so far were mostly isolated timed events generated by blocks activated by other isolated timed events. These events were programmed in particular for given times in the future using the event delay blocks **EventDelay** and **EventVariableDelay**. When activated, **Activate** blocks can program output events on their activation output ports. The block in particular specifies the event firing delay, i.e., how much time after the block execution the event should fire, for each of its output activation ports.

The block may also program initial output events. The block **EventGenerate** for example programs only initial events. It does nothing during simulation.

The time of a programmed event may be modified and the event may even be deprogrammed by the block before the firing time. This is done simply by programming a new output event by indicating the new firing delay. This operation overwrites the previously programmed event because only one event can be programmed at any time for any output activation port. Giving a negative delay value deprograms the event. When an event is re- or de-programmed, a warning is issued by the simulator.

### 1.2.2 Zero-crossing events

Activation signals may also be produced by blocks activated in continuous-time. The **EdgeTrigger** block seen previously is an example where an activation signal is produced based on a zero-crossing test. Such events are generated when a signal crosses a given surface, in general the surface corresponding to the value 0. A number of blocks in **Activate** palettes implement zero-crossing functionalities. Figure 1.25 represents the simple model of a thermostat and its results (figure 1.26).

Zero-crossing blocks are used to turn on the heater or the cooler when the temperature falls below -10 or rises above 10. The  $+t_0-$  and  $-t_0+$  blocks generate events when the input of the block crosses zero respectively with a negative and a positive slope. The events activate the **SelectInput** block, which depending on the activation port through which it has been activated copies its first or second input (value -6 or +6) to its output. This output is the heat flow that is added to a random signal and fed to an integrator, the output of which represents the temperature. The simulation results show how the thermostat functionality is implemented by the zero-crossing blocks.

### 1.2.3 Continuous-time activation signal

The activation signals so far encountered are series of events, which are isolated activation signals in time. But activation signals are more general and include time intervals. The simplest activation signal of this type is the *always active* activation signal. The **SampleClock** block with period zero generates the always active activation signal; blocks that should be always active may be activated using this signal.

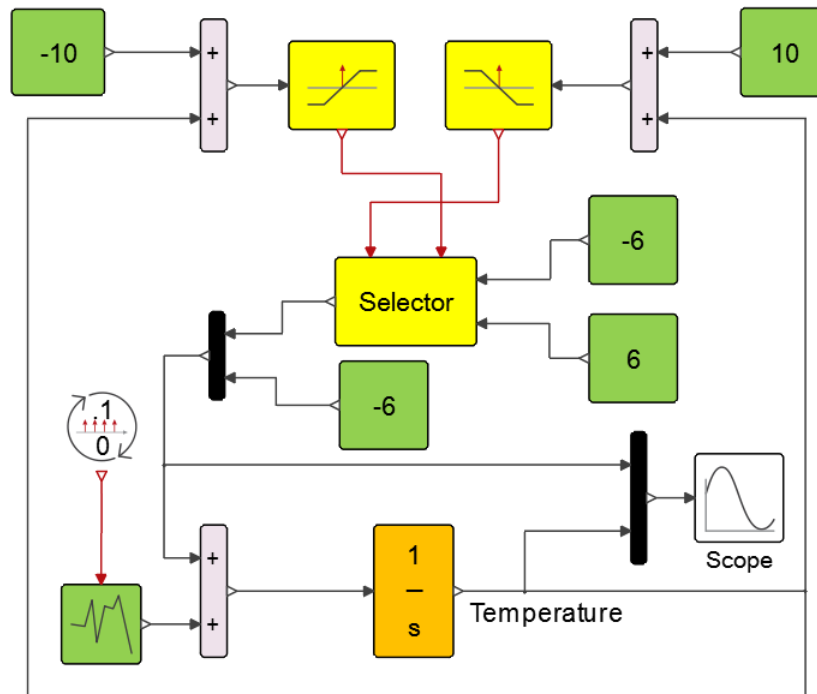


Figure 1.25: Simple Thermostat model (Simple\_Thermostat.scm)

**Activate** also provides a facility to declare a block “always active”, without having to draw an explicit link from the **SampleClock** block. Such a link is added by the compiler to the virtual activation port numbered 0 of the block when the block is declared “always active”. Many basic blocks in **Activate** palettes, such as the **SineWaveGenerator** or the **Integral** block, are “always active”.

#### 1.2.4 Initial-time activation signal

Some blocks are only activated at the start of the simulation. This is the case for example of the **Constant** block with default parameters. Other blocks may need to perform specific tasks at initial time. The “initial activation” signal is used to activate blocks at the start of the simulation. This signal may be generated using the **InitialEvent** block (which contains a **SampleClock** block with period set to  $-1$ ). As for the “always active” signal, the “initial activation” may be declared as a block property. The **Constant** block is for example declared initially active. Note that an always active block is also initially active.

#### 1.2.5 Periodic activation signals

In the first model considered in Section 1.1, a train of events firing at regular intervals was created using the **EventDelay** block. This was done in particular by programming an event on a regular basis. The Activation signal produced resembles the signal produced by the **SampleClock** block but it is not of the same type. The one produced by the **SampleClock** block is of type *periodic*. The compiler knows

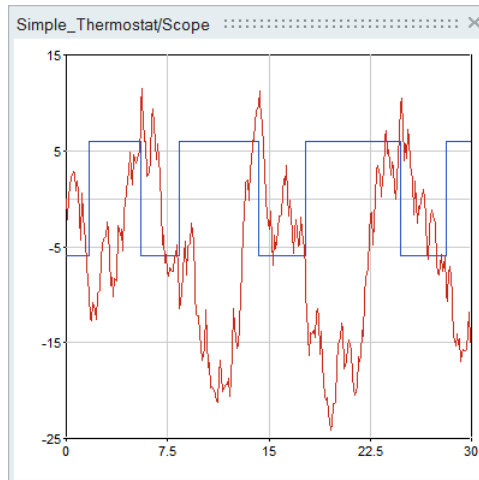


Figure 1.26: Results from Simple Thermostat model (Simple\_Thermostat.scm)

that this signal is periodic contrary to the signal produced by the **EventDelay** block, which happens to contain events firing periodically thanks to the way the model is constructed.

An Activation signal is said to be periodic only if it is of type periodic.

Strictly speaking a **SampleClock** block is not a regular **Activate** block. Not only does it produce Activation signals that are periodic but also synchronous (see Section 1.4).

The **Activate** compiler has a special treatment for periodic activation signals. Some **Activate** blocks operate only on periodic activation signals. When a block is activated by a periodic signal, it has access to the period and offset information at compile time. Thanks to this information, the block can adapt its behavior by computing specific block simulation parameters. The **SampledData** block for example computes the discrete-time linear system matrices corresponding to the discretization of a continuous-time linear system for the operating frequency. This frequency, which is the inverse of the sampling (activation) period, is available at compile time. This block cannot function if it is activated by a non-periodic activation signal.

The only blocks producing periodic activation signals are the **SampleClock** and the **ReSampleClock** blocks.

### 1.3 Activation inheritance

Specifying explicitly the activation signals of the blocks in the model is useful for specifying the model behavior unambiguously. But doing it systematically could make the model labyrinthian. In some cases the explicit graphical presence of activation links is not necessary for the comprehension of the model behavior. The ability to declare blocks initially active or always active is one mechanism to simplify the model by removing some activation links. The other mechanism is inheritance.

The inheritance mechanism is an editor facility that can be used to reduce significantly the number of activation links in models. Most basic blocks in **Activate** may be both explicitly activated or activated by inheritance. Whether the block works by inheritance or external activation is specified via the “External activation” checkbox in the block parameter GUI. When the box is checked, the input activation port is added; unchecked, the block inherits its activation. For example, see the Scope block.

A block inherits its activation if it has no activation input ports and it is not declared always active. A block that inherits, either receives the union of the activations of all the blocks that provide its inputs on its only (invisible) input activation port, or receives the activations associated with each of its input on a separate (invisible) input activation port. The former is the default behavior. Indeed most blocks don't behave differently depending on the port through which they have been activated.

Consider the following model described in figure 1.27.

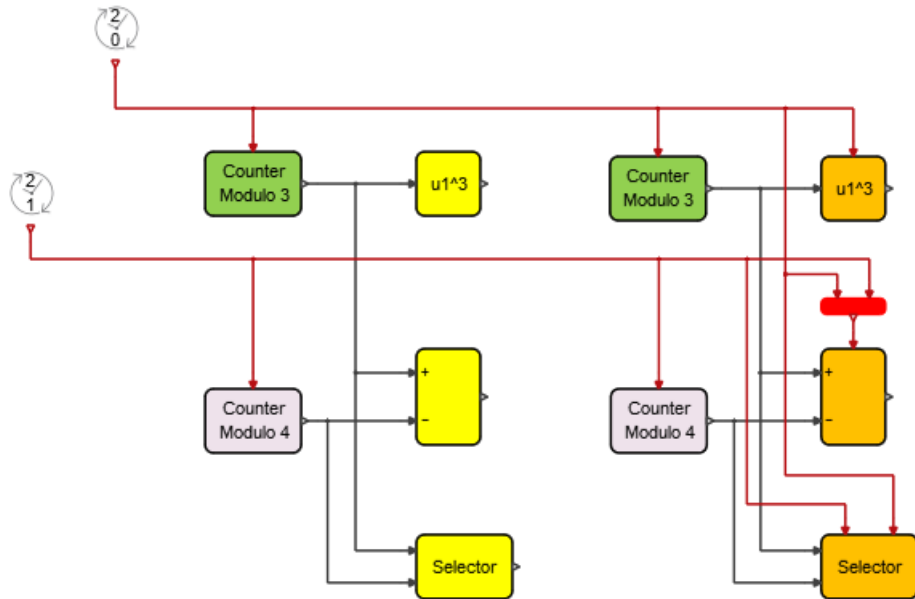


Figure 1.27: Model illustrating inheritance

The yellow colored blocks inherit their activations. The yellow **Power** block has one input so its inheritance yields the exact behavior as that of the orange colored **Power** block. The yellow **Sum** block has two inputs and has default inheritance property so it is activated by the union of the activations generated by the two **SampleClock** blocks, as in the case of the orange colored **Sum** block. This is the proper inheritance property for the **Sum** block since no matter which input of the block is modified, the same summation operation must be performed.

The **SelectInput** block on the other hand has naturally multiple activation input ports. The block copies one of its inputs to its output depending on the port by which it has been activated. So the inheritance property of the **SelectInput** block is necessarily different from that of the **Sum** block. Its behavior is that of the orange colored **SelectInput** block.

## 1.4 Synchronous vs. asynchronous activations

Activation signals are characterized by time periods and instances. An event for example defines an isolated point in time specifying the time instant when the block(s) receiving the event should be activated. The time of the event however does not completely characterize the event, and in particular its relationship with other events. In particular, two events may have identical times but not be *synchronous*. The distinction between synchronicity and simultaneity is a subtle but important point to understand since it plays a fundamental role in the semantic specification of the graphical language.

When two blocks are activated by the same event the compiler must compute the order in which they must be activated depending on the way they are connected to each other. In particular if one of the blocks requires the value on one of its inputs for computing its output and this input is connected to the output of other block, then the latter block should be executed first. In general, for any activation signal the compiler computes a table to specify the list of the blocks to be executed and the order in which they have to be executed. The list includes the blocks receiving the activation signal directly or indirectly through inheritance or the **ExtractActivation** block.

Each “distinct” activation source then has its own list of blocks and is treated independently of other activation sources. Even if two events produced by two “distinct” activation sources happen to have identical times, they are treated independent event. At run time, the two events are treated sequentially. Two “distinct” activation sources produce *asynchronous* activation signals. The notion of “distinct” activation sources however needs clarification. In general any output activation port on a **Activate** is considered a distinct activation source. There are however exceptions:

### 1.4.1 Conditional blocks

Two **Activate** blocks play a very special role in the construction of models. Strictly speaking conditional blocks are not **Activate** blocks but are graphically represented as such to facilitate their usage. To make an analogy with classical programming languages such as C where blocks represent function calls, conditional blocks represent the conditional constructs: **if** and **switch case** statements. The **IfThenElse** block has one activation input port and two activation output ports. Depending on the value of the signal on its regular input port, the “block” redirects its input activations to one of its output activation ports. In this case, the output activation signal is synchronous with the input activation signal. So the compiler cannot treat the output activation ports of the **IfThenElse** block as “distinct” activation sources.

The **SwitchCase** block is the other conditional block in **Activate**. This block may have more than two output activation ports and the input activation signal is redirected to one of them depending on the value of the regular input of the block at the time of execution.

### 1.4.2 Sample and Resample Clock blocks

The **SampleClock** and **ReSampleClock** blocks are not regular blocks either. These “blocks” produce a train of events with specified phase and period, just as the **EventClock** block does. The **EventClock** is essentially a Super Block containing an **EventDelay** with feedback. But unlike the **EventClock**, the activation signals produced by multiple **SampleClock** and **ReSampleClock** blocks may be synchronous. The compiler replaces the synchronous **SampleClock** and **ReSampleClock** blocks with a unique **EventClock** and subsampling mechanisms using conditional blocks to produce synchronous activation signals corresponding to the outputs of all the **SampleClock** and **ReSampleClock** blocks.

**SampleClock** and **ReSampleClock** blocks are considered synchronous throughout the model unless the model contains **SyncTag** blocks. If a diagram contains a **SyncTag** block, then the **SampleClock** and **ReSampleClock** blocks inside the diagram are not considered synchronous with **SampleClock** and **ReSampleClock** blocks outside the diagram (but they remain synchronous among themselves in the absence of other **SyncTag** blocks).

Note that conditional and Sample blocks may be hidden inside Super Blocks. In particular the **FrequencyDivider** and **EdgeTrigger** blocks in **Activate** palettes generate synchronous activation signals



because they include conditional blocks.



# Bibliography

- [1] D. Brown et al., Modelling the Luteinizing Hormone-Releasing Hormone Pulse Generator. Neuroscience, Vol 63, no 3, 1994.
- [2] S.L. Campbell, J.Ph. Chancelier and R. Nikoukhah, Modeling and Simulation in Scilab/Scicos with ScicosLab 4.4. Springer, 2010.



## Chapter 2

# Altair Activate Matrix Expression Block

### 2.1 Introduction

The Matrix Expression block is a **Activate** block with multiple inputs and a single output. The input/output function is expressed in terms of a matrix formula called *matrix expression*. This document specifies the syntax and the semantics of the expressions that can be used in this block as matrix expressions.

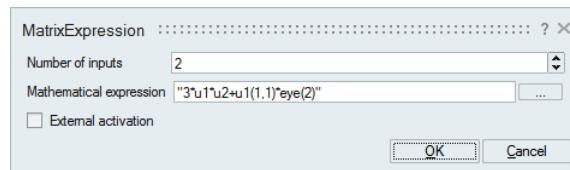


Figure 2.1: Matrix Expression Block

The inputs of the block are always denoted  $u_1, u_2, \dots, u_{<n>}$ , where  $n$  is the number of input ports. These variable names are keywords that take precedence in the matrix expression over any parameter or function with the same name defined in the context of the **Activate** diagram.

In addition to inputs, matrix expressions may include parameters. Unlike inputs, the parameters remain constant during simulation. Only parameters defined in the context of the **Activate** diagram can be used in the definition of the matrix expression. Inputs and parameters are matrices of arbitrary dimensions. Only real double matrices are currently supported.

A large set of matrix operators and functions can be used in a matrix expression. For the most part these matrix operators and functions follow **OML**'s syntax and semantics. For example the  $+$  and  $-$  operators are used to add and subtract matrices of the same size but they accept also operations between matrices and scalars. The  $*$  operator denotes the matrix multiplication but also the product of a scalar and a matrix. The  $\backslash$  and  $/$  operators denote left and right matrix divisions. If the multiplication and division operators are preceded with a dot then the operation is performed element-wise. For example  $A .* B$  denotes the operation that consists in multiplying each entry of the  $A$  matrix with the corresponding entry of the  $B$  matrix and producing a matrix of the same size as  $A$  and  $B$ .

**OML** operators for matrix constructions and extractions are also generally supported. There are some differences, for example the arguments of the **OML** column concatenation operator may be separated by

commas or spaces whereas only comma-separated arguments are supported in the matrix expression block (for example in the matrix expression block, the valid **OML** concatenation operation  $[A \ B]$  must be expressed as  $[A, B]$ , which is also a valid **OML** expression). In some cases the matrix expression block is less restrictive than **OML**.

Most mathematical functions such as  $\sin$ ,  $\cos$ ,  $\exp$ , accept matrix arguments in which case the function is applied to each entry of the matrix.

The list of these operators and supported functions are given in Sections 2.6 and 2.7.1. In addition to supported functions, any **OML** function available in the context of the diagram (both basic **OML** functions and user defined functions) can be used as long as their arguments do not depend on the values of the block inputs. These functions are evaluated once at compilation time. String arguments of such functions must be expressed with double quotes and not single quotes as in **OML**.

## 2.2 Examples

Matrix Expressions are similar to **OML** expressions with some restrictions and some extensions.

### 2.2.1 Basic operators and functions

**Matrix addition and multiplication** The expression

$u1' * u2$

is the matrix product of  $u1$  transpose and  $u2$ . This is a valid expression if  $u1$  and  $u2$  have the same number of rows, or if one is one-by-one, i.e., is a **scalar**.

The expression

$A * u1 + u2$

when the parameter  $A$  is defined in the context of the **Activate** diagram containing the block is valid if the dimensions of matrices  $A$ ,  $u1$  and  $u2$  are compatible. Note that the product  $A * u1$  is considered valid if the number of columns of  $A$  equals the number of rows of  $u1$ , or if  $A$  or  $u1$  is scalar. Similarly, the sum  $A * u1 + u2$  is valid if either both terms of the sum have identical dimensions, or one is scalar.

**Element-wise operators and functions** If matrix inputs  $u1$  and  $u2$  have equal size, the expression

$(u1 .* u2) ./ (u1.^2 + u2.^2)$

returns a matrix of the same size. The  $(i, j)$  entry of the result is equal to the product of the  $(i, j)$  entries of  $u1$  and  $u2$  divided by the sum of their squares.

The expression

$u1 > 3$

generates a matrix having the size of  $u1$  and including ones and zeros depending on whether the corresponding entry in  $u1$  is larger than 3 or not.

The expression

$\sin(u1)$

returns a matrix having the size of `u1` with entries equal to the sine of the corresponding entry in `u1`.

**If expression** Conditional statements in **OML** are not expressions and are not supported. However matrix expression supports expressional if statements, and in particular their extension to the matrix case, where the conditions are applied elementwise. The keywords `.if`, `.then` and `.else` are used in the expressional if statements. The condition and the values returned in the `.then` and `.else` branches are normally matrices of identical dimensions however a scalar can also be used in which case it is (conceptually) expanded to a matrix.

For example the expression:

```
.if u1<0 .then u1 .else 0 .end
```

returns a matrix equal to `u1` in which the negative entries are set to zero.

**Switch expression** Similarly the 'switch' clause is expressional and is extended to the matrix case. For example the expression:

```
.switch u1  
  .case a, u1  
  .case b, -u1  
  .otherwise 0  
.end
```

implements an element-wise switch expression.

## 2.2.2 Matrix construction, extraction and assignment

**Matrix construction** Commas (for column concatenation) and semi-colons (for row concatenation) are used to construct matrices from other matrices (or scalars which are just one-by-one matrices). This is similar to constructing matrices in **OML**. For example:

```
[u1,1;u2]
```

is valid provided `u1` has one row and that the number of columns of `u2` exceeds that of `u1` by one.

**Extracting a sub-matrix** A submatrix of a matrix can be specified by indicating the row and column indices to be extracted. For example

```
u1([1,3],2)
```

returns the two-by-one matrix `[u1(1,2);u1(3,2)]`.

The extraction operator can also be used for repeating rows and columns or changing their positions. For example if `u1` is a two-by-two matrix

```
u1([1,1,1,2,2,2],[1,2])
```

returns a six-by-two matrix where each column is repeated three times and the rows are switched.

Single index extraction is also possible:

```
u1(2:n)
```

The matrix `u1` is seen as a vector containing the entries of the matrix aligned column-wise so the result is a column vector, unless `u1` is a row vector in which case the output is a row vector.

**Special operators** The use of the keyword `end` and the symbol `' : '` simplify matrix expressions by allowing to refer to the last element or all the elements of a row or a column of a matrix without having to explicitly compute its size. For example

```
[u1 (1, end) ; u2 (end-1, : ) ]
```

returns the concatenation of the first row of `u1` with the one-before-the-last row of `u2`.

The `'end'` indicates the last row or column element depending on whether it is used in the first or second argument of the extraction operation, and `' : '` denotes all the elements of the row or column. In the case of single index extraction, `'end'` indicates the last element (last row element of the last column) and `' : '` all the elements of the matrix.

The symbol `'$'` may also be used in place of `'end'`.

**Matrix assignment and row column deletion** The `'='` sign is not used in the matrix expression since unlike in a **OML** script, an expression does not define intermediary variables. The matrix assignment operator `':='` however allows modifying entries of an existing matrix similar to the **OML** instruction `A (exp1, exp2) =exp3`. For example the following expression

```
u1 (:, 1) :=3
```

returns a matrix identical to `u1` with all the elements of its first column set to 3. The right hand side of the `':='` operator should have the same size as the submatrix designated by the left hand side, or be a scalar.

Row and columns can be deleted by assigning an empty matrix to them. For example

```
u1 (:, [1, 3]) :=[]
```

removes the first and third columns. Note that

```
u1 (:, u2) :=[]
```

is compiled but results in run-time error if any entry of `u2` becomes less than or equal to zero, or becomes bigger than the number of columns of `u1`. A run-time error is also generated if two entries of `u2` (casted into integers) become identical. The reason is that the size of the result would then change during simulation. This violates the fixed-sized matrix principle discussed later.

### 2.2.3 Use of OML functions

Functions other than those listed in Section 2.6 may be used if they are **OML** functions available in the context of the **Activate** diagram containing the Matrix Expression block. These **OML** functions can be used only if their arguments do not depend the block inputs; for example the expression

```
[u1 ; HammWin (L) ]
```

is valid (the argument of the Hamming Window function, `L` must be defined, for example in the context of the diagram). In this case, this parameter is available at compile time and its value does not change during the simulation, contrary to `u1`. Note however that even though `u1` may vary in time, its size



cannot. So

```
[u1;HammWin(size(u1,2))]
```

is also valid.

Only memoryless **OML** functions should be used because these functions are evaluated by **OML** only once. So for example the expression

```
UnifRnd()
```

does not produce a random signal during simulation but rather a random output that remains constant during the simulation, contrary to what the user may expect.

## 2.3 Parser

When the **Activate** diagram is compiled, the matrix expressions of the Matrix Expression blocks present in the model are parsed and converted into pseudo-codes used for evaluation during simulation<sup>1</sup>. The parser is similar to **OML**'s parser, in particular it uses the same rules of precedence, and allows for compositions of functions and operators. This is an important feature for the definition of matrix expressions for which it is not possible to define intermediary variables. For example the **OML** code

```
a=C*u1*u2  
=a(1,2)
```

can be expressed (as in **OML**) as a single expression as follows

```
(C*u1*u2)(1,2)
```

There is no limit as to the level of composition allowed, for example expressions such as

```
[inv([u1,u2])(1,:) *u2,u2(1)]'(2)
```

are valid.

Matrix expressions can be expressed over multiple lines. But line breaks and spaces are not separators: in particular spaces cannot be used as matrix column separators and line breaks cannot be used as row separators. The valid **OML** expression

```
[u1 2  
 1 3]
```

is not valid as a matrix expression and must be expressed as follows

```
[u1, 2;  
 1, 3]
```

---

<sup>1</sup>In the future, they will also be used for C code generation

## 2.4 Limitations

### 2.4.1 Fixed-sized matrix principle

Matrix sizes of all intermediary matrices obtained in the course of evaluating the matrix expression, and of course the final result, are determined at compile time so that no dynamic allocation is used during simulation. This places limitations on the types of expressions allowed. For example

```
[1:u1]
```

is not allowed since the size of the result depends on the value of `u1` (the first input), which may change during simulation.

In some cases, even if the the size of the result does not depend on an input, the compiler cannot determine the size. For example the following expression is not accepted

```
[u1(end)+1:u1(end)+5]
```

and has to be re-written as follows

```
u1(end)+[1:5]
```

This means in particular that the expression

```
u2(u1(end)+[1:5])
```

is valid (but may produce a run-time out of index error during simulation depending on the value of `u1(end)`).

### 2.4.2 All the branches of conditional expressions evaluated

The 'if' and 'switch' clauses are extended to the matrix case and applied element-wise, thus the output may depend on the expressions in multiple branches. That is why all the branch expressions are always evaluated. For the same reason, logical or (`|`) and logical and (`&`) operators are not short circuited.

### 2.4.3 No support for logical type

The logical data type is not supported so the output of the relational and logical operations are considered regular matrices (which happen to contain only zeros and ones). This means in particular that matrix extraction based on logical tests cannot be used. For example

```
u1(u1>3) :=3
```

cannot be used to select the values of `u1` that are larger than three and set them to 3. This can be done using the element-wise if expression

```
.if u1>3 .then 3 .else u1 .end
```

Note that in this simple example the expression can also be implemented as follows

```
min(u1,3)
```

## 2.5 Data types

### 2.5.1 Supported data type

The basic numerical data type used in the evaluation of a matrix expression is the matrix of real numbers coded as doubles (float64). The inputs, the output and all intermediary matrix evaluations during simulation use this data type. In particular, no distinction is made between a scalar and a one-by-one matrix.

Parameters defined in the diagram context can have any **OML** data types if in the matrix expression they are only used in the definition of the argument of a **OML** function that produces a matrix of real numbers. Strings can also be used explicitly in defining the expression under the same condition. For example the expression

```
f(1, "row") * u1
```

is valid provided the **OML** function  $f$  is defined in the context and returns a real matrix with proper size. Note the usage of double-quotes for defining a string. **OML** uses single quotes to define strings.

The reason for which **OML** data types can be used under the above condition is that **OML** functions and in general expressions that do not depend on block inputs are evaluated at compile time by **OML**. Such expressions are then replaced with the resulting matrices of real numbers in the matrix expression.

### 2.5.2 Unsupported data types

As previously stated, only matrices of real numbers are supported by supported functions. In particular it should be noted that there is no support for the logical data type. The output of logical and relational operators are considered matrices of real numbers containing zeros and ones. In a logical operation, zero is considered false and any non-zero number is true.

Currently complex numbers are not supported.

### 2.5.3 Data coding

The matrix data is stored in column-major format in memory. All the operations and supported functions use the IEEE standard for 64-bit floating point arithmetic. The block however generates an error if the result of the overall evaluation of the matrix expression, which is the output of the block, contains a NaN (Not a Number).

## 2.6 Supported operators

### 2.6.1 Arithmetic operators

The arithmetic operators available for matrix expressions are similar to those available in **OML**.

In the following tables,  $\langle \text{exp} \rangle$  denotes any valid expression.

Matrix algebra		
-	matrix subtraction	<exp>-<exp>
	negation	-<exp>
+	matrix addition	<exp>+<exp>
*	matrix multiplication	<exp>*<exp>
/	matrix right division	<exp>/<exp>
\	matrix left division	<exp>\<exp>
^	matrix power	<exp>^<exp>
'	matrix transpose	<exp>'

The second operand of the matrix power operation must evaluate to a scalar.

Element-wise matrix operations		
.*	element-wise multiplication	<exp>.*<exp>
./	element-wise left division	<exp>./<exp>
.\	element-wise right division	<exp>.\<exp>
.^	element-wise power	<exp>.^<exp>

The operand expressions must evaluate to matrices with compatible sizes. For matrix algebra operators, the compatibility follows the standard matrix operations rules. For element-wise operations, both operands must have identical dimensions. In both cases an operand may be scalar, in which case it is “expanded” to a matrix of proper size<sup>2</sup>.

## 2.6.2 Matrix construction and structuring

The following operators and functions can be used to construct matrices out of other matrices but also to construct special matrices. The size of the matrix cannot depend on the values of the inputs but may depend on their sizes since the sizes remain constant during simulation. Note that in some cases the compiler cannot determine that the size of the matrix does not depend on the values of the input; see Section 2.4.1.

Matrix construction		
[ ]	matrix constructor	[ <exp>, ..., <exp>;
,	column concatenation	..... ;
;	row concatenation	<exp>, ..., <exp>]
:	numeric sequence with step one	<exp>:<exp>
	numeric sequence (start:step:end)	<exp>:<exp>:<exp>
eye	identity matrix	eye (<exp>)
ones	matrix of ones	ones (<exp>, <exp>)
zeros	matrix of zeros	zeros (<exp>, <exp>)

An existing matrix can be manipulated using the following functions

<sup>2</sup>The scalar is not really expanded for efficiency reasons but the behavior of the operator is as if it did.

Matrix extraction and assignment		
( , )	sub-matrix extraction	<exp> (<exp>, <exp>)
()	vector extraction	<exp> (<exp>)
:	all rows	<exp> (:, <exp>)
	all columns	<exp> (<exp>, :)
	all elements	<exp> (:)
end	last row	<exp> (end, <exp>)
	last column	<exp> (<exp>, end)
	last element	<exp> (end)
:=	entry assignment	<exp> (<exp>) :=<exp>
	column assignment	<exp> (:, <exp>) :=<exp>
	row assignment	<exp> (<exp>, :) :=<exp>
	row deletion	<exp> (:, <exp>) := [ ]
	column deletion	<exp> (<exp>, :) := [ ]

### 2.6.3 Relational and logical operators

The relational operators can be used to compare two matrices of the same size or a matrix and a scalar. In the latter case, the elements of the matrix are each compared with the scalar. In both cases, the result is a matrix including ones and zeros.

Relational operators		
<	element-wise less than	<exp> < <exp>
>	element-wise greater than	<exp> > <exp>
<=	element-wise less than or equal	<exp> <= <exp>
>=	element-wise greater than or equal	<exp> >= <exp>
==	element-wise equal	<exp> == <exp>
~=	element-wise not equal	<exp> ~= <exp>
<>		<exp> <> <exp>

The logical operators produce ones and zeros but their arguments can be any number. The number zero is considered false and any other number, true.

Logical operators		
	element-wise logical or	<exp>   <exp>
&	element-wise logical and	<exp> & <exp>
~	element-wise logical not	~<exp>
and	logical and of all matrix elements	and (<exp>)
or	logical or of all matrix elements	or (<exp>)

## 2.6.4 Conditional operators

Conditional operators 'if' and 'switch' are extended to the matrix case. Note that expressions in all branches of conditional operators are evaluated; see Section 2.4.2.

**Element-wise if expression** The element-wise if expression operates on matrices and behaves like a standard 'if' expression for each entry of the matrix.

```
.if <cond_exp> .then <exp> .else <exp> .end
```

Normally all three expressions must have identical sizes. But one or two may be scalar. The result is a matrix with entries coming from the expressions in the 'then' and 'else' branches. In particular the  $(i, j)$  entry of the resulting matrix is equal to the  $(i, j)$  entry of the matrix obtained by evaluating the expression in the 'then' branch (or the value of the expression if it is a scalar) if the  $(i, j)$  entry of the matrix obtained by evaluating the condition expression `<cond_exp>` (or the value of the expression if it is a scalar) is not zero. Otherwise it is equal to the  $(i, j)$  entry of the matrix obtained by evaluating the expression in the 'else' branch (or the value of the expression if it is a scalar).

**Element-wise switch expression** The element-wise switch expression is an extension of the element-wise if expression. The resulting matrix is composed of the entries of the matrices corresponding to expressions returned in different cases.

```
.switch <cond_exp>
  .case <case_exp>, <exp>
  ...
  .case <case_exp>, <exp>
  .otherwise <exp>
.end
```

The  $(i, j)$  entry of the resulting matrix is equal to the  $(i, j)$  entry of the matrix obtained by evaluating the expression in the branch where the  $(i, j)$  entry of the matrix obtained by evaluating the case expression `<case_exp>` is equal to the  $(i, j)$  entry of the matrix obtained by evaluating the condition expression `<cond_exp>`.

In case this  $(i, j)$  entry matches more than one cases, the first case is taken into account. If it matches no case, the  $(i, j)$  entry of the resulting matrix is set to the  $(i, j)$  entry of the matrix obtained by evaluating the expression following `.otherwise`. The `.otherwise` "case" must always be present.

## 2.7 Supported functions

The syntax of a function call is

```
<fun> (<exp>, ..., <expr>)
```

The list of arguments may be empty.

## 2.7.1 Supported functions

Supported functions listed in this section can be used in matrix expressions. Their arguments can be any valid expressions. If all the arguments of a function do not depend on the values of the inputs, the function is evaluated only once at compile time.

### Element-wise functions

The following functions apply to matrices by applying the corresponding scalar function to each element of the matrix.

Trigonometric functions		
acos	inverse cosine	acos (<exp>)
acosh	inverse hyperbolic cosine	acosh (<exp>)
asin	inverse sine	asin (<exp>)
asinh	inverse hyperbolic sine	asinh (<exp>)
atan	inverse tangent	abs (<exp>)
atan2	four-quadrant inverse tangent	atan2 (<exp>, <exp>)
atanh	inverse hyperbolic tangent	atanh (<exp>)
cos	cosine	cos (<exp>)
cosh	hyperbolic cosine	cosh (<exp>)
hypot	hypotenuse	hypot (<exp>)
sin	sine	sin (<exp>)
sinh	hyperbolic sine	sinh (<exp>)
tan	tangent	tan (<exp>)

Rounding functions		
ceil	round up	ceil (<exp>)
floor	round down	floor (<exp>)
int	round toward zero	int (<exp>)
round	round to nearest integer	round (<exp>)

Other functions		
abs	absolute value	abs (<exp>)
exp	exponential	exp (<exp>)
log	natural logarithm	log (<exp>)
log10	base 10 logarithm	log10 (<exp>)
max	maximum	max (<exp>, <exp>)
min	minimum	min (<exp>, <exp>)
sign	sign (return -1,0 or 1)	sign (<exp>)
sqrt	square-root	sqrt (<exp>)

## Matrix functions

The following matrix functions are available

Matrix functions		
det	determinant of a square matrix	det (<exp>)
diag	create diagonal matrix from vector or vice-versa	diag (<exp>)
expm	matrix exponential of a square matrix	expm (<exp>)
inv	matrix inversion of a square matrix	inv (<exp>)
logm	matrix log of a square matrix	logm (<exp>)
max	maximum along rows unless argument is a row vector in which case maximum of all entries	max (<exp>)
min	minimum along rows unless argument is a row vector in which case minimum of all entries	min (<exp>)
size	vector including both matrix dimensions	size (<exp>)
	row dimension	size (<exp>, 1)
	column dimension	size (<exp>, 2)
sum	sum along rows unless argument is a row vector in which case sum of all entries	sum (<exp>)
	sum along rows	sum (<exp>, 1)
	sum along columns	sum (<exp>, 2)
svd	singular values	svd (<exp>)

### 2.7.2 Using OML functions

Any **OML** function available in the context of the **Activate** diagram containing the Matrix Expression block can be used in the definition of its matrix expression provided the arguments of the function do not depend on the values of the inputs. Note that `size (<exp>)`, `size (<exp>, 1)` and `size (<exp>, 2)` may be used in constructing the arguments of the **OML** function regardless of whether or not the argument of the size functions depend on the inputs. The reason is that the output of `size` does not depend on the value of its argument.

Unlike for supported functions, a block parameter defined in a diagram context hides a **OML** function if it has the same name.



## Chapter 3

# C and OML Custom Blocks in Altair Activate

**Activate** contains a large set of blocks organized in libraries and made available to the user through various palettes. These blocks provide basic mathematical functions needed to construct complex models of dynamical systems and also provide access to some input/output functionalities for files, graphical widgets, etc. In some cases however, it may be required or more convenient to express dynamical behavior of a block programmatically in **C** or in (**OML**) language , for example to take advantage of existing code or gaining access to OS functionalities in special ways. Constructing new **Activate** blocks is possible in **Activate** and the procedure is presented in the Block Builder documentation. An alternative to constructing a new block is using Custom blocks.

Custom blocks (**C** or **OML**) allow user to specify the dynamical behavior of the block in a block parameter. The actual simulation source code of the block (**C** or **OML**) or a reference to corresponding shared library or **OML** file becomes a parameter value, which is stored in the model where the block is used, and saved in the corresponding **scm** file. The model is thus self-contained -in other words, a library is not referenced by the block to define its behavior as is the case for most blocks. Indeed, **CCustomBlock** and **OmlCustomBlock** blocks are not new blocks.

So compared to creating and using new blocks, Custom blocks have the advantage of not requiring any library management operations. But even though their look and feel may be customized to some extent (in particular by encapsulating them in a masked Super Block) Custom blocks are not always the best solution. With Custom blocks, there is no mechanism for automatically upgrading the code of Custom blocks as there is for library blocks. Custom blocks are often used as a first step to develop and test **C** simulation codes destined for new blocks.

Both **CCustomBlock** and **OmlCustomBlock** blocks also provide the option of not including the source code of the simulation function. The simulation function of a **CCustomBlock** block may be provided by a shared library (dll file under Windows operating system). Instead of the **C** code, the block parameters include the path to the shared library and the name of the entry point. This option is particularly interesting for preserving intellectual property but note that the model is no longer self-contained. The shared library file must be distributed along with the model.

Similarly for **OmlCustomBlock** blocks the **OML** code does not need provided. An **OML** file defining the function and its name may used as block parameters instead. If a file is not specified **Activate** assumes that the function is already defined in the environment.

The **MathExpression** and **MatrixExpression** blocks should be considered before using Custom blocks since they provide even simpler ways to express the dynamical behavior of a block. They are however limited to expressing stateless dynamics: block outputs depend on block inputs alone, and the expressions support a fixed set of functions that cannot be extended by the user.

### 3.1 Activate block simulation function

A **Activate** block is defined by its graphical properties (specified in XML), an **OML** code for its instantiation and parameterization, and its simulation routines which can be in **C** or **OML**. The Block Builder tool provides a user interface for interactively designing a new block, in particular its graphics, and the **OML** code for its instantiation and parameterization. For using the **CCustomBlock** and **OML** custom blocks however users need not worry about these aspects; Custom blocks provide generic graphics and parameters allowing users to focus on developing the simulation function.

Block simulation functions are called by the **Activate** simulator during simulation to perform different tasks, such as updating the output or the state. Simulation functions have two arguments: `block` and `flag`. The task to be realized is specified by the argument `flag`. The argument `block` is the block's data structure to be passed to the APIs to get and set block data. Block simulation code may be in **C** or **OML**.

### 3.2 Custom block parameter GUI

**CCustomBlock** and **OmlCustomBlock** blocks' parameter GUIs provide an interactive user interface to define block properties. Users can define the number and types of inputs and outputs, the states and simulation parameters (if any) and other properties. The interface also gives access to an editor for writing the block's simulation code.

Developing a simulation code requires a knowledge of the APIs needed to get and set block data, for example for reading block inputs and writing block outputs. But Custom blocks provide a utility to generate a skeleton of the simulation code by taking into account the block properties specified by the user. This significantly facilitates the development of the simulation code.

Block inputs, outputs and simulation parameters are named and the code skeleton generator takes into account this information in order to provide a code where the user, in most cases, simply needs to add instructions (in **C** or **OML**) involving the provided names as program variables. These variables are already declared and initialized when needed.

The Custom block parameter GUI, illustrated in Fig. 3.1 (the **C** block; **OML** block is similar), could be considered as an interactive interface. It contains five tabs for customizing the properties of the block.

- Ports tab is used to specify the inputs and outputs of the block. For regular ports (not activation ports), the port sizes, types, feedthrough properties and names can in particular be specified.
- States tab is used to initialize block states, both discrete and continuous-time. Discrete object states *oz* may be typed and named.

If the simulation function is declared to be implicit in the SimFunctions tab, the initial value of the continuous-time state derivative should also be specified.

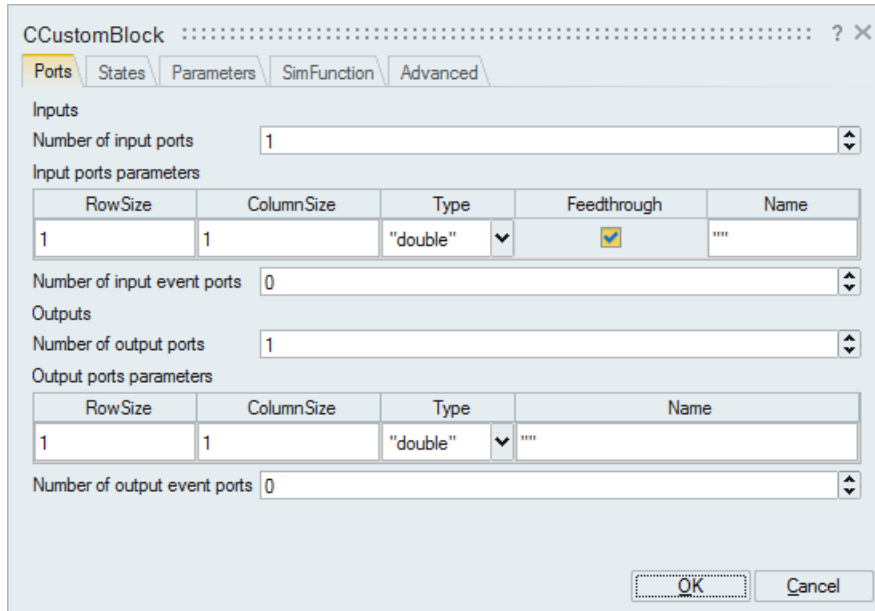


Figure 3.1: Block parameter GUI of **CCustomBlock** block.

- Parameters tab is used to set the value of *rpar* and *ipar* parameters; a list of *opar* parameters may also be defined, typed and named.

These parameters are simulation parameters that are available to the simulation function. The object parameters (*opar*) may be matrices of various sizes and data types (including complex and various integer data types). Parameters *rpar* and *ipar* are vectors of real and integer data types.

Unlike for the **CCustomBlock** block, the use of integer data types in an **OmiCustomBlock** block is not useful since all integer parameters are converted to doubles in the **OML** code. The **OML** language does not currently support integer data types.

- SimFunction tab is used to specify the simulation function of the block. The code may be inlined, in which case the interface can be used to define the simulation code in **C** for **CCustomBlock** blocks and in **OML** for **OmiCustomBlock** blocks. A skeleton of the code may be generated based on information provided in other tabs. See Fig. 3.2.

If the code is not inlined, then its location must be specified. In case of a **CCustomBlock** block, the path to the shared library and the name of entry point are given. In case of an **OmiCustomBlock** block, the path to an **OML** file and the name of the **OML** function are provided.

- Advanced tab may be used to define the number of zero-crossings and modes; the activation mode of the block is also defined in this tab. The activation mode options are *Always active*, *Initially active* and *Standard*. In *Standard* mode, the block is activated by inheritance if it does not contain any input activation ports.

In most cases, if the block does not contain a continuous-time state, the *Standard* mode should be used. On the other hand, if the block contains continuous-time states, the *Always active* mode is often more appropriate. *Initially active* blocs are rarely constructed as custom blocks.

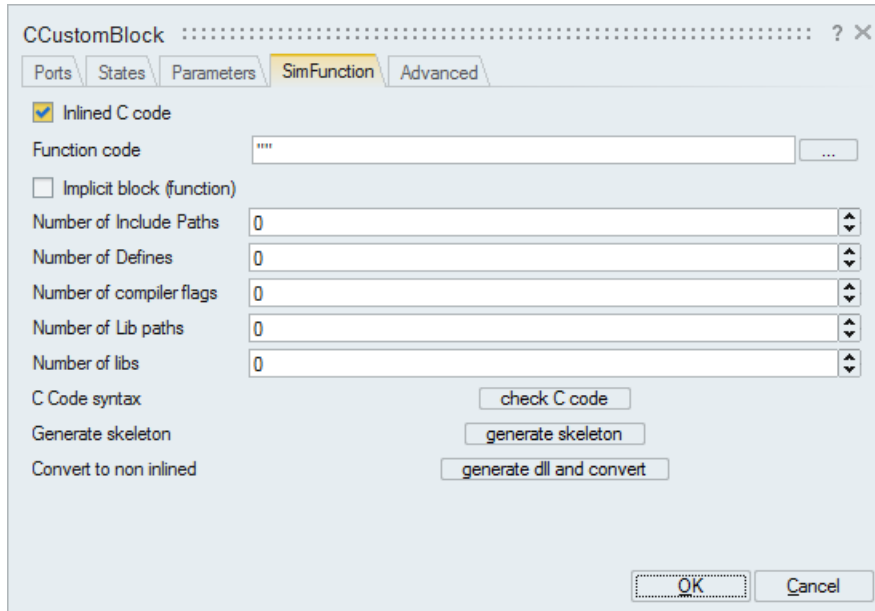


Figure 3.2: SimFunction tab used to specify the simulation function.

### 3.3 Examples

The usage of the Custom block parameter GUI is illustrated through a number of examples in this section.

#### 3.3.1 Median block

A block that computes the median of the entries of its input matrix is considered. This block is constructed using Custom blocks. This is a simple exercise since the block has no states and no parameters. It has one input and one output, both assumed to be of type double. The size of the input is set to  $[-1, -2]$  indicating that its size is arbitrary. The size of the output is  $[1, 1]$ . The names of the input and output are respectively set to `mat` and `med`.

Using a **OmlCustomBlock** block, generating the skeleton of the simulation code provides the following code. Note that the instruction computing the output is commented so that the simulation can be run without error even prior to the modification of the code. It should be uncommented when the code to compute the variable `med` is added.

**OML** contains already a function `Median` that computes the median of each column of the matrix argument as a vector. To use this function to implement the **Median** block, the input may be converted to a column vector using the column operator. The skeleton code can thus be edited as follows:

The code has been simplified by excluding the initialization and terminal phases. In this case the block does not perform any special operations in these phases.

The resulting block is tested in this model

```

OML Script Editor ..... X
[Icons]
1 function OmlBlockFunction(block, flag)
2   u1=vssGetInputData(block,1);
3   nevprt=vssGetEventCode(block);
4   if flag == vssBlockInitializeFlag
5
6   elseif flag == vssBlockReinitializeFlag
7
8   elseif flag == vssBlockTerminateFlag
9
10  elseif flag == vssBlockOutputUpdateFlag
11  %%vssSetOutputData(block,1,y1,vssGetOutputDataType(block,1));
12
13  end
14 end
15
[OK] [Cancel]

```

Figure 3.3: Skeleton of OML code

```

OML Script Editor ..... X
[Icons]
1 function OmlBlockFunction(block, flag)
2   u1=vssGetInPortData(block,1);
3   nevprt=vssGetEventCode(block);
4
5   if flag==vssBlockOutputUpdateFlag
6     med=median(u1(:));
7     vssSetOutPortData(block,1,med,vssGetOutputDataType(block,1));
8
9   end
10 end
11
[OK] [Cancel]

```

Figure 3.4: Edited Skeleton of OML code

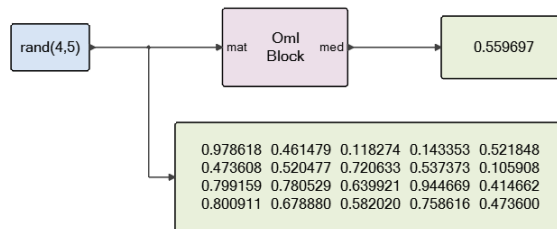
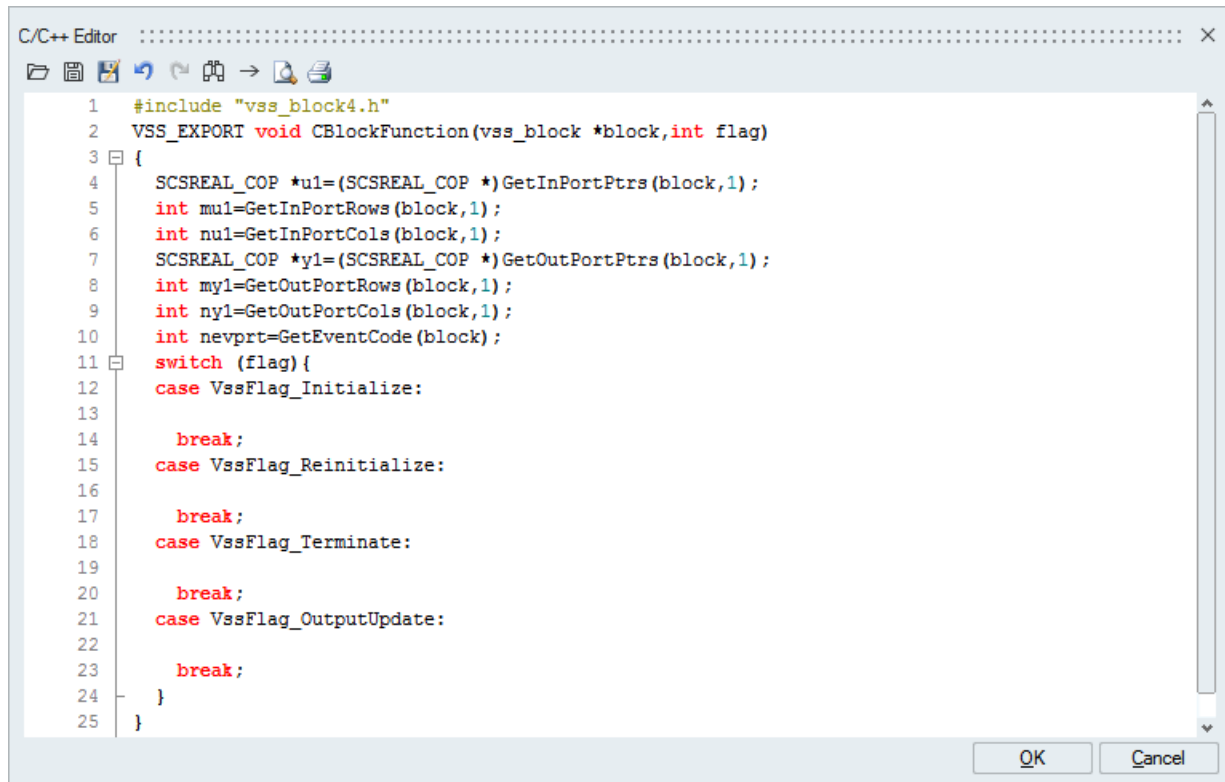


Figure 3.5: Median OML model (median\_OML.scm)

The **OmlCustomBlock** block may be encapsulated in a masked Super Block to hide its generic parameter GUI, making it look like a basic block. In this case the block has no parameter.

The **CCustomBlock** block may be used in a very similar way. Generating the skeleton of the simulation

code for the same example provides the following C code



```
1 #include "vss_block4.h"
2 VSS_EXPORT void CBlockFunction(vss_block *block,int flag)
3 {
4     SCSREAL_COP *u1=(SCSREAL_COP *)GetInPortPtrs(block,1);
5     int mu1=GetInPortRows(block,1);
6     int nu1=GetInPortCols(block,1);
7     SCSREAL_COP *y1=(SCSREAL_COP *)GetOutPortPtrs(block,1);
8     int my1=GetOutPortRows(block,1);
9     int ny1=GetOutPortCols(block,1);
10    int nevprt=GetEventCode(block);
11    switch (flag){
12        case VssFlag_Initialize:
13
14            break;
15        case VssFlag_Reinitialize:
16
17            break;
18        case VssFlag_Terminate:
19
20            break;
21        case VssFlag_OutputUpdate:
22
23            break;
24    }
25 }
```

Figure 3.6: Skeleton of C code

There are different algorithms for computing the median, see for example [1]. The public domain C code `torben.c` from [1] is used here to implement the block's simulation code. See Fig. 3.7. This code has the advantage of not allocating runtime memory and leaving the input matrix intact.

```

C/C++ Editor ..... X
1  typedef double elem_type ;
2
3  elem_type torben(elem_type m[], int n)
4  {
5      int      i, less, greater, equal;
6      elem_type  min, max, guess, maxltguess, mingtguess;
7
8      min = max = m[0] ;
9      for (i=1 ; i<n ; i++) {
10         if (m[i]<min) min=m[i];
11         if (m[i]>max) max=m[i];
12     }
13
14     while (1) {
15         guess = (min+max)/2;
16         less = 0; greater = 0; equal = 0;
17         maxltguess = min ;
18         mingtguess = max ;
19         for (i=0; i<n; i++) {
20             if (m[i]<guess) {
21                 less++;
22                 if (m[i]>maxltguess) maxltguess = m[i] ;
23             } else if (m[i]>guess) {
24                 greater++;
25                 if (m[i]<mingtguess) mingtguess = m[i] ;
26             } else equal++;
27         }
28         if (less <= (n+1)/2 && greater <= (n+1)/2) break ;
29         else if (less>greater) max = maxltguess ;
30         else min = mingtguess;
31     }
32     if (less >= (n+1)/2) return maxltguess;
33     else if (less+equal >= (n+1)/2) return guess;
34     else return mingtguess;
35 }
36
37 #include "vss_block4.h"
38 VSS_EXPORT void CBlockFunction(vss_block *block,int flag)
39 {
40     SCSREAL_COP *mat=(SCSREAL_COP *)GetInPortPtrs(block,1);
41     int mmat=GetInPortRows(block,1);
42     int nmat=GetInPortCols(block,1);
43     SCSREAL_COP *med=(SCSREAL_COP *)GetOutPortPtrs(block,1);
44     switch (flag){
45     case VssFlag_OutputUpdate:
46         med[0]=torben(mat, mmat*nmat);
47         break;
48     }
49 }
50

```

Figure 3.7: C simulation code of the Median block.

### 3.3.2 Variable discrete delay

A discrete delay block is considered where the amount of delay is variable and is given by an additional block input. Let  $u_1$  and  $u_2$  be the block inputs, then the output is given by

$$y(k) = u_1(k - u_2(k)),$$

The delay input  $u_2(k)$  is supposed to be receive values between 1 and  $N$ . But if  $u_2(k)$  is less than 1 then the delay is set to 1; if it is larger than  $N$  then it is set to  $N$ .

The block is implemented using a circular buffer of size  $N$  placed in the discrete state  $z$  of the block. An integer state  $oz(1)$  is used to track the position of the last entry of  $z$  used to store the latest value of the input.

The **CCustomBlock** block is used to implement the Variable delay block. The block has two inputs. For simplicity, we assume both are scalars but the extension to vector case is straightforward. The first input  $u_1$  is the delayed signal, which contrary to the second input representing the amount of delay, does not have feedthrough property. See Fig. 3.8 for the parameterization of the ports of the **CCustomBlock** block.

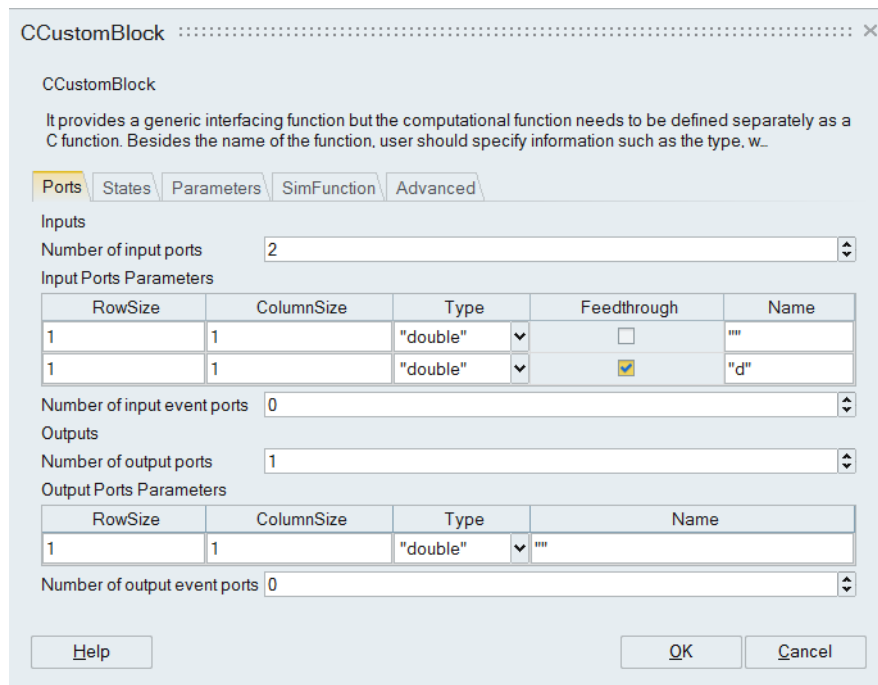


Figure 3.8: Parameterization of the ports of the Variable delay block.

The states are initialized as illustrated in Fig. 3.9. The buffer entries are initially set to zero and an integer state is used to indicate the current position in the buffer.

The block has no parameters and the simulation code is given in Fig. 3.10.

The following model is used to test the Variable delay block:



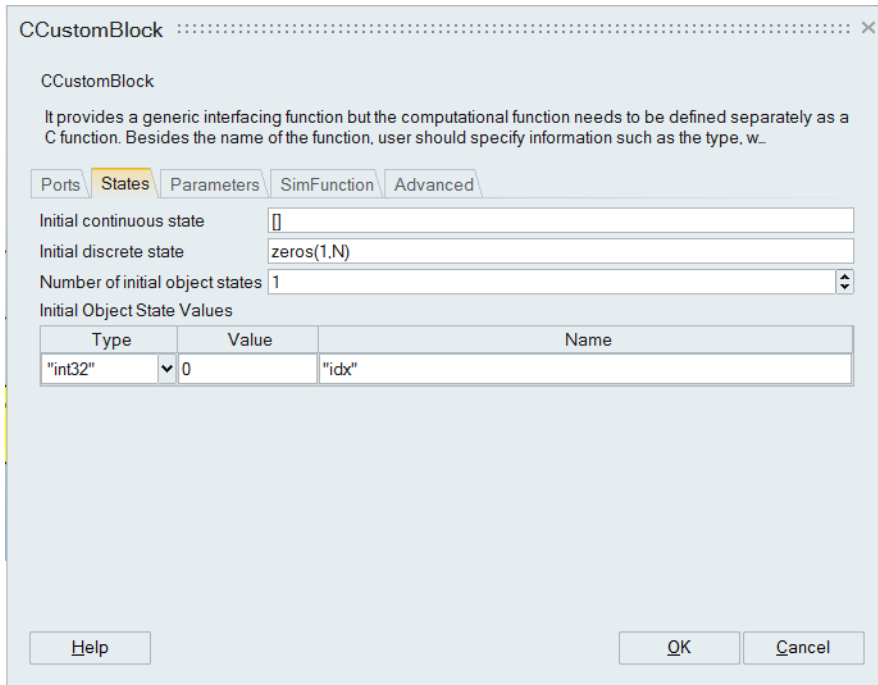


Figure 3.9: Parameterization of the states of the Variable delay block.

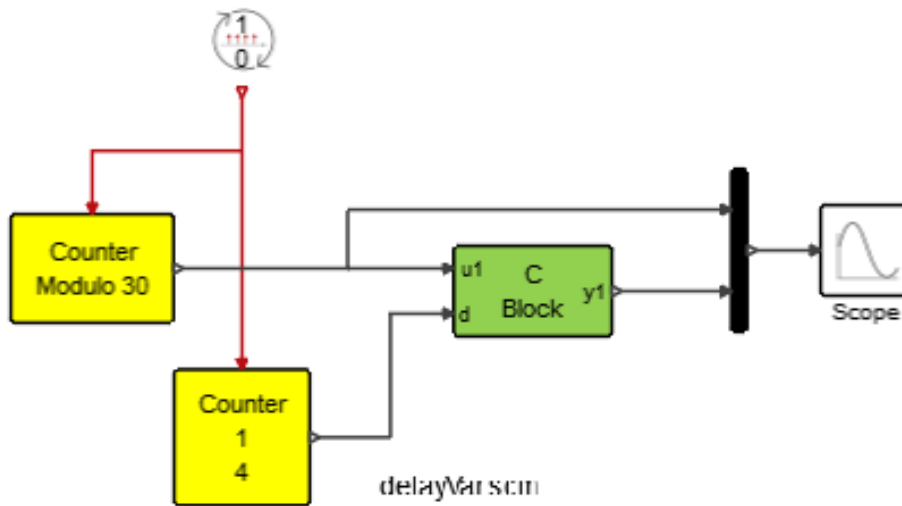


Figure 3.11: Variable Delay model (delayVar.scm)

The simulation result is given in Fig. 3.12.

```

C/C++ Editor ..... X
[Icons]
1  int mod(int a, int b)
2  {
3      int r = a % b;
4      return r < 0 ? r + b : r;
5  }
6
7  #include "vss_block4.h"
8  VSS_EXPORT void CBlockFunction(vss_block *block,int flag)
9  {
10     SCSREAL_COP *u1=(SCSREAL_COP *)GetInPortPtrs(block,1);
11     SCSREAL_COP *d=(SCSREAL_COP *)GetInPortPtrs(block,2);
12     SCSREAL_COP *y1=(SCSREAL_COP *)GetOutPortPtrs(block,1);
13     int nevpnt=GetNevin(block);
14     int nz=GetNdstate(block);
15     SCSREAL_COP *z=GetDstate(block);
16     SCSINT32_COP *idx=(SCSINT32_COP *)GetOzPtrs(block,1);
17     int i,n;
18     switch (flag){
19     case VssFlag_OutputUpdate:
20         n=(int) (*d-1);
21         if(n>nz-1) n=nz-1;
22         else if(n<0) n=0;
23         i=mod(*idx- n,nz);
24         y1[0]=z[i];
25         break;
26     case VssFlag_StateUpdate:
27         *idx = mod(*idx +1,nz);
28         z[*idx]=*u1;
29         break;
30     }
31 }
[OK] [Cancel]

```

Figure 3.10: Parameterization of the states of the Variable delay block.

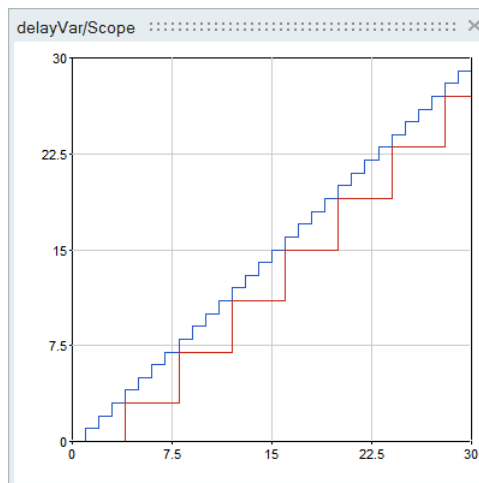


Figure 3.12: Simulation results show that the input signal is delayed differently at each step.

### 3.3.3 Fluid flow

The fluid flow between two tanks is modeled. The fluid levels in the tanks are denoted by  $h_1$  and  $h_2$  satisfying the following equations:

$$\begin{aligned} \dot{h}_1 &= -\frac{A_d}{A_t}V \\ \dot{h}_2 &= \frac{A_d}{A_t}V \end{aligned}$$

where  $V$  represents the speed of flow.  $A_t$  represents the cross-sectional areas of the tanks.  $A_d$  represents the cross-sectional area of the pipe connecting the two tanks:

$$A_d = \frac{\pi D^2}{4} \quad (3.1)$$

where  $D$  is the diameter of the pipe.

The speed  $V$  satisfies the Bernoulli equation:

$$h_1 + H = h_2 + f \frac{2L}{D} \frac{V^2}{2g} + \frac{V^2}{2g}$$

with  $H$  denoting the contribution of the pump.  $H = 0$  if the pump is off; when the pump is on:

$$H = 4 - 8.0 \times 10^4 Q^2. \quad (3.2)$$

The flow  $Q$  is given by the equation of continuity:

$$Q = A_d V. \quad (3.3)$$

The friction factor  $f$  satisfies one of following relations depending on the flow phase: turbulent, laminar or transitional:

$$\left\{ \begin{array}{ll} \frac{1}{\sqrt{f}} = -2 \log_{10} \left( \frac{e}{3.7} + \frac{2.51}{Re \sqrt{f}} \right) & \text{if } Re > Re_T \\ f = \frac{K_s}{Re} & \text{if } Re < Re_L \\ f = f_L + \frac{Re - Re_L}{Re_T - Re_L} (f_T - f_L) & \text{Otherwise} \end{array} \right.$$

The phase depends on the value of  $Re$ , the Reynold's number:

$$Re = \frac{DV\rho}{\mu} \quad (3.4)$$

This system cannot be readily expressed as a system of ordinary differential equations and must be modeled as an implicit system, i.e., a system of differential algebraic equations (DAE). Such systems may be modeled as **Activate** blocks, called implicit blocks. **C** and **OML** Custom Blocks can be used to implement implicit blocks.

The difference between an explicit block and implicit block is that in an explicit block the continuous-time dynamical behavior is an explicit system

$$\begin{aligned} \dot{x} &= F(t, x, u) \\ y &= G(t, x, u) \end{aligned}$$

Here  $u$  and  $y$  represent respectively the inputs and the outputs of the block. The block simulation code provides the functions  $F$  and  $G$ .

The continuous-time dynamical behavior of an implicit block is

$$\begin{aligned} 0 &= E(t, x, \dot{x}, u) \\ y &= H(t, x, \dot{x}, u) \end{aligned} \quad (3.5)$$

The block simulation code provides the functions  $E$  and  $H$ . Unlike the explicit case where the simulation code must compute  $\dot{x}$ , in the implicit case the code computes a residual, which may depend on  $\dot{x}$ .

Since algebraic relationships may be included in (3.5), it is possible to include all system equations in (3.5). In this case, the size of the state  $x$  (so the size of the residual) would be equal to the number of equations in the system. But for better performance and precision, it is better to reduce the size of residual by computing explicitly system variables, when possible, and thus removing as many algebraic equations as possible. Here in particular instead of including Equations (3.1), (3.2), (3.3) and (3.4) in (3.5),  $A_d$ ,  $H$ ,  $Q$  and  $R_e$  are computed explicitly and the state of the block is reduced to

$$x = \begin{pmatrix} h_1 \\ h_2 \\ V \\ f \end{pmatrix}.$$

The **C** simulation code in **CCustomBlock** block for this example is given in Fig. 3.13.

All system parameters are defined as block parameters, so they can be modified without having to edit the **C** code. The block has one input (`ump`) taking values 0 and 1 indicating respectively that the pump is off or on. The block outputs are the fluid levels in the tanks.

Note that in the **CCustomBlock** block parameter GUI, the **Implicit block** property checkbox must be checked, the initial state specified and the **Activation mode** be set to **Always Active mode**.

The following model is used to test the two-tank block:

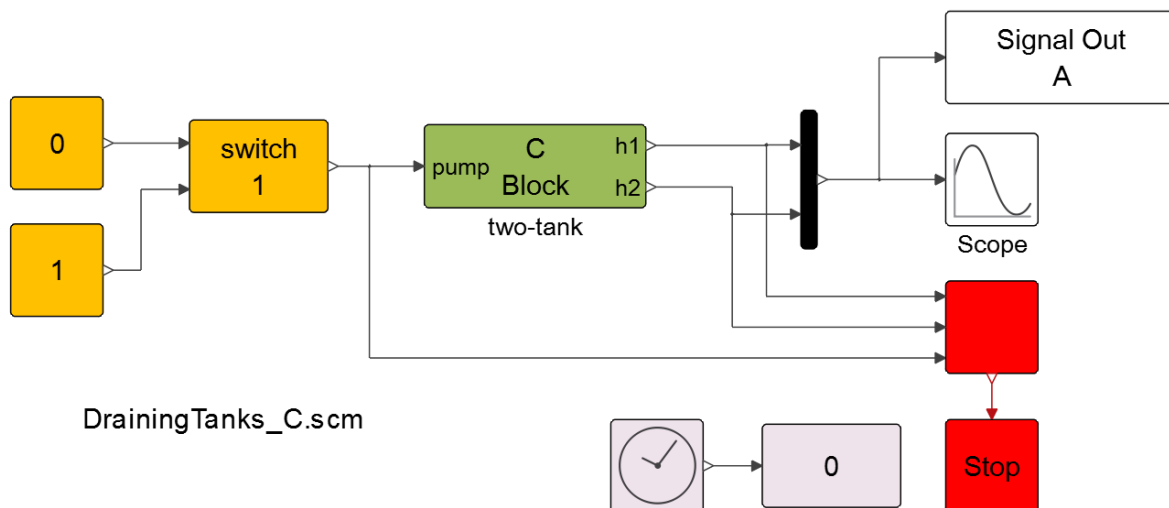


Figure 3.14: DrainingTanks C model (DrainingTanks\_C.scm)

```

C/C++ Editor .....
1 #include "vss_block4.h"
2 VSS_EXPORT void CBlockFunction(vss_block *block,int flag)
3 {
4     SCSREAL_COP pump=(SCSREAL_COP *)GetInPortPtrs(block,1);
5     SCSREAL_COP *h1=(SCSREAL_COP *)GetOutPortPtrs(block,1);
6     SCSREAL_COP *h2=(SCSREAL_COP *)GetOutPortPtrs(block,2);
7     SCSREAL_COP *x=GetState(block);
8     SCSREAL_COP *xd=GetDerState(block);
9     SCSREAL_COP *res=GetResState(block);
10    SCSREAL_COP L=(SCSREAL_COP *)GetOparPtrs(block,1);
11    SCSREAL_COP D=(SCSREAL_COP *)GetOparPtrs(block,2);
12    SCSREAL_COP rho=(SCSREAL_COP *)GetOparPtrs(block,3);
13    SCSREAL_COP mu=(SCSREAL_COP *)GetOparPtrs(block,4);
14    SCSREAL_COP At=(SCSREAL_COP *)GetOparPtrs(block,5);
15    SCSREAL_COP e=(SCSREAL_COP *)GetOparPtrs(block,6);
16    SCSREAL_COP Ks=(SCSREAL_COP *)GetOparPtrs(block,7);
17    SCSREAL_COP ReL=(SCSREAL_COP *)GetOparPtrs(block,8);
18    SCSREAL_COP ReT=(SCSREAL_COP *)GetOparPtrs(block,9);
19    SCSREAL_COP fT=(SCSREAL_COP *)GetOparPtrs(block,10);
20    SCSREAL_COP pi=3.14, g=9.81, V, f, Q, H, Re, Ad, fL;
21    switch (flag){
22        case VssFlag_OutputUpdate:
23            *h1=x[0];
24            *h2=x[1];
25            break;
26        case VssFlag_Derivatives:
27            Ad=D*pi/4;fL=Ks/ReL;
28            V=x[2];f=x[3];
29            Q=pi*V*D*D/4;
30            H=pump*(4-8.0e4*Q*Q);
31            Re=D*V*rho/mu;
32            res[0]=x[1]+f*2*L*V*V/(D*2*g)+V*V/(2*g)-x[0]-H;
33            res[1]=xd[0]+V*Ad/At;
34            res[2]=xd[1]-V*Ad/At;
35            if (Re>ReT) res[3]=1/sqrt(f)+2*log10(e/3.7+2.51/(Re*sqrt(f)));
36            else if (Re<ReL) res[3]=f-Ks/Re;
37            else res[3]=fL+(fT-fL)*(Re-ReL)/(ReT-ReL)-f;
38            break;
39        }
40    }
41
OK Cancel

```

Figure 3.13: C simulation code of the two-tank block.

The pump may be turned on and off by double clicking on the **Switch** block. With the pump off, the simulation stops when the fluid levels are equal and the simulation time is indicated in the **Display** block.

The simulation results for both the case the pump is on and off are given in Fig. 3.15:

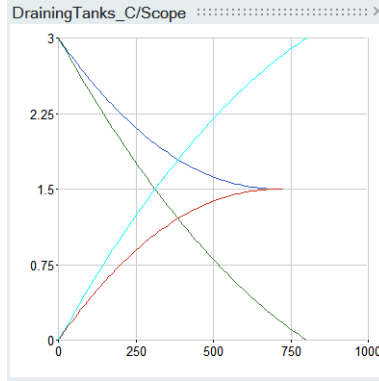


Figure 3.15: Simulation results:  $h_1$  and  $h_2$  with pump off and on.

### 3.3.4 Simplified explicit automaton block

In chapter 16, the implementation of hybrid automata in **Activate** has been illustrated. The Automaton block<sup>1</sup> is an implicit block which allows modeling general hybrid automata with Differential algebraic equations (DAE). In this section we try to demonstrate the way a simplified Automation block for explicit hybrid automata with ordinary differential equation (ODE) is built. The sticky balls example (16.3.3) will be implemented using **CCustomBlock** block with the new explicit automation block.

As explained in (16.3.3), the sticky balls model works in two modes, stuck and separate. In each mode the dynamical model is described as an ODE of size four (see 16.11, 16.12, and 16.19). In order to simulate the sticky balls models, the **CCustomBlock** block contains the state vector and outputs it. The state vector, in separate mode and stuck mode is  $[x_1, x_2, v_1, v_2]$  and  $[x_3, v_3, s, q]$ , respectively. The block outputs the state vector at the second output port whatever the mode is. The derivative of the states should be computed as a function of states and given back to the **CCustomBlock** block as input. The derivative of state in separate mode is fed to the first input and that of the stuck is given to the second input. During the mode transition, the initial value of state vector should be given to **CCustomBlock** block to reinitialize the state vector. The initial value of states are concatenated with the state derivative of the destination mode. In order to detect a mode transition, a zero-crossing function is used in each mode. In the separate mode, the zero-crossing function in separate and stuck modes are 16.18 and 16.20, respectively. The zero-crossing functions are concatenated with the state-derivative and initial vector for each mode. The overall model of the sticky balls are shown in Fig. 3.16.

The first output of the **CCustomBlock** block contains the current mode and the second output is the state vector.

See Fig. 3.17 for the parameterization of the ports of the **CCustomBlock** block.

The states are initialized as illustrated in Fig. 3.9.

The block has two integer parameters, i.e., number of modes of the automaton, the initial mode. There are also two object parameters of type integer, i.e., the destination modes, when the zero-crossing happens. In the case of sticky balls model, the destination modes is simple, when the mode is One, the destination mode when a zero-crossing happens, is Two and when the mode is Two the destination mode is One. The parametrization is given in Fig. 3.19.

The number of zero-crossing function is one and the block has no internal mode for automatic handling

<sup>1</sup> Available in the Hybrid palette.

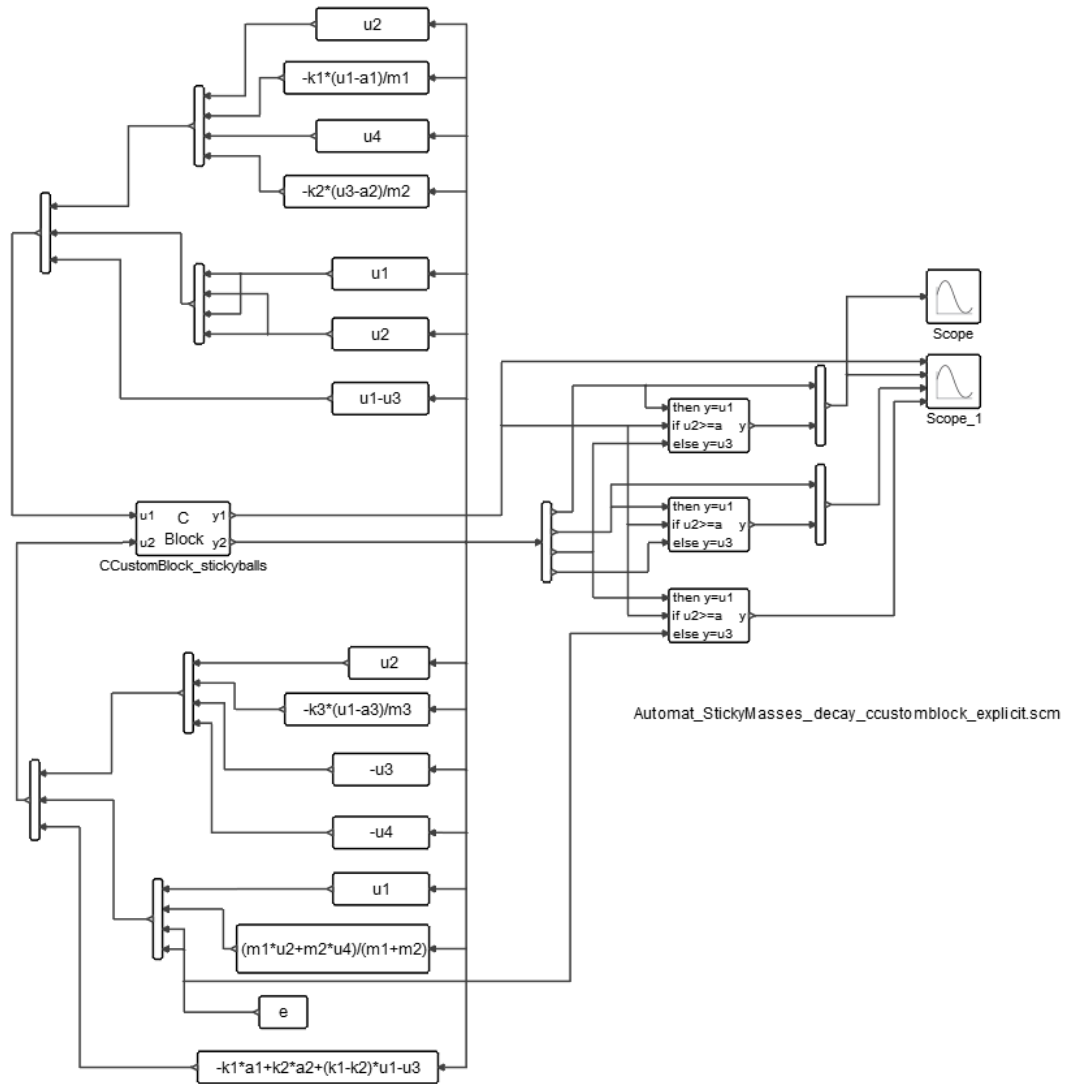


Figure 3.16: Complete model of sticky masses  
(Automat\_StickyMasses\_decay\_ccustomblock\_explicit.scm)

the zero-crossing. The **C** simulation code in **CCustomBlock** block for this example follows:

```
#include "vss_block4.h"

typedef struct {
int cMode, pMode;
} automat_t;

#define WD ((automat_t *) GetWorkPtrs(block))
#define _cMode (WD->cMode)
#define _pMode (WD->pMode)

VSS_EXPORT void CBlockFunction(vss_block *block,int flag)
{
double * y0, *y1, *ui;
double* g=GetGPtrs(block);
double* x=GetState(block);
double* xd=GetDerState(block);
double* evout= GetNevOutPtrs(block);
```

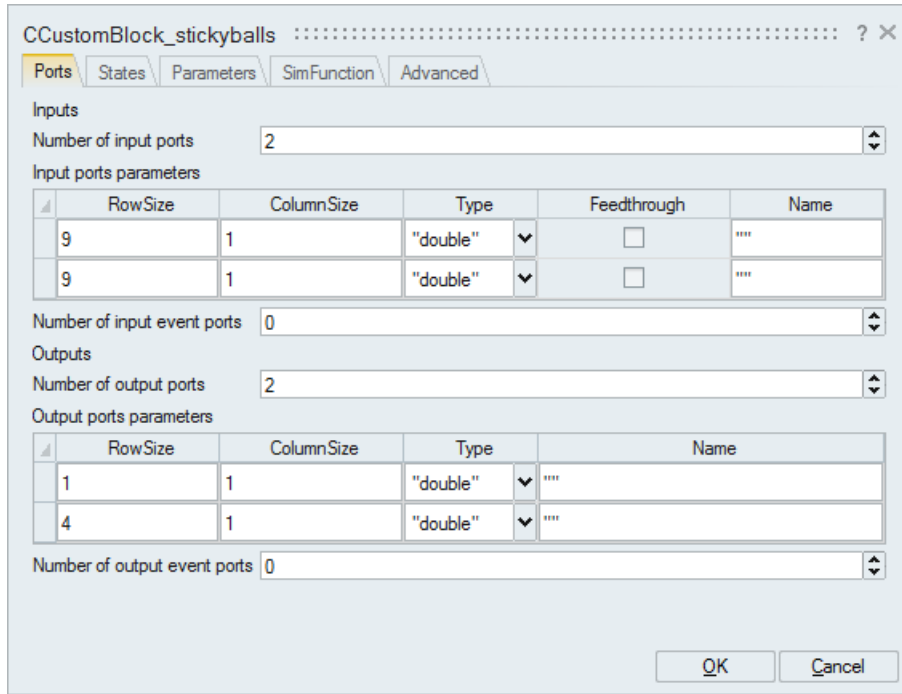


Figure 3.17: Parameterization of the ports of the explicit automaton block for sticky balls model.

```

int* ipar=GetIparPtrs(block);
double* rpar=GetRparPtrs(block);
int* jroot=GetJrootPtrs(block);
int ng=GetNg(block);
int NX=GetNstate(block);
int* tmodev, NMode, Minitial,j,k;
double time=GetVssTime(block);

NMode=ipar[0];
Minitial=ipar[1];

switch(flag)
{
case VssFlag_Initialize:
{
if ((GetWorkPtrs(block)=(automat_t*) vss_malloc(block,sizeof(automat_t)))== NULL )
{ SetBlockError(block,1); return; }
_pMode=_cMode=Minitial;
}
break;
case VssFlag_OutputUpdate:
{
y0=GetRealOutPortPtrs(block,1);
y1=GetRealOutPortPtrs(block,2);
y0[0]=(double) _cMode;
memcpy(y1,x,sizeof(double)*NX); y1+=NX;
}
break;

case VssFlag_Derivatives:
{
ui=GetRealInPortPtrs(block,_cMode);
memcpy(xd,ui,sizeof(double)*NX);
}
break;
case VssFlag_ZeroCrossings:

```



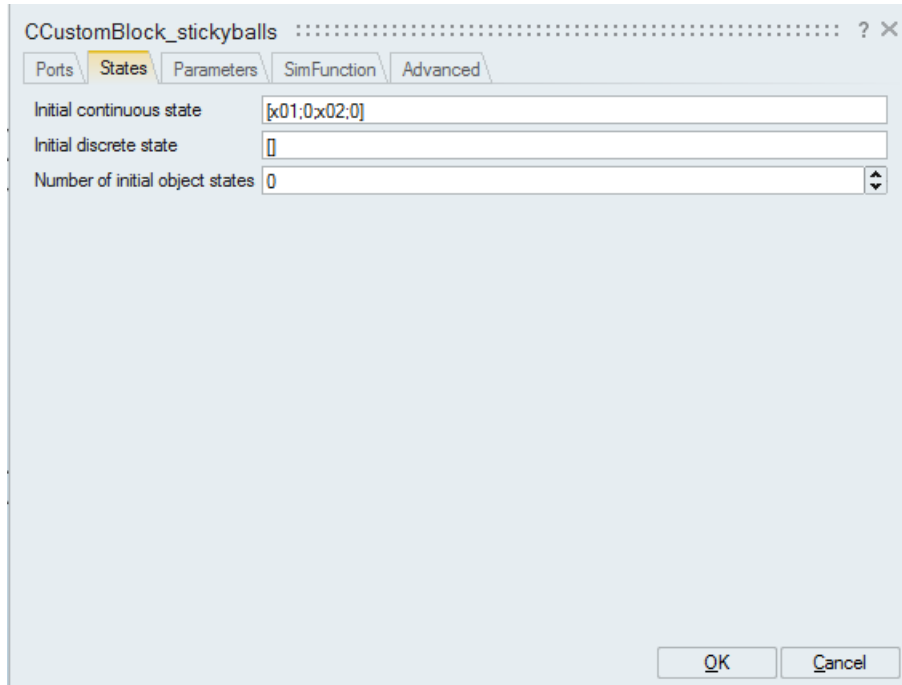


Figure 3.18: Parameterization of the states of the explicit automaton block for sticky balls model.

```

{
ui=GetRealInPortPtrs(block,_cMode);
for (j=0;j<ng;j++) g[j]=0;
for (j=0;j<GetInPortRows(block,_cMode)-2*NX;j++) {
g[j]=ui[j+2*NX];
}
}
break;

case VssFlag_StateUpdate:
{
if (!( isZeroCrossing(block)
|| GetSimulationPhase(block)==PHASE_DISCRETE
|| GetSimulationPhase(block)==PHASE_MESHPOINT )) return;

tmodev=GetInt32OparPtrs(block,_cMode);
ui=GetRealInPortPtrs(block,_cMode);

for (k=0;k<GetInPortRows(block,_cMode)-2*NX;k++) {
ui=GetRealInPortPtrs(block,_cMode);
if( ( isZeroCrossing(block) && jroot[k]==1) ||
(GetNevIn(block)==1 && ui[k+2*NX]>0 ) ||
(GetSimulationPhase(block)==PHASE_DISCRETE && ui[k+2*NX]>0) ||
(GetSimulationPhase(block)==PHASE_MESHPOINT && ui[k+2*NX]>0)
) {
_pMode=_cMode;
_cMode=tmodev[k];
ui=GetRealInPortPtrs(block,_cMode);ui+=NX;
memcpy(x,ui,sizeof(double)*NX);
DoColdRestart(block);
break;
}
}
}
break;
default:

```

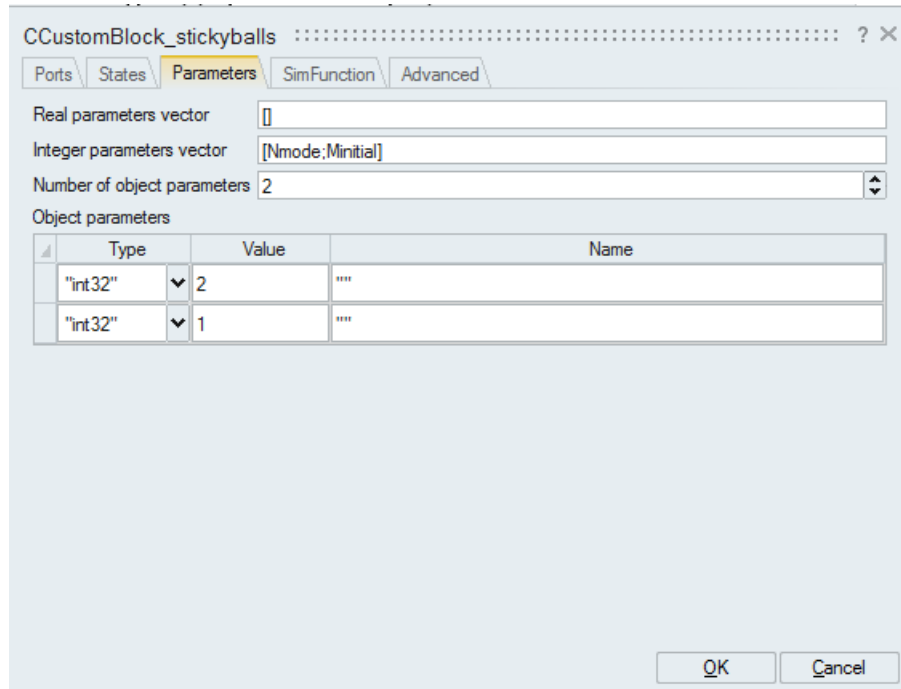


Figure 3.19: Parameterization of the parameters of the explicit automaton block for sticky balls model.

```
break;
}
}
```

In order to hold the current and previous modes, they are put into a structure and stored in the block's internal workspace. The workspace is allocated and initialized in `VssFlag_Initialize` flag using block integer parameters.

In the `VssFlag_OutputUpdate` flag, the outputs are computed. The first output is straightforward and in order to compute the second output, the state vector `x=GetState(block)` is copied into the output using the `C memcopy` command. The size of the state vector is obtained using `nx=GetNstate(block)`.

In the `VssFlag_Derivatives` flag, first, based on the current mode `_cMode` the pointer to the appropriate input port is grabbed, then the first `nx` elements are copied into the state-derivative vector (`xd=GetDerState(block)`).

In the `VssFlag_ZeroCrossings` flag, first, based on the current mode `_cMode` the pointer to the appropriate input port is grabbed. The first `2nx` elements of the input port are state derivative and initial value of states the next `nz=GetNg(block)` elements are the zero-crossing functions. Hence, these elements are copied into the state-derivative vector (`g=GetGPtrs(block)`).

In the `VssFlag_StateUpdate` flag, the mode transition is performed. The mode can change at zero-crossing instants or at non-try simulator points such as discrete events or simulator meshpoints. At other instants, the call to the `VssFlag_StateUpdate` flag is ignored. If a valid transition happens, the zero-crossing surface of the next mode will be positive. Based on this, the positive zero-crossing is found and the corresponding mode is found. Then the internal state of the **CCustomBlock** block is updated.

The simulation result of the model 3.16 is given in fig. 3.20.

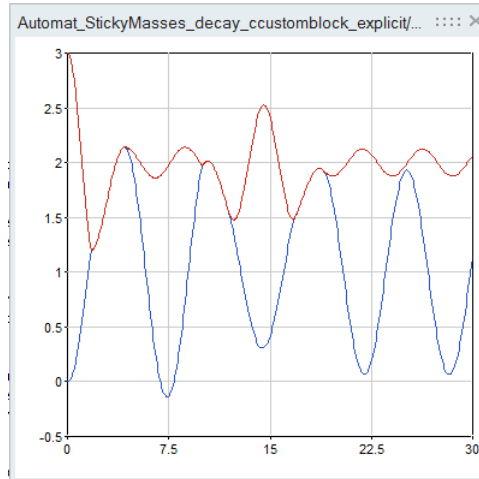


Figure 3.20: Simulation result of the model 3.16.

### 3.4 Non-inlined simulation functions

The **C** and **OML** functions used in custom blocks need not necessarily be provided in the block (be inlined). They can be provided in a shared library (dll file under Windows) for **CCustomBlock** blocks or **OML** files for **OmlCustomBlock** blocks. The choice of inlining the function is made using a checkbox in the SimFunction tab. For example in Fig. 3.21 the simulation function of the **CCustomBlock** block is defined in the file **foolib.dll**. Since the path of the file is not specified, the file is supposed be in the same directory as the model.

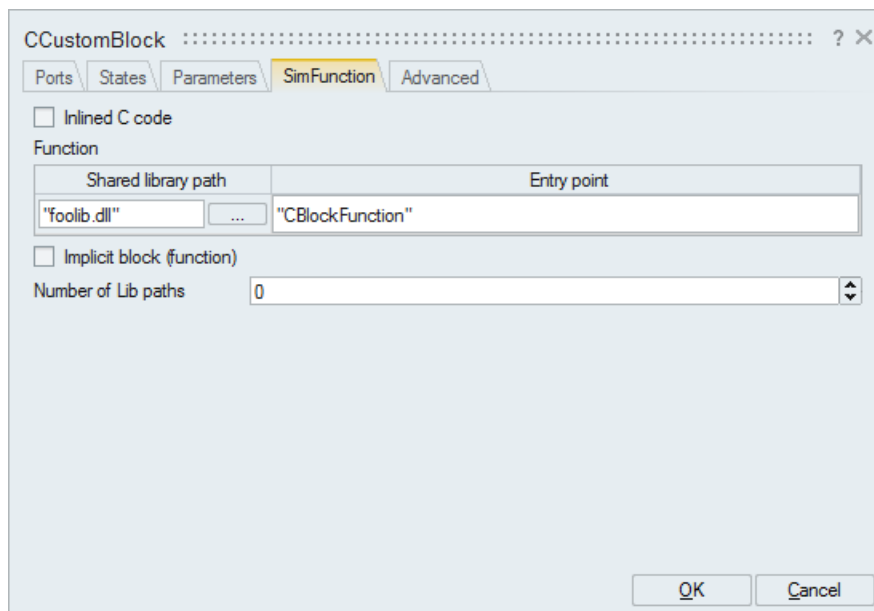


Figure 3.21: SimFunction tab used to specify the simulation function. In this case the function is not inlined.

A convenient way to create a **CCustomBlock** block without inlined code is first to create one with an inlined code. This way, the skeleton generator facility can be used and the code can be tested.

Moreover, the shared library is automatically created by **Activate**. Once the code is validated, the block may use the shared library instead of the inlined code. This can be done automatically using the convert to non inlined facility. In this case the inlined checkbox is unchecked and the library information is provided in the SimFunction tab. Note that the “Function code” becomes an invisible parameter but its content, the original **C** code, can be retrieved by marking the inlined checkbox. So if the objective is to hide the code (in particular for IP protection), make sure to erase the content of the “Function code” parameter.

If the shared library of the non-inlined **CCustomBlock** block has dependency on other shared libraries, then the location of these libraries must be provided. These libraries may include both external dll files that must accompany the model and **Activate** dll files already available in the software distribution. In particular if the **CCustomBlock** block is created using the **Activate** code generation facility, the libraries associated with the blocks used in the diagram are included in the list of libraries.

# Bibliography

- [1] Nicolas Devillard, Fast median search: an ANSI C implementation, 1998.  
<http://ndevilla.free.fr/median/median/>.



## Chapter 4

# Simulation restarts: implementing iterations

In some applications, a single time simulation does not provide a solution to the problem. This is particularly the case where multiple simulations are needed to solve problems such as parameter fitting, solving nonlinear systems, optimizing, etc. Time simulation iterations in **Activate** are implemented through the **End** block, which can either stop the simulation for good, or stop and restart a new time simulation. These actions are controlled by activation signals in **Activate**.

Activation signals represent the key element in **Activate** formalism. They determine in particular when, how and in what order the block simulation functions are called during simulation. The ability to explicitly manipulate Activation signals, gives users modeling capabilities that are seldom present in other modeling and simulation tools.

The use of Activation signals go beyond the framework of a single simulation run. Within the formalism of **Activate**, there exists the possibility of creating models where their simulations lead to multiple time simulations. Indeed in **Activate** an activation signal can end or restart a time simulation. This ability to conditionally restart simulations is particularly useful for parameter optimization, strategy assessment, Monte Carlo simulation, etc. Traditionally, such applications are developed using scripts calling the simulator in batch mode. By integrating the complete formulation within the simulation model (which actually is a misnomer since the model contains more than a simulation model), the implementation becomes more concise and easy to maintain. There is of course always the possibility to use **OML** scripts to code the optimization part and run **Activate** simulations in batch mode.

To illustrate possible applications of conditional restart in **Activate**, an example is presented here. In particular the implementation of the shooting method for solving boundary value systems is considered. Boundary value systems are for example obtained when solving trajectory optimization problems. A boundary value system is often a system of ODEs defined over a time interval  $[T_i, T_f]$  where the states of the system are not fully specified at the initial time of the interval. The problem is well posed thanks to conditions specified on the states at the final time of the interval.

The simplest implementation of the shooting method for solving boundary value systems consists of guessing values for the missing states of the system at initial time, simulating the system up to the final time, comparing the final states with the constraints imposed on them and based on this comparison generate new guess values for the initial states. The iterations are continued until the final states satisfy the constraints up to desired precision.

To implement the shooting method in **Activate**, it is then necessary to model the system and implement a mechanism to compute new guess values and, until the desired precision is not achieved, to restart the simulation after changing the initial conditions. The system modeling is done as usual without any special consideration for the shooting method. The system model should provide a function of the final state of the system representing the constraints (the value of the function is zero if the constraints are satisfied).

For well-posed boundary value problems, the number of free initial states is equal to the number of final constraints. Let  $p$  denote the subset of initial state which are free and  $f$  the final constraint function, i.e.,

$$f(x(T_f)) = 0 \quad (4.1)$$

where  $x$  represents the states of the system. The final state  $x(T_f)$  depends of course on the initial state  $x(T_i)$  and in particular on  $p$ :  $x(T_f) = H(T_f, p)$ . The function  $H$  can only be evaluated numerically by simulating the system from  $T_i$  to  $T_f$ . For a given  $T_f$ , the constraint (4.1) can be expressed as

$$f(x(T_f)) = f(H(T_f, p)) = g(p) = 0.$$

The solution  $p$  of the nonlinear system  $g(p) = 0$ , which has as many equations as unknowns, can be obtained using various methods. For example the **OML** function `fsolve` is provided specifically for solving such problems. This function can be used in an **OML** script to solve the boundary value problem by running the simulation model programmatically. The objective however here is to include the solution of the problem within the model, obtaining a self contained formulation of the problem in terms of an **Activate** diagram without any dependency on external code. So a nonlinear solver similar to `fsolve` must be made available in **Activate**, as a block.

## 4.1 Nonlinear solver block

A nonlinear solver block cannot function the way `fsolve` does. In `fsolve`, the nonlinear solver iterates by calling an external function provided by the user evaluating the function  $g(p)$  for the given  $p$  until an end condition is satisfied, in which case the function returns the solution  $p$ , or an error message. `fsolve` is the master and the function evaluating  $g(p)$  the slave. This master-slave configuration is not appropriate for implementing a solver block in **Activate**. In **Activate**, the **Activate** simulator is the master and the blocks are slaves. For that, the solver algorithm must be implemented in reverse-communication mode. Many nonlinear solver and optimization codes are available both in normal and reverse-communication modes.

### 4.1.1 A reverse-communication solver

In reverse-communication mode, the solver is provided with the values of  $p$  and  $g(p)$  and returns a new value of  $p$  indicating also whether this value is the solution or a value to be used for the next iteration. If we make the assumption that the master calls back systematically the solver function with  $p$  provided by solver at the previous iteration, then it is fairly straightforward to reprogram any solver in reverse-communication mode. The structure of the program must be changed so that the internal state of the solver is maintained from one call to the next. The solver algorithm however is not modified.

If the solver function is not recalled with the value it returns, then specific reverse-communication algorithms are required. This is the case considered here for the implementation of a nonlinear solver



block. The function used here is a simple nonlinear solver implemented as toy code to illustrate the approach. The internal state of the function contains a number of previous  $(p, g(p))$  pairs and a weight function associated with each one. At each iteration, the new pair  $(p, g(p))$  replaces the “least interesting” (having the smallest weight) pair and the eights are updated. The weight associated to a pair  $(p, g(p))$  depends on the norm of  $g(p)$  and how long ago the pair has been received. The weight is reduced at every iteration.

Let  $p$  be close to the solution  $p^*$  satisfying  $g(p^*) = 0$ , then by neglecting higher order terms,

$$g(p) = -J(p - p^*) \quad (4.2)$$

where  $J = \frac{\partial g}{\partial p}(p^*)$  is the Jacobian matrix. This relation is the basis of the Newton’s method when  $J$  is known. Using this relation for the pairs  $(p_i, g(p_i))$ ,  $i = 1, \dots, N$ , yields

$$p^* = p_i - J^{-1}g(p_i), \quad i = 1, \dots, N. \quad (4.3)$$

The unknowns are  $p^*$  and  $J$  so the system has enough equations if  $N = n + 1$  where  $n$  is the size of  $p$ , and it is over-determined if  $N > n + 1$ . In this latter case, a least-squared problem is formulated as follows

$$p^* = p_i - J^{-1}g(p_i) + v_i, \quad i = 1, \dots, N, \quad (4.4)$$

with a cost function to minimize:

$$\mathcal{J} = \sum_{i=1}^N w_i \|v_i\|^2. \quad (4.5)$$

$w_i^{-1}$ ’s are weights associated with each equation.

The set of equations (4.4) can be expressed in matrix form as follows

$$p^*T = P - J^{-1}G + V \quad (4.6)$$

where

$$T = [1 \quad 1 \quad \dots \quad 1], \quad P = [p_1 \quad p_2 \quad \dots \quad p_N]$$

and

$$V = [v_1 \quad v_2 \quad \dots \quad v_N], \quad G = [g(p_1) \quad g(p_2) \quad \dots \quad g(p_N)].$$

The value of interest here is  $p^*$ , so the matrix  $J$  can be removed from the system by multiplying (4.6) on the right by  $G^\perp$ , a maximum rank right-annihilator of  $G$ :

$$p^*TG^\perp = PG^\perp + VG^\perp \quad (4.7)$$

which can be expressed as

$$Ap^* = b + Cv \quad (4.8)$$

where  $A = (TG^\perp)^T \otimes I$ ,<sup>1</sup>  $b$  is the vector obtained by concatenating the columns of the matrix  $PG^\perp$  and  $C = (G^\perp)^T \otimes I$ . The vector  $v$  is obtained by concatenating the  $v_i$ ’s.

The cost  $\mathcal{J}$  in (4.5) can be expressed as follows

$$\mathcal{J} = v^T W v \quad (4.9)$$

<sup>1</sup>The  $\otimes$  operation represents the Kronecker product.

where

$$W = \begin{bmatrix} w_1 I & & & \\ & w_2 I & & \\ & & \dots & \\ & & & w_N I \end{bmatrix}.$$

The solution  $p^*$  minimizing  $\mathcal{J}$  in (4.9) subject to (4.8) satisfies

$$\begin{bmatrix} A & -C^T W^{-1} C \\ 0 & A^T \end{bmatrix} \begin{bmatrix} p^* \\ \lambda \end{bmatrix} = \begin{bmatrix} b \\ 0 \end{bmatrix}$$

from which the solution can be explicitly obtained as follows

$$p^* = [I \ 0] \begin{bmatrix} A & -C^T W^{-1} C \\ 0 & A^T \end{bmatrix}^{-1} \begin{bmatrix} b \\ 0 \end{bmatrix}.$$

This algorithm can be coded as an **OML** function as follows:

```
function [pnext, TT]=nsolve(p, gp, TT)
    n=length(p); N=size(TT, 2);
    G=TT(1:n, :); P=TT(1+n:2*n, :); wi=TT(2*n+1, :);
    wi=wi*(1+1/n);
    [_x, idx]=max(wi);
    G(:, idx)=gp; P(:, idx)=p; wi(idx)=gp'*gp;
    [Q, R]=qr(G'); r=rank(R); Gperp=Q*[zeros(N-r, r); eye(r)];
    At=ones(1, N)*Gperp; Bt=P*Gperp; Ct=Gperp;
    A=kron(At', eye(n)); b=Bt(:); C=kron(Ct', eye(n));
    Wi=kron(diag(wi), eye(n));
    m=size(A, 1);
    pnext=[eye(n), zeros(n, m)]*...
        inv([A, -C*Wi*C'; zeros(n, n), A'])*[b; zeros(n, 1)];
    if norm(p-pnext)<eps
        pnext=pnext+2*eps*rand(n, 1);
    end
    TT=[G; P; wi];
end
```

The arguments of the function are  $p$ , the value of  $g(p)$  and a matrix `_TT` used as an internal state of the function used to store the previous values of  $p$ ,  $g(p)$  and the associated weights (more specifically the inverse of the weight).

The function returns a new value of  $p$  to be used at the next iteration and the updated internal state matrix `_TT`. This matrix could be initialized and retained by the function itself (and not be part of its input output arguments) from one call to the next by defining it as `persistent`. In the way this function will be used in the construction of a **Activate** block, it is easier to have the calling function keep the value of `_TT`.

The stopping tests based on the value of  $g(p)$  and the maximum number of iterations are not implemented in this function and are handled externally.

## 4.1.2 Activate block implementation

The nonlinear solver block is implemented as a Super Block where the central piece is an **OmlCustomBlock** block implementing the solver function `nsolve` developed in the previous section. This block has 3 regular inputs and 2 regular outputs corresponding to the arguments of `nsolve`. It also has an activation input port for its activation.

The Super Block should also implement the logic to stop the iterations and propose a user friendly interface for the user. The block is shown in Fig. 4.1. The block is a Super Block, with 2 inputs and 3 outputs: the inputs correspond to the pairs  $(p, g(p))$  and the outputs are the new value of  $p$  proposed for iteration, a stop signal set to 1 if no further iterations are required and finally an error output, set to 1 if the iterations should stop on an error (the number of iterations exceeding the maximum allowed).

The block is masked as shown in Fig. 4.2. The parameters `ftol`, `maxiter`, `sz` are used to parameterize the solver. The last parameter, `count`, may be made invisible; the value of the variable `__counter__` corresponds to the iteration number; the first iteration is numbered one.

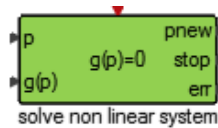


Figure 4.1: The new nonlinear solver block.

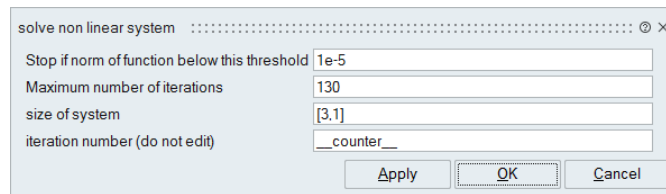


Figure 4.2: The mask of the Super Block.

The content of the Super Block is shown in Fig. 4.3. And its Context contains the following script

```
if count==1
    n=prod(sz);N=2*n;
    G=[eye(n,n),eye(n,n)];G=G(:,1:N);
    P=[eye(n,n),eye(n,n)];P=P(:,1:N);
    wi=ones(1,N)*1e32;
    TT=[G;P;wi];
else
    TT=GetFromBase('_TT',[]);
end
```

So at the first iteration, the value of `TT` is initialized in the Context. In the subsequent iterations, the value is read from the Base environment, in particular the variable `_TT`. This is done thanks to the **OML** function `GetFromBase`. The second argument of this function is optional and is used only if the variable named in the first argument does not exist in the Base environment. In this case, the variable `_TT` exists at the second iteration because it has been defined by the block **ToBase** block in the diagram. Variables in the Base environment are used to hold and pass values from one time

simulation to the next.

Note that the value of  $N$  is set to  $2n$ . The first tab of the parameter GUI of the **OmiCustomBlock** block is shown in Fig. 4.4.

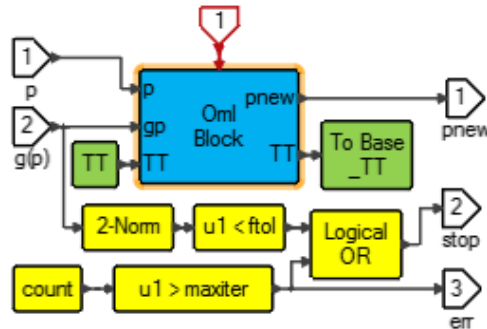


Figure 4.3: The content of the Super Block.

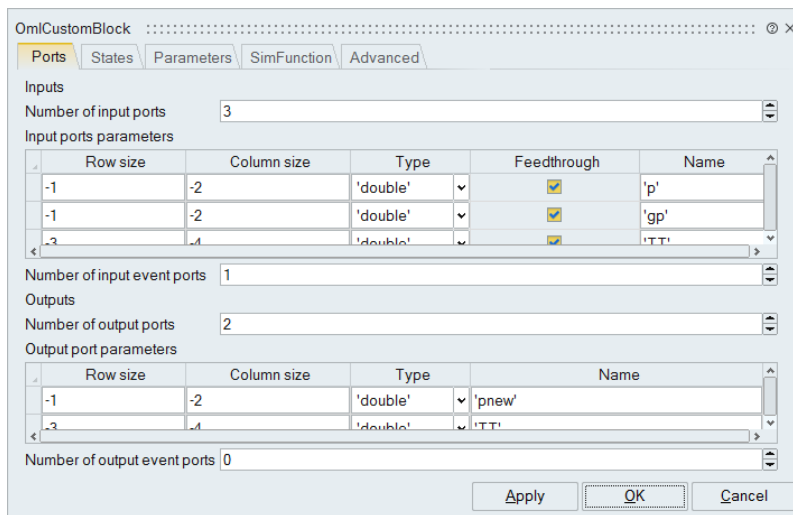


Figure 4.4: The first tab of the parameter GUI of the **OmiCustomBlock** block.

### 4.1.3 Examples

#### Static model

The model in Fig. 4.5 is a test example. There is no dynamical system and the simulation consists simply of evaluating a function at time zero, and the objective is to find the vector  $p$  for which this function is close enough to zero.

The initial guess value for  $p$  is defined in the context of the top diagram:

```
if __counter__==1
p=-ones(4,1);
else
p=GetFromBase('p');
```

end

The variable `__counter__` is provided by the system and represents the number of times the time simulation is performed. At the first iteration, the value of  $p$  is initialized in the Context. In the subsequent iterations, the value is read from the Base environment, in particular the variable `p`. This is done thanks to the **OML** function `GetFromBase`. The variable exists at the second iteration because it has been defined by the block **ToBase** in the model.

The time simulation restarts when the **End** block with option `Restart` is activated. This block is activated at the event at time zero only if the start signal has value zero. The simulation is no longer restarted if start has value 1, in which case depending on the value of the signal `err`, the simulation ends with a success message or an error message.

The use of Base variables and simulation restarts will be further explored when discussing optimization in the Chapter 6.

## Shooting model

This example corresponds literally to a shooting problem. Projectile is fired with a given initial velocity  $V$  and the objective is to find the firing angle so that the projectile lands at a distance  $d$ .

Denoting the position and the velocity of the projectile in 2D by  $s = (x, y)$  and  $v = (v_x, v_y)$ , the equations of motion are given by

$$\begin{aligned}\dot{s} &= v, & x(0) &= 0, \\ \dot{v} &= -\alpha(|v|^{\beta-1})v + \begin{bmatrix} 0 \\ -g \end{bmatrix}, & v(0) &= \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} V.\end{aligned}$$

The problem is then finding the initial firing angle  $\theta$  such that when  $y$  crosses zero,  $x$  is close to  $d$ . So the nonlinear function to solve is  $f(\theta) = x - d$ . The problem is formulated as an **Activate** model in Fig. 4.6. The system parameters are defined in the Initialization script of the model:

```
a=.1;
b=2.5;
g=9.8;
V=1000;
d=9;
```

and the initial guess value of  $\theta$  is defined in the Context:

```
if __counter__==1
p=pi/3;
else
p=GetFromBase('p');
end
```

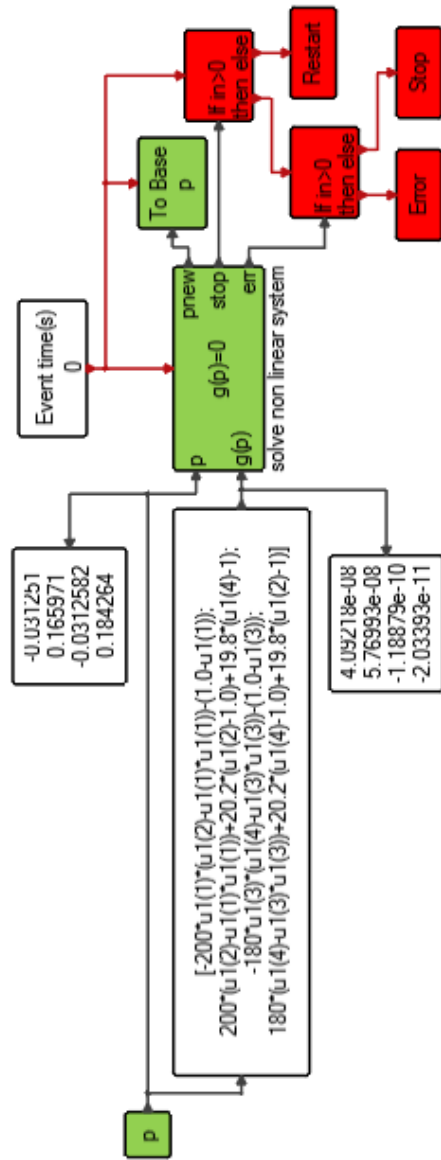


Figure 4.5: The **End** block is used in different configurations to restart the simulation, stop it with a success message and stop it with an error.

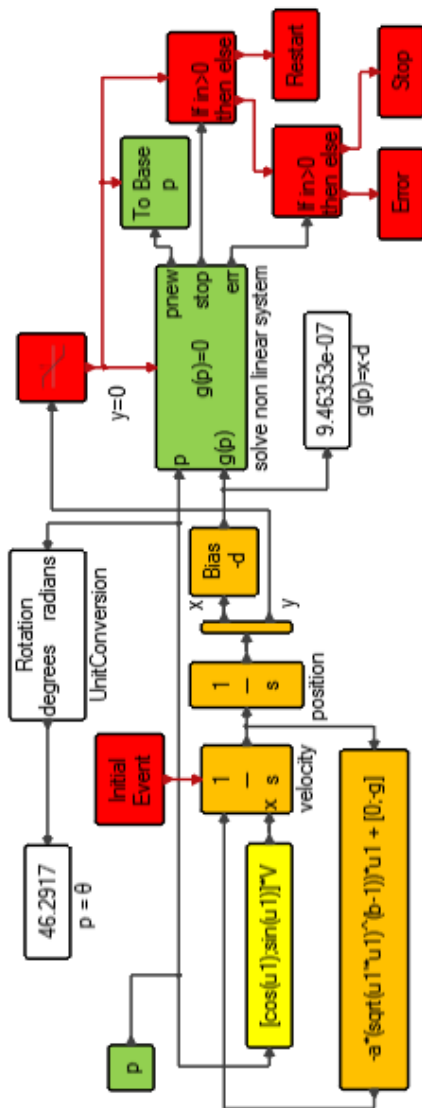


Figure 4.6: The implementation of the shooting method. In this case the final simulation time is not fixed and is determined by the time of the zero-crossing detected by the **ZeroCrossDown** block.

#### 4.1.4 Solving nonlinear system without a specific block

The new nonlinear solver block is convenient to use for solving nonlinear systems in **Activate** but the iterative method presented in the previous sections can be implemented without it. The function `nsolve` introduced earlier can be used directly in the Context of the diagram, as it is shown below. The Model in Fig. 4.7 is an alternative implementation of the model in Fig. 4.6. Assuming the file

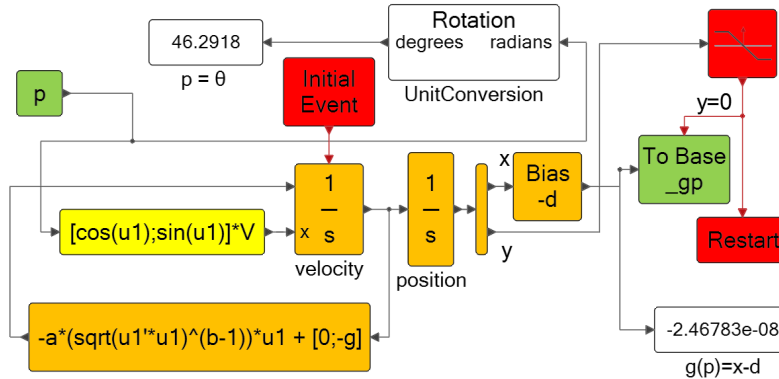


Figure 4.7: The implementation of the shooting method without using a specific block.

defining `nsolve` is in the path, the solver part is implemented in the Context of the diagram as follows:

```

if __counter__==1
    n=length(p0);N=2*n;
    G=[eye(n,n),eye(n,n)];G=G(:,1:N);
    P=[eye(n,n),eye(n,n)];P=P(:,1:N);
    wi=ones(1,N)*1e32;
    TT=[G;P;wi];
    p=p0;
else
    TT=GetFromBase('_TT',[ ]);
    p=GetFromBase('_p',[ ]);
    gp=GetFromBase('_gp',[ ]);
    if norm(gp)<1e-5
        printf('succesfull convergence, p=%g, g(p)=%g',p,gp);
        return
    end
    [p,TT]=nsolve(p,gp,TT);
    if __counter__>100
        error('Max nb. of iterations reached.');
```



## Example

The problem considered here is an optimal control problem where the objective is to find a function optimizing a cost function. The example consists of the computation of geodesics for the Grusin's metric. See [1] for details. The problem considered here is the following:

$$J = \min \int_0^1 u_1^2 + u_2^2 dt$$

subject to

$$\dot{x}_1 = u_1 \quad (4.10)$$

$$\dot{x}_2 = u_2 x_1 \quad (4.11)$$

with boundary conditions  $x_1(0) = x_2(0) = x_1(1) = 0$  and  $x_2(1) = 1/(2\pi)$ .

The Lagrangian is defined as follows

$$\mathcal{L} = \frac{1}{2}(u_1^2 + u_2^2) + \lambda(\dot{x}_1 - u_1) + \mu(\dot{x}_2 - u_2 x_1)$$

from which the following optimality conditions are obtained

$$\dot{\lambda} = -\mu^2 x_1 \quad (4.12)$$

$$\dot{\mu} = 0 \quad (4.13)$$

with optimal controls  $u_1 = \lambda$  and  $u_2 = \mu x_1$ .

This system can be solved analytically but here it is used to illustrate the usage of the shooting method. Since  $\mu$  is a constant, the unknowns are considered to be  $p = (\lambda(0), \mu)$  and the system of equations to solve is

$$g(p) = \begin{bmatrix} x_1(1) \\ x_2(1) - 1/(2\pi) \end{bmatrix}$$

where  $x_1(1)$  and  $x_2(1)$  are obtained by integrating the system

$$\begin{aligned} \dot{x}_1 &= \lambda \\ \dot{x}_2 &= \mu x_1^2 \\ \dot{\lambda} &= -\mu^2 x_1 \end{aligned}$$

with initial conditions  $x_1(0) = x_2(0) = 0$ . These equations are obtained from (4.10), (4.11), (4.12) and using the optimal controls  $u_1 = \lambda$  and  $u_2 = \mu x_1$ .

The solution is implemented as the **Activate** model in The Initialization script defines the final conditions of  $x$  and an initial guess for the unknown  $p$ :

```
xT=[0;1/(2*pi)];  
p0=[1;1];
```

The Context is the following script:

```
n=length(p0);N=2*n;  
if __counter__==1
```

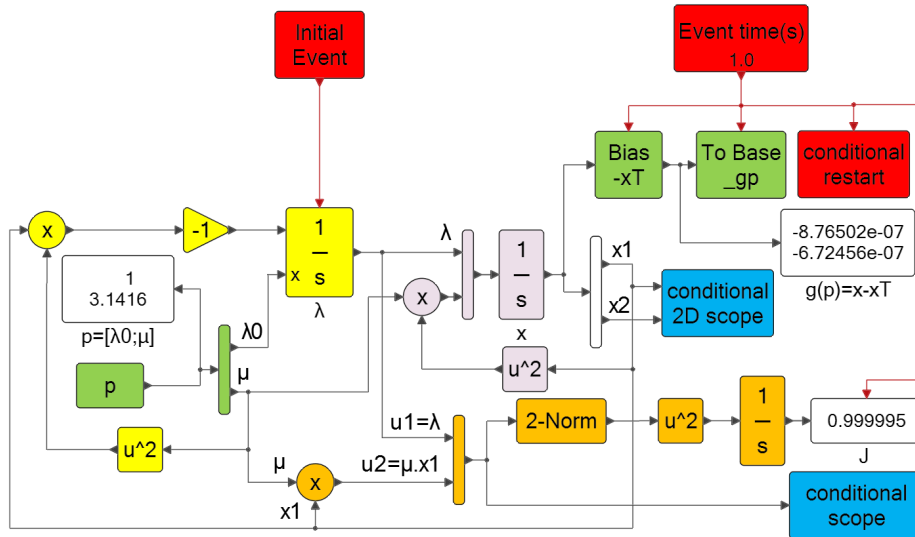


Figure 4.8: Model implementing the computation of geodesics for the Grusin's metric.

```

done=false;
G=[eye(n,n), eye(n,n), eye(n,n)]; G=G(:, 1:N);
P=[eye(n,n), eye(n,n), eye(n,n)]; P=P(:, 1:N);
wi=ones(1,N)*1e32;
TT=[G;P;wi];
p=p0;
else
TT=GetFromBase('_TT', []);
p=GetFromBase('_p', []);
gp=GetFromBase('_gp', []);
if norm(gp)<1e-5
    printf('succesfull convergence, p=%g, g(p)=%g', p, gp);
    done=true;
elseif __counter__>100
    error('Max nb. of iterations reached.');
```

The simulation results are shown in Fig. 4.9 and 4.10.

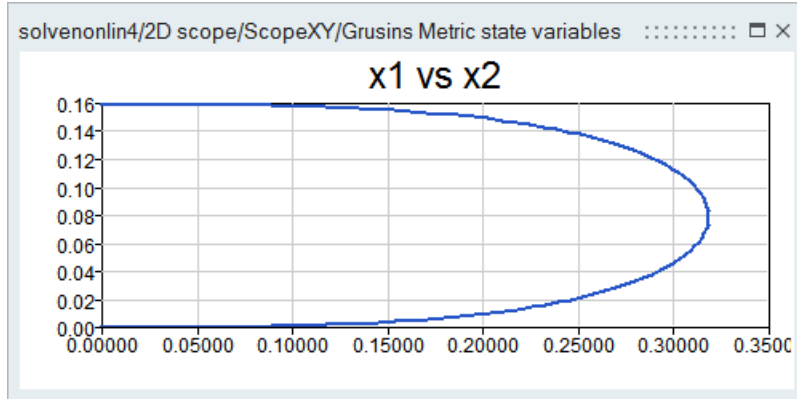


Figure 4.9: Optimal solution  $x_1$  versus  $x_2$ .

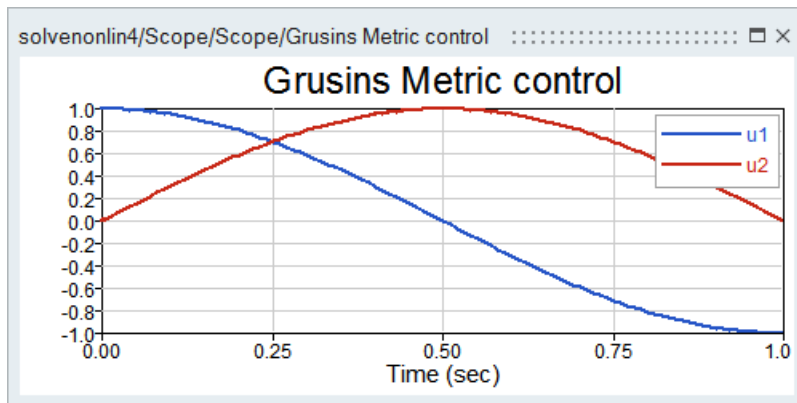
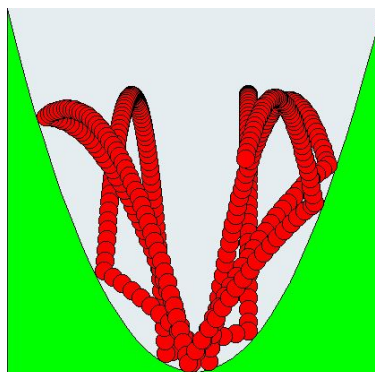


Figure 4.10: Optimal controls.

## 4.2 Repeating activations

The **RepeatActivation** block in **Activate** can be used to implement iterative computations at single event times. A simple example is presented to show its usage. In particular, the computation of the solution of a nonlinear equation is implemented using the Newton's method.

The **RepeatActivation** block plays also an important role in the implementation of state machines in **Activate**.



### 4.2.1 Problem setup

The problem considered here is a 2D version of a ball bouncing on a not-necessarily-flat surface. The surface is defined by a curve

$$y = f(x). \quad (4.14)$$

User defines the function  $f$  and its derivative  $f'$ . The ball is supposed to be a point-mass, and the collision with the surface elastic.

The objective is to model and simulate this system in **Activate**.

### 4.2.2 System equations

The state of the system contains the position  $(x, y)$  and the velocity  $(w, v)$  of the ball. The ball is subject to gravity, so its movement can be expressed as follows

$$\dot{x} = w \quad (4.15a)$$

$$\dot{w} = 0 \quad (4.15b)$$

$$\dot{y} = v \quad (4.15c)$$

$$\dot{v} = -g. \quad (4.15d)$$

The collisions occur when  $(x, y)$  hits the curve, i.e, when (4.14) is satisfied.

The collision of the ball with the surface creates a jump in the velocity of the ball. The new velocity can be obtained using the law of conservation of momentum and the assumption of the conservation of energy (elastic collision), resulting in the following

$$\begin{pmatrix} w \\ v \end{pmatrix} \leftarrow \begin{pmatrix} 1 & f'(x) \\ -f'(x) & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & f'(x) \\ f'(x) & -1 \end{pmatrix} \begin{pmatrix} w \\ v \end{pmatrix}.$$

For this particularly simple example, it is possible to explicitly solve the differential equations. Given the current position and velocities of the ball, its new position and velocity,  $t$  seconds in the future, assuming no collisions occur during this period, can be expressed as follows

$$w \leftarrow w$$

$$v \leftarrow v - gt$$

$$x \leftarrow x + wt$$

$$y \leftarrow y + vt - \frac{1}{2}gt^2.$$

The time delay  $t$  is the time to the next collision. So  $t$  is the smallest positive value satisfying

$$h(t) \triangleq y + vt - \frac{1}{2}gt^2 - f(x + wt) = 0. \quad (4.16)$$

The solution to this nonlinear equation is obtained using Newton's iterations

$$t_{k+1} = t_k - \frac{h(t_k)}{h'(t_k)}. \quad (4.17)$$

The iterations are ended when  $|h(t_k)|$  falls below a given threshold. The function  $h'(t)$  can be expressed as follows

$$h'(t) = v - gt - f'(x + wt)w. \quad (4.18)$$



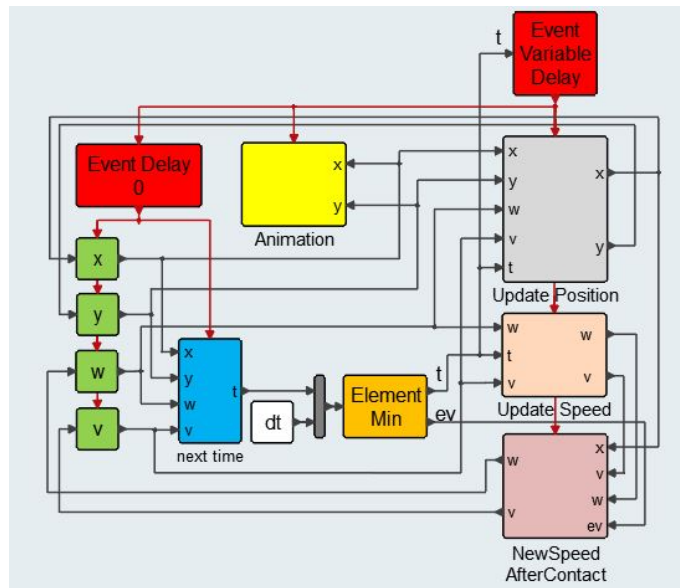


Figure 4.12: Top diagram of the model.

# Bibliography

- [1] Ugo Boscain and Benedetto Piccoli, “An introduction to optimal control” in Contrôle Non Linéaire et Applications, January 2005. <http://www.cmapx.polytechnique.fr/~boscain/AUTOMATICS/introduction-to-optimal-control.pdf>





## Chapter 5

# Example: Implementation of an Extended Kalman Filter in Activate

Kalman filter is an algorithm that uses several measurements which are polluted by noise and produces estimates of the unknown signal. The estimated signal tends to be more precise than those based on a single measurement. The Kalman filter has numerous applications and is a widely applied concept in time series analysis used in fields such as signal processing, navigation and econometrics. The extended Kalman filter (EKF) is the nonlinear version of the Kalman filter which linearizes about an estimate of the current mean and covariance. In the case of well defined transition models, the extended Kalman filter is the de-facto method of state estimation for non-linear systems. This technique essentially linearizes a non-linear system at an operating point to be able to use the linear Kalman Filter methods of state estimation.

In this chapter, the implementation of an Extended Kalman Filter (EKF) in **Activate** is illustrated through an example. The problem considered has been presented in [1]. It consists of constructing an estimator for the airspeed of a projectile using the measurements of its axial acceleration. The dynamics of the projectile is modeled as a system of nonlinear differential equations and noisy observations are supposed to be available periodically. The estimator is implemented using an EKF.

The model of the projectile is the following:

$$\begin{pmatrix} \dot{V} \\ \dot{z} \\ \dot{\theta} \end{pmatrix} = f \begin{pmatrix} V \\ z \\ \theta \end{pmatrix} = \begin{cases} \frac{1}{2m} \rho V^2 S_{\text{ref}} C_d - g \sin(\theta) \\ -V \sin(\theta) \\ -\frac{g}{V} \cos(\theta) \end{cases}$$

where the model states  $V$ ,  $z$  and  $\theta$  are air-relative speed, altitude, and angle of attack of the projectile respectively. The model parameters  $m$ ,  $\rho$ ,  $S_{\text{ref}}$ ,  $C_d$ , and  $g$  are the projectile mass, the air density, the reference surface of the projectile, the drag coefficient, and the gravity acceleration respectively. The sensor provides the axial acceleration of the projectile. Thus the measurement equation used as the measurement equation of the Kalman filter is:

$$h(V, z) = a_x = \frac{1}{2m} \rho V^2 S_{\text{ref}} C_d.$$

The value of  $C_d$  is usually provided through a table as a function of the Mach number  $M$ . Here the Mach number is given by the formula  $M = V/a$  where  $a$  is the speed of sound. The speed of sound is

a function of the air density  $\rho$ , which in turn is given by the following empirical equation given in [2]:

$$\rho(z) = (\rho_0 + 0.00651 \frac{z}{T_0})^{4.256} \quad (5.1)$$

where  $\rho_0$  and  $T_0$  represent the air density and air temperature at sea level.

## 5.1 Activate model

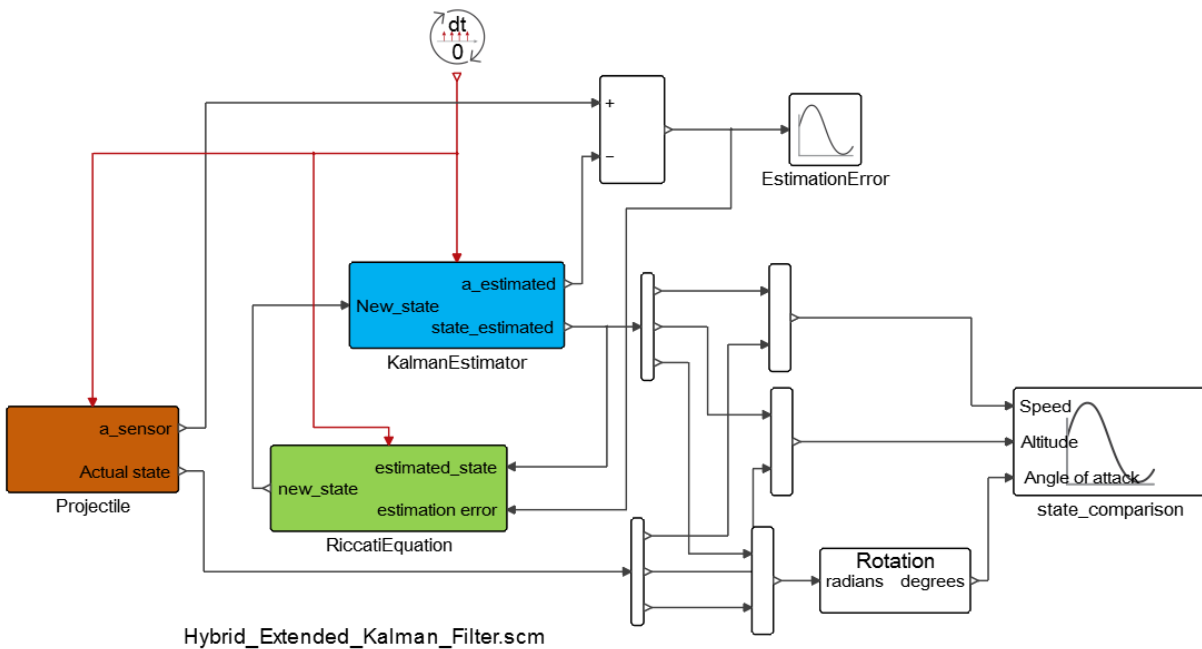


Figure 5.1: Projectile speed estimation model (Hybrid\_Extended\_Kalman\_Filter.scm)

The above model is implemented in **Activate** as illustrated in Figure 5.1. This diagram contains the whole system including the projectile model and the EKF. The parameters of the model are defined in the Context of the diagram as shown in Figure 5.2:

In the Context Editor of **Activate**, the projectile parameters, the drag coefficient of the projectile as a function of the Mach number  $C_d(M)$  as well as expressions for its derivative  $\frac{dC_d}{dM}$  are given. In the projectile model, in order to compute the Mach number, the air density and the sound speed are first computed as a function of altitude and the projectile speed. These values are computed using two `MathExpression` blocks. Once the Mach number is computed, a lookup table is used to find out the drag coefficient of the projectile. Two noise sources with Normal distribution are used to simulate the process and the measurement noises. The Covariance of these sources are defined in the context editor.

```

Model - Hybrid_Extended_Kalman_Filter
Initialization Properties Statistics
1 dt=0.01;
2 g=9.81;
3
4 V0=490;
5 z0=0;
6 Theta0=pi/6;
7 V0_hat=510;
8 z0_hat=0.1;
9 Theta0_hat=pi/6.1;
10
11 Sref=0.001;
12 m=10;
13 A0=831;
14 Rho0=1.1922;
15 T0=273;
16 Q=diag([0.5,0.1,0.0001]);
17 R=0.25;
18 P0=diag([10,0.01,0.001]);
19
20
21
22 M=[0.000,0.400,0.500,0.600,0.700,0.800,0.825,0.850,0.875,0.900,0.925,0.950,0.975,1.000,1.025,1.050,1.075,1
23 CX=[0.230,0.229,0.200,0.171,0.164,0.144,0.141,0.137,0.137,0.142,0.154,0.177,0.236,0.306,0.334,0.341,0.345,1
24 CX=-CX;
25 n=length(M);
26 dCXdM=zeros(1,n);
27 dCXdM(1)=(CX(2)-CX(1))/(M(2)-M(1));
28 dCXdM(n)=(CX(n)-CX(n-1))/(M(n)-M(n-1));
29 for i = 2:n-1
30     dCXdM(i)=(CX(i+1)-CX(i-1))/(M(i+1)-M(i-1));
31 end
32
33 RhoExp='(Rho0+0.00651*abs(u2)/T0)^4.256';
34 dRho_dExp='0.0277*(Rho0/T0)^4.256*(T0+0.00651*abs(u2))^3.256';
35 soundSpeedExp='30*sqrt(u1/Rho0)';
36 Sensor='1/2*m*u4*u1*Sref*u5';
37
OK Cancel

```

Figure 5.2: Parameter definition in the **Activate** context editor

## 5.2 EKF model

The Kalman filter is essentially a set of equations that implement an optimal predictor-corrector type estimator. The optimality is in the sense that it minimizes the estimated error covariance. Although conditions to reach optimality rarely exist, the filter is used in many applications. The standard Kalman filter is limited to a linear system, the projectile model is nonlinear and the nonlinearity is associated with both the process model and the observation model. As a result we need to choose an extended Kalman filter for state estimation. Unlike standard Kalman filter which is discrete-time, this systems is represented as continuous-time models and discrete-time measurements are frequently taken for state estimation and state update. Therefore, we used a hybrid Extended Kalman filter where the state dynamics is represented by a nonlinear system and the state updated is done through a nonlinear discrete-time update. The system model and measurement model are given by

$$\begin{cases} \dot{\hat{x}}(t) &= f(\hat{x}, t) + w(t) \\ y(t_k) &= h(\hat{x}) + v(t_k) \end{cases}$$

where

$$\hat{x} = \begin{pmatrix} \hat{V} \\ \hat{z} \\ \hat{\theta} \end{pmatrix}$$

$\hat{x}$  is the Kalman filter estimation for actual process state  $x$ . In the Figure 5.4, the Kalman filter has been implemented in **Activate**. The estimator model is very similar to the projectile model with the difference that the state of the integral block is updated at each new sensor reading. The Kalman estimator equation itself is integrated using a numerical solver. In **Activate**, any numerical solver available in the solver pane can be used. The solver integrates between each two measurements. At the end of each integration, the predicated state is used for computing estimated error.

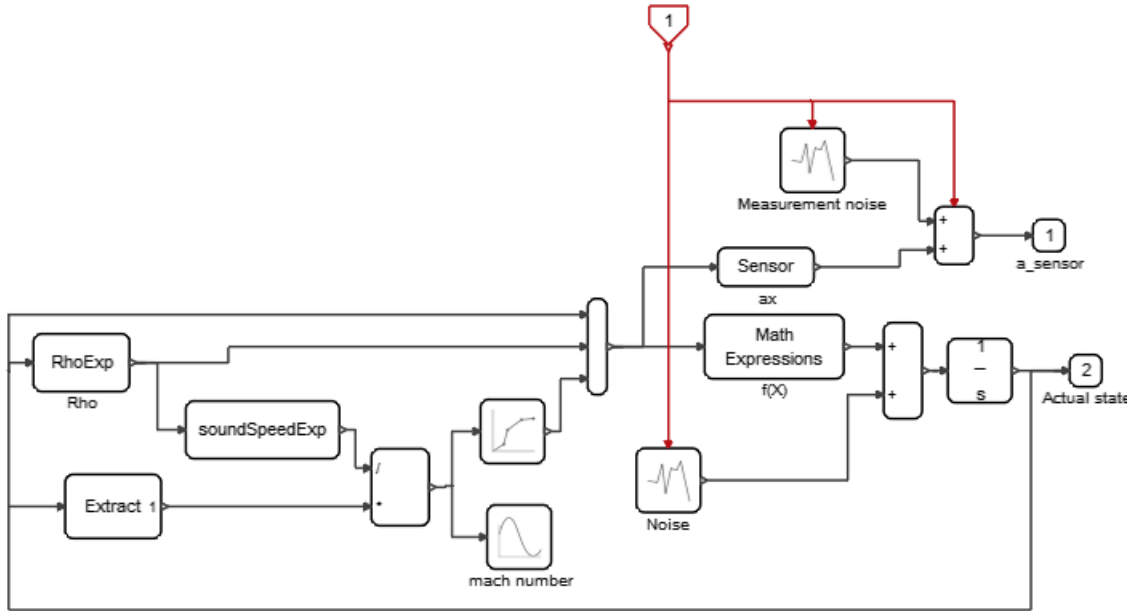


Figure 5.3: Model of the projectile

### 5.3 Covariance prediction

In Kalman filter the error covariance matrix ( $P$ ) containing the filter's estimated error covariance for the Kalman states ( $\hat{x}$ ). The following relationship defines the way this matrix is evaluated as a function of the system Jacobian ( $A$ ) and process noise ( $Q$ ).

$$\dot{P} = AP + PA^T + Q$$

The system process noise is a way to tell the filter how much the system dynamic model should be trusted. The system Jacobian matrix  $A$  is the partial derivative of  $f(x)$  with respect to  $x$ . In computing the matrix  $A(x)$ , we need to compute  $\frac{\partial C_d}{\partial z}$  and  $\frac{\partial C_d}{\partial V}$ . These two values can be computed as follows:

$$\begin{cases} \frac{\partial C_d}{\partial V} = \frac{\partial C_d}{\partial M} \frac{\partial M}{\partial V} \\ \frac{\partial C_d}{\partial z} = \frac{\partial C_d}{\partial M} \frac{\partial M}{\partial z} \end{cases}$$

$\frac{\partial C_d}{\partial M}$  is a table as a function of  $M$  whose values are computed by brute force differentiating of the table  $C_X(M)$ . The value of  $\frac{\partial M}{\partial V}$  is simply  $\frac{1}{a}$ , where  $a$  is the speed of the sound. The value of  $\frac{\partial M}{\partial z}$  is however more tricky to compute. The sound speed ( $a$ ) as a function of air density ( $\rho$ ), see level air-speed ( $a_0$ ), and see level air density ( $\rho_0$ ) is given by

$$a = a_0 \sqrt{\frac{\rho}{\rho_0}}$$

Thus, we can get

$$\frac{\partial M}{\partial z} = -\frac{V}{a^2} \frac{\partial a}{\partial z} = -\frac{V a_0}{2a^2 \sqrt{\rho \rho_0}} \frac{\partial \rho}{\partial z}$$

The value of  $\frac{\partial \rho}{\partial z}$  can be obtained by differentiating equation (5.1) with respect to  $z$ . The computation of these values in **Activate** are illustrated in Figure 5.6.

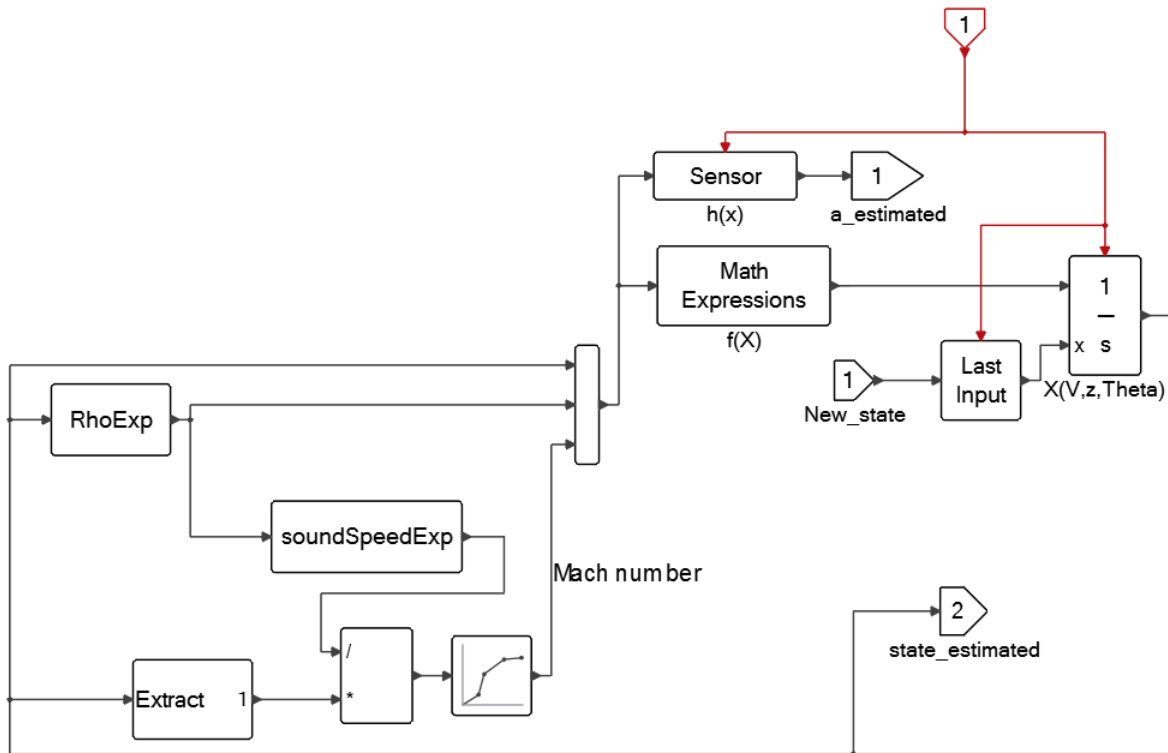


Figure 5.4: Kalman estimator

## 5.4 State update

At each sensor observation (an event activation in **Activate**), the a posteriori Kalman state and the error covariance state are updated with the following relation:

$$\begin{cases} \hat{x}^+ &= \hat{x}^- + K(a_{\text{measurement}} - h(\hat{x}^-)) \\ P^+ &= (I - KH)P^- \end{cases}$$

where  $a_{\text{measurement}}$  is the measured value at  $t_k$ . The gain  $K$  is computed as follows:

$$K = P^- H^T (H P^- H^T + R)^{-1}$$

where  $H = \frac{\partial h(x)}{\partial x}$ . In Figure 5.5, the implementation of the error covariance matrix and the state update computation in **Activate** are illustrated.

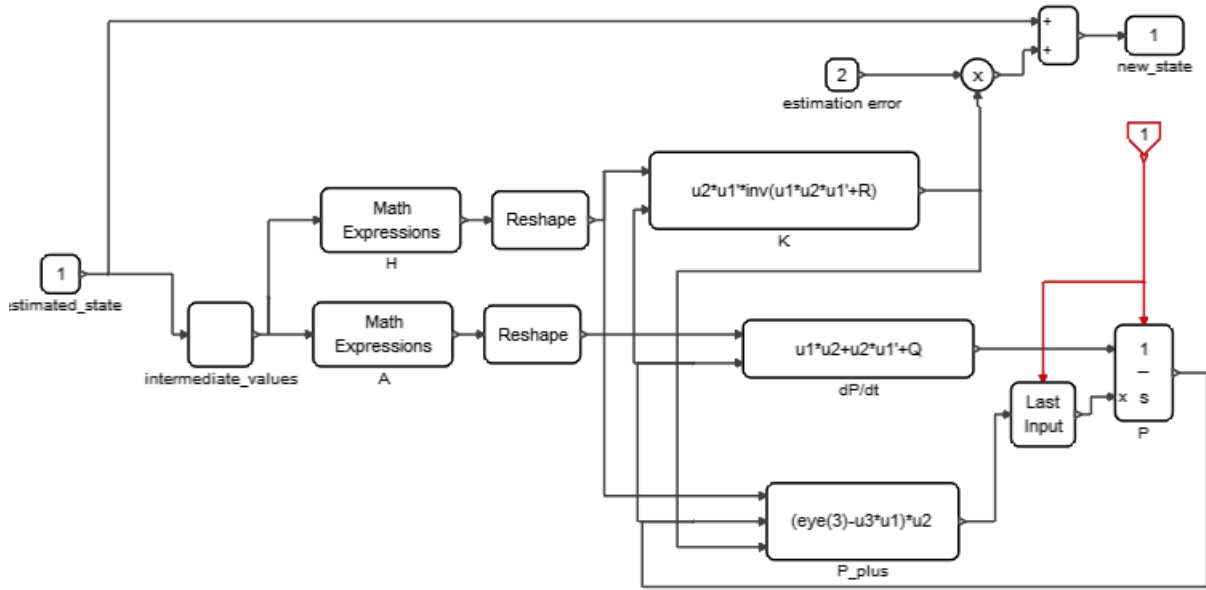


Figure 5.5: Kalman state and error covariance state Update

## 5.5 Simulation

In Figure 5.7 the actual process states (in red) as well as the Kalman estimation of the states are shown. In Figure 5.8, the Kalman estimation error is shown.

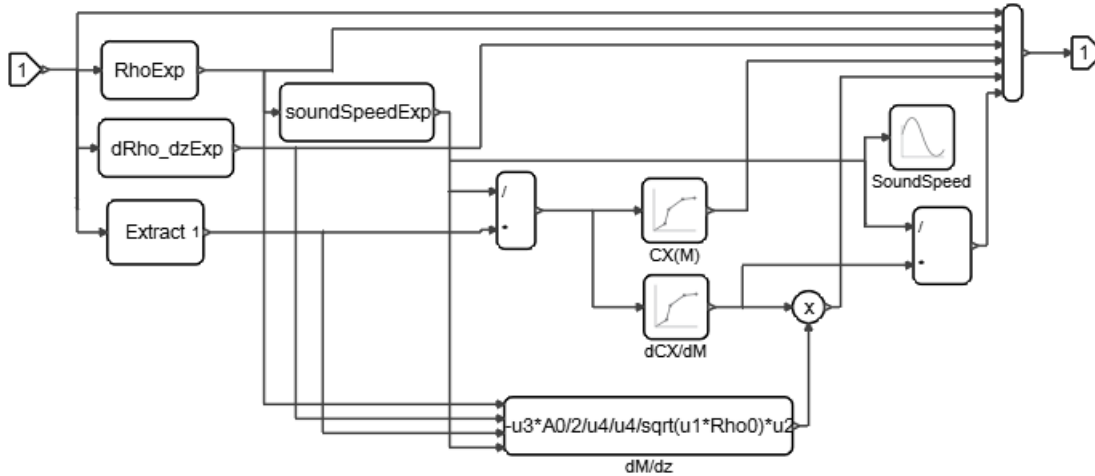


Figure 5.6: Computing required partial derivatives

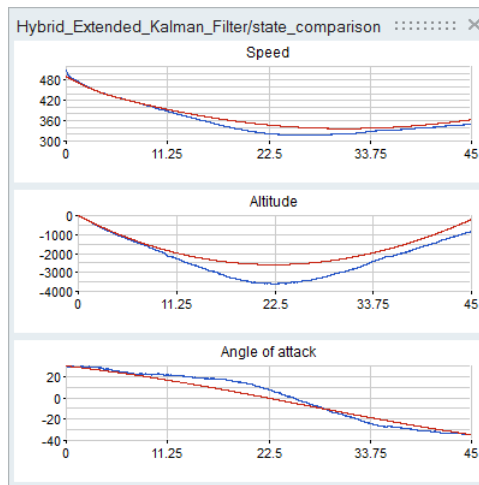


Figure 5.7: Kalman state and error covariance state Update

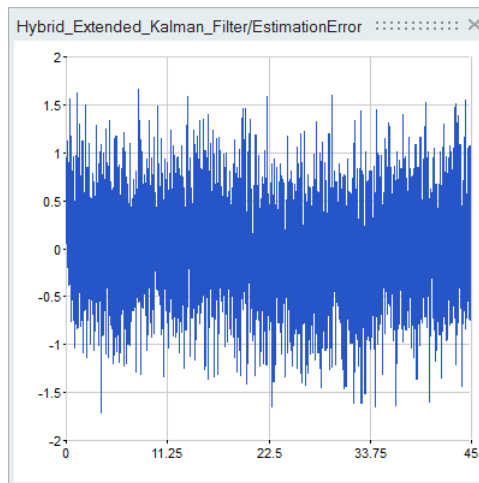


Figure 5.8: Kalman state and error covariance state Update





# Bibliography

- [1] Thomas Recchia, Projectile velocity estimation using aerodynamics and accelerometer measurements: a kalman filter approach, US Army research center, New Jersey, 2010.
- [2] Barnes W. McCormick, Aerodynamics Aeronautics and Flight Mechanics. 2nd ed., John Wiley & Sons, Inc., New York, NY, 1995.



# Chapter 6

## Optimization

In many applications, modeling and simulation are used to find optimal values for system parameters or optimal control strategies. This can be done by formulating an optimization problem with proper cost function associated with the simulation model. There are various ways to define and solve such optimization problems in **Activate**. This document describes some of the available methods.

The outline of this chapter is as follows: first the **Optimization** block is presented. This block may be used to formulate optimization problems fully integrated within **Activate** models. Then batch simulation from the **OML** environment is presented and its application to optimization is illustrated via several examples. Finally, a graphical tool specifically developed for optimization in **Activate** is presented. The graphical tool is probably the simplest way to formulate and solve optimization problems in **Activate** but understanding its operations requires some knowledge of batch simulation that is why it is presented last.

### 6.1 Use of Optimization block in Activate

The **Optimization (BobyqaOpt)** block, available in the Optimization palette, may be used to solve optimization problems using the **BOBYQA software** written by Prof. Powell, implemented in reverse communication mode.

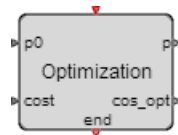


Figure 6.1: BOBYQA block

Unlike the usual optimization procedures where the optimization code is the master code calling the simulator for cost evaluation by providing the value of the variable, in reverse communication mode the simulator runs the main program and calls the optimizer with the cost value. The optimizer then returns a new value of the variable indicating whether the optimization should be continued or that this value corresponds to the optimal value of the variable.

The **Optimization** block parameters are the optimization parameters and the inputs and outputs of the block are used to exchange variable values (vectors in general), cost and control signals between the

optimizer and the simulator. The block restarts the simulation as long as necessary until the optimal solution is found.

This method requires a way to maintain the value of the optimization variable from one simulation run to the next. This may be done using the **ToBase** and **FromBase** blocks. The **ToBase** block, when activated, copies the value of its input into a variable in the base **OML** workspace. The name of the variable is given by the block parameter. The **FromBase** block, when activated does the opposite: it copies the value of the variable to its output. If the variable name specified by the **FromBase** block parameter does not exist in the base **OML** workspace, a default value is copied.

The first input of the **Optimization** block is used to pass the initial/current value of the variable to the optimizer. The second input is used to pass the cost associated with the value produced by the **Optimization** block on its first output in the previous simulation run. The block is activated when a new value of the cost is available, usually at the end of every simulation run. The block then produces the optimization variable on its first output, and either restarts the simulation or generates an event indicating that the optimization has ended and that the variable provided on the output is the solution to the optimization problem. In general the activation output is connected to an **End** block with parameter “Stop” but other configurations are possible. The optimal cost is provided by the second output of the block.

## 6.2 Example

The following is a simple example to illustrate the use of the **Optimization** block. The problem considered is that of finding the optimal gear ratios for a 5 speed vehicle given its torque curve as a function the engine RPM. This should be considered a toy example: the model is very simple and in particular does not consider the clutch mechanism. Multiple optimization criteria will be examined.

The simple model used for the dynamics of the vehicle is given by the following equations:

$$\begin{aligned}\dot{\omega} &= g_i^2 T(\omega)q/c - a\omega - bv^2 \\ v &= c\omega/g_i \\ \dot{x} &= v.\end{aligned}$$

The variable  $\omega$  designates the engine angular velocity,  $g_i$  is the gear ratio for gear number  $i$ ,  $v$  is the speed of the vehicle and  $x$  the traveled distance.  $T(\omega)$  is the torque as a function of the angular velocity, and  $a$  and  $b$  are constant friction coefficients.  $q$  and  $c$  are also constant coefficients depending on vehicle characteristics. At the time of gear shift,  $\omega$  jumps so that  $v$  remains a continuous-time function. The system is modeled as a Super Block named car the content of which is illustrated in Fig. 6.2.

The 5 activation links entering the diagram on the top are used to model the shifting events; the Selector block outputs the corresponding gear ratio. The Torque curve is modeled using a Lookup table block.

The first optimization problem consists of finding the maximum traveled distance in 30 seconds starting with a given small initial speed. The optimization variables are the gear ratios and the shift times. Even though given the gear ratios it is possible to find the up-shift times without optimization in this simple case, we consider them as optimization variables for simplicity. So there are 9 optimization variables: a vector  $\mathbb{T}$  of size 4 representing the time lapses from one up-shift to the next, and a vector  $gear$  of size 5 that represents the gear ratios associated with the gearbox.

The **Optimization** block is used with the configuration described in Fig. 6.3.

At  $t = 30$ , the **Optimization** block is activated. When this happens, the **Optimization** block reads the cost value and generates a new optimization vector  $p$ . Vector  $p$  is the concatenation of  $T$  and  $gear$ . The values of  $T$  and  $gear$  are saved in the base environment from one simulation run to the next. **SetSignal** blocks are used to make their values available in the rest of the diagram without having to draw long links through the model.

The parameters of the **Optimization** block are given below (see Fig. 6.4). Note that the bounds are set very large but the lower bounds guarantee that the variables cannot take zero or negative values. The model is not valid if they do.

The shift times vector  $T$  generates events using the Event Variable Delay block. The Event Variable Delay block delays its input activation by the value given at its regular input port at the time of activation. Note that time values given in  $T$  represent incremental times, i.e., the time from one gear shift to the next, not absolute shift times.

The usage of Event Variable Delay block may seem unnecessary since the amount of delay does not change during a single simulation run. In such cases, the Event Delay block is commonly used where its block parameter is set to the desired delay. But since in this case the amount of delay comes from the base, the Event Variable Delay block is used and the value of delay is obtained from the base using a **FromBase** block. This is not however the only possible implementation. Later, an alternative method for obtaining variables from the base will be presented allowing the usage of the Event Delay block, simplify considerably this diagram.

The model parameters are defined in the Initialization script of the diagram:

```
v0=10/3.6;  
a=.004;  
b=0.005^2;  
c=.05;  
q=.003;  
Torque=[0,600,800,1000,2000,3000,4000,5000,6000,8000;  
0,10,40,100,200,280,360,350,200,0];
```

Running the simulation actually starts the optimization process. The optimum gear ratios and the maximum traveled distance are seen through the use of **Display** blocks in the model. The traveled distance and speed as a function of time are also given below along with trajectory followed on the Power/RPM curve which shows, as expected, that the optimal gear shifting times correspond to the strategy where the engine RPM is beyond the maximum power RPM and the up-shift reduces the engine RPM to a point that has the same power output as before the shift (Fig. 6.5):

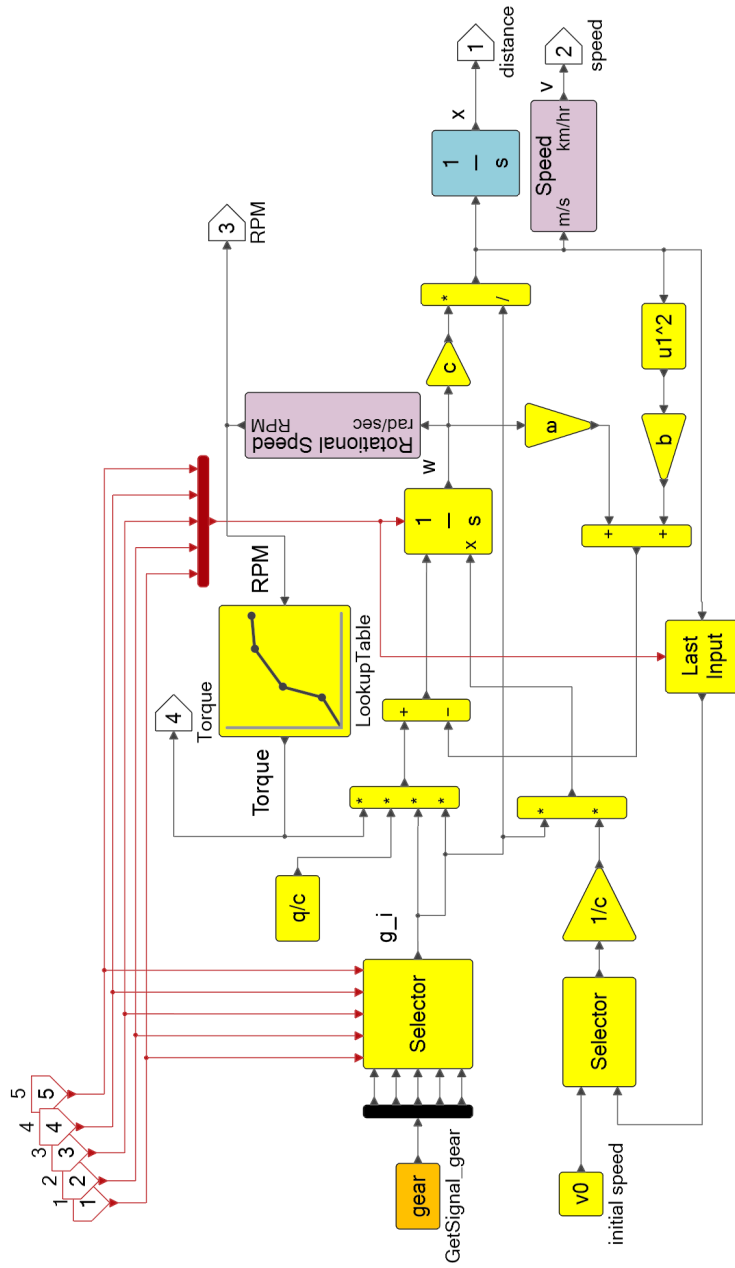


Figure 6.2: Superblock in Car model

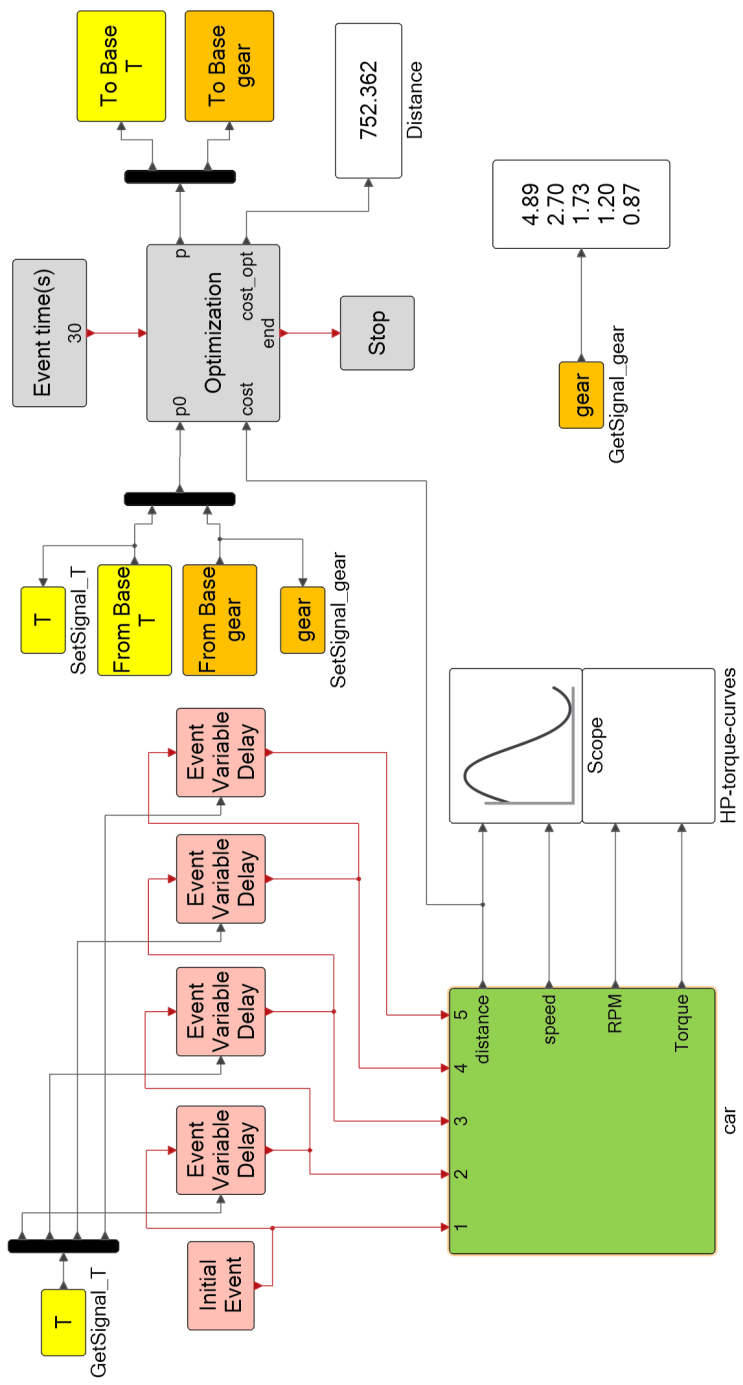


Figure 6.3: Optimization block used in Car model

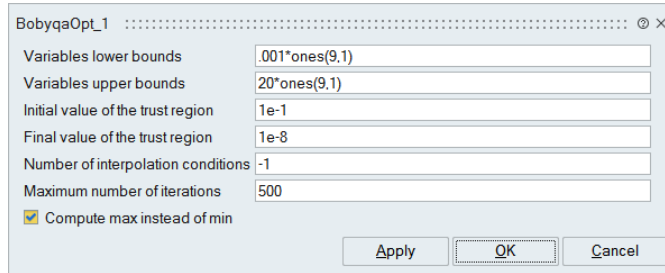


Figure 6.4: BobbyQA block dialog

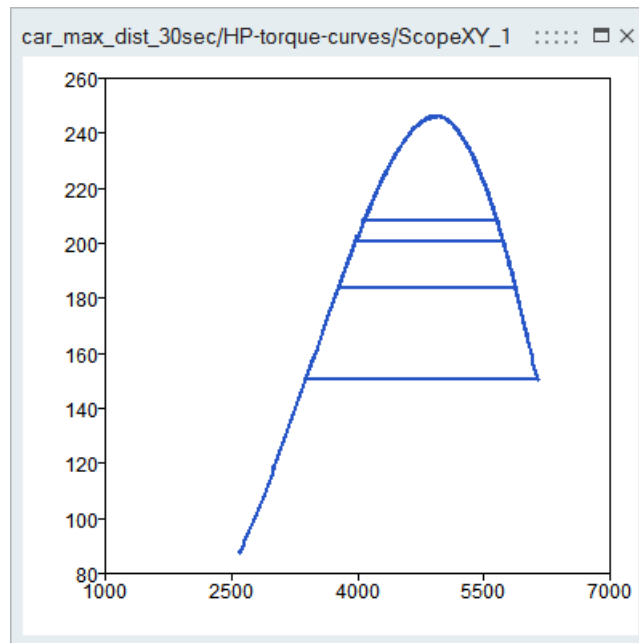


Figure 6.5: Optimal path illustrated on the curve representing the engine power versus RPM



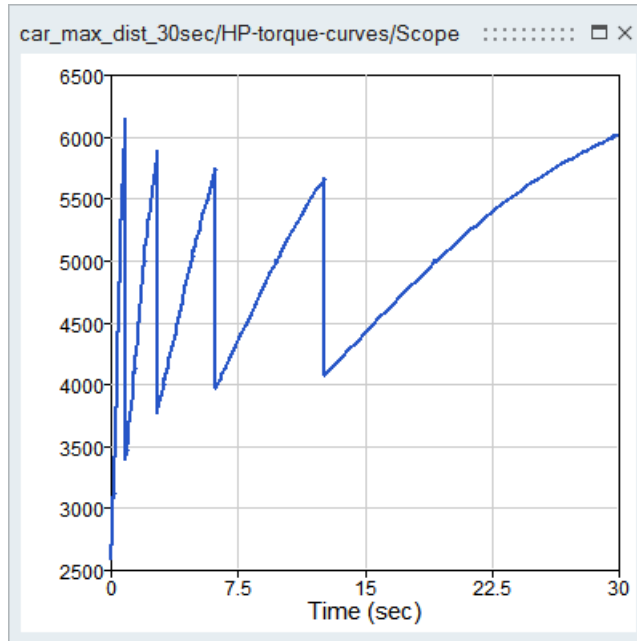


Figure 6.6: The RPM as a function time.

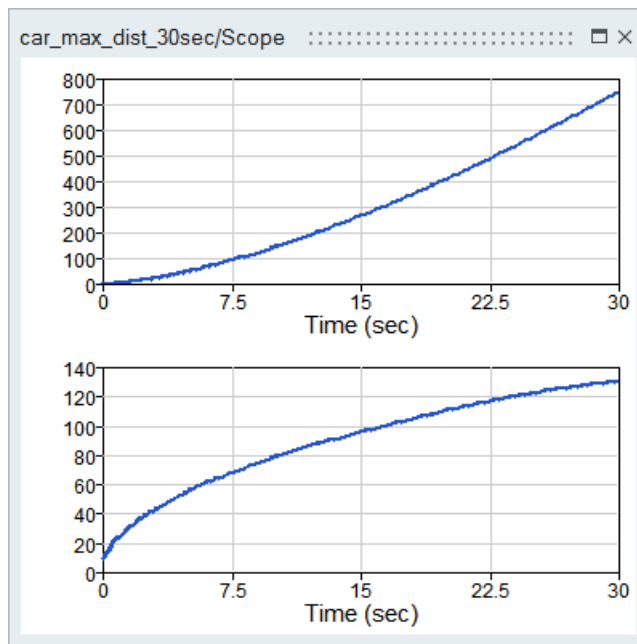


Figure 6.7: Distance and Speed results for the model car\_max\_dist\_30sec.scm

The second optimization problem consists of minimizing the time to reach 1000m. In this case each simulation run is ended when the traveled distance goes beyond 1000m. The zero-crossing block **ZeroCrossUp** generates an event when this happens, activating the **Optimization** block, which reads the cost value (in this case the time) and generates a new optimization vector  $p$ . Vector  $p$  is again the concatenation of  $T$  and  $gear$ . The full model is shown in Fig. 6.8.

The optimization result shows that the vehicle reaches 1000m in 36.5 seconds. The optimal gear ratios are different from the solution of the first problem.

In another problem formulation, reaching a given speed in the shortest possible time is considered. This problem is similar to the previous problem and the corresponding model is found in the file `car_min_time_speed_vtarget.scm` and is illustrated in Fig. 6.9.

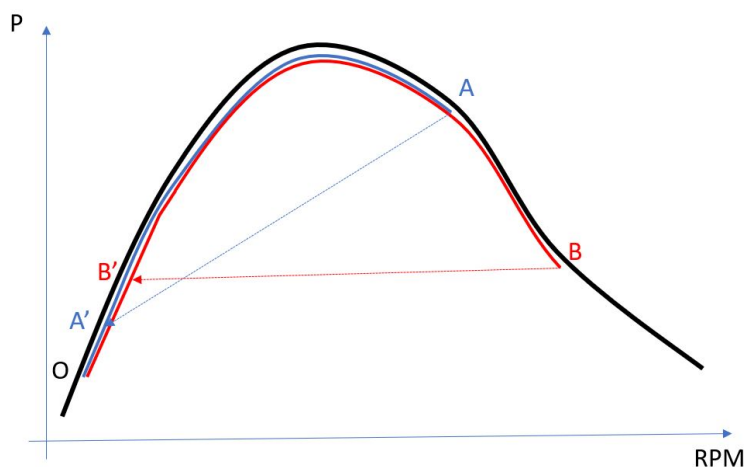
Note that an event at time 100 (final simulation time) is added to the event corresponding to reaching  $v_{target}$  since depending on the gear ratios, the car may never reach the speed of  $v_{target}$ . This problem is considered again later to illustrate the use of an **OML** function to read data from the base workspace. Using this function, leads to simpler models as explained in the following section.

## Optimal gear-change timing

It is interesting to note that the optimal gear changing instants are the time instants where before and after the gear change, the engine power has the same value, i.e., the transition is horizontal (see Fig. 6.5). This of course is not a coincidence and can be shown to hold under very general assumptions, as it is shown below. The solution found by the optimizer having this property is an indication that the optimization process has succeeded in finding the optimal solution.

To show that the optimal solution corresponds to a horizontal transition, consider the simpler problem where the car has only two gears, so the problem is to find the timing of the only possible gear change. The extension to the case of more than two gears is then straightforward.

The following image compares two gear shifting strategies corresponding to different gear changing times:



For both strategies (blue and red) the starting point is  $O$ . As time goes on, the RPM goes up with the power following the black curve (representing the maximum engine horse power as a function of RPM). For the blue strategy the gear is shifted at point  $A$  transitioning the state of the system to point  $A'$ . For the red strategy, the gear shifting happens later, at point  $B$ , transitioning to  $B'$ . So both strategies follow

the exact same path except over the intervals  $[A, B]$  and  $[A', B']$ : in the red strategy, the path includes  $[A, B]$ , whereas the blue strategy includes  $[A', B']$ .

To compare the two strategies, the time it takes to go from  $A$  to  $B$  must be compared to the time to go from  $A'$  to  $B'$ . In both cases the starting and ending speeds are the same, so the time is shorter if the power over the time interval is higher. In the image above, the part of the black curve from  $A$  to  $B$  is clearly higher than that between  $A'$  and  $B'$ . So more power is supplied over  $[A, B]$  compared to  $[A', B']$ , which means that the time for going from  $A$  to  $B$  is shorter. Thus the time for the red strategy is smaller. Now by considering the limiting case as  $A$  gets closer to  $B$  (and so  $A'$  to  $B'$ ), it is easy to see that the optimal transition occurs when  $A, B, A'$  and  $B'$  are aligned.

### 6.3 GetFromBase and AddToBase OML functions

The usage of the **Optimization** block requires a method for keeping the values of the optimization variables from one simulation run to the next. The **FromBase** and **ToBase** blocks were used to communicate with the base. The use of **FromBase** however implies that the optimization variables can only be used as input to other blocks and not as block parameter values. To be able to use optimization variables in the definition of block parameter values, the **OML** function **GetFromBase** may be used either directly in the definition of parameters or, in the Initialization script or a Context.

To illustrate the use of this function consider again the problem of reaching `vtarget` in minimum time but in this case the values of `T` and `gear` are retrieved from the base in the Initialization script:

```
v0=10/3.6;
a=.004;
b=0.005^2;
c=.05;
q=.003;
Torque=[0,600,800,1000,2000,3000,4000,5000,6000,8000;
        0,10,40,100,200,280,360,350,200,0];
vtarget=100;
T=GetFromBase('T',ones(4,1)*4);
gear=GetFromBase('gear',[3;2;1.5;1;.8]);
```

`T` and `gear` can now be used in the definition of block parameter values or in diagram Contexts. Here for example `T` is used to obtain the absolute times of gear shifts in the Context of the top diagram:

```
engine_par=[a,b,c,q];
T2=T(1);
T3=T2+T(2);
T4=T3+T(3);
T5=T4+T(4);
```

The values of `T2`, `T3`, `T4` and `T5` represent the shift times and may be used as block parameters. This is used to simplify the model as shown in Fig. 6.10.

Variables `gear` and `T` are available in the scope of the diagram so there is no need to use **FromBase** blocks to obtain their values. The latter are replaced with **Constant** blocks. The car Super Block may now be masked (see Fig. 6.11).

The masking is done by the auto masking operation followed by mask edition

Note that the optimization variables remain in the base after an optimization is run. So in order to restart an optimization process with the original initial values, the optimization variables must be removed from the base. This can be done using the `clear` function.

The **OML** function `AddToBase` may be used in the Initialization script or the Context to define or redefine a variable in the base.

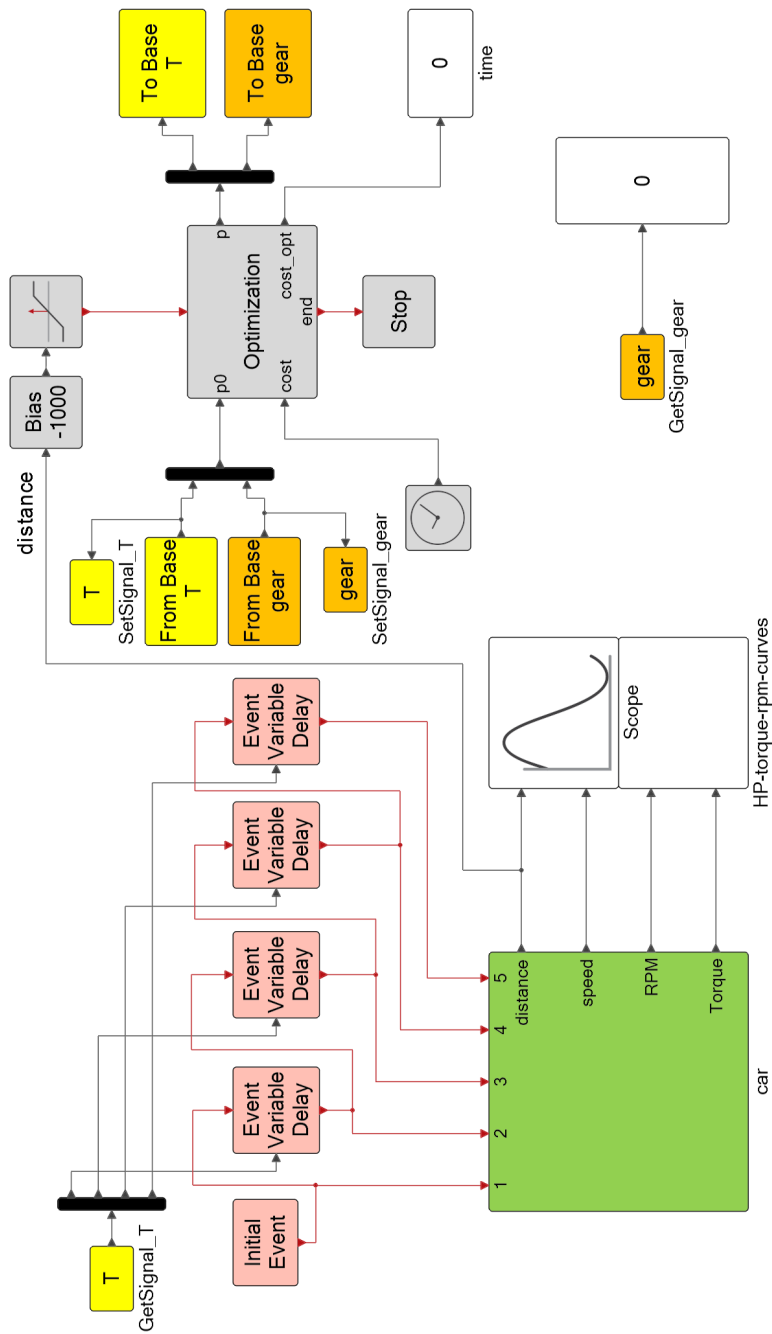


Figure 6.8: Car model for minimizing time to distance (car\_min\_time\_dist\_1000m.scm)

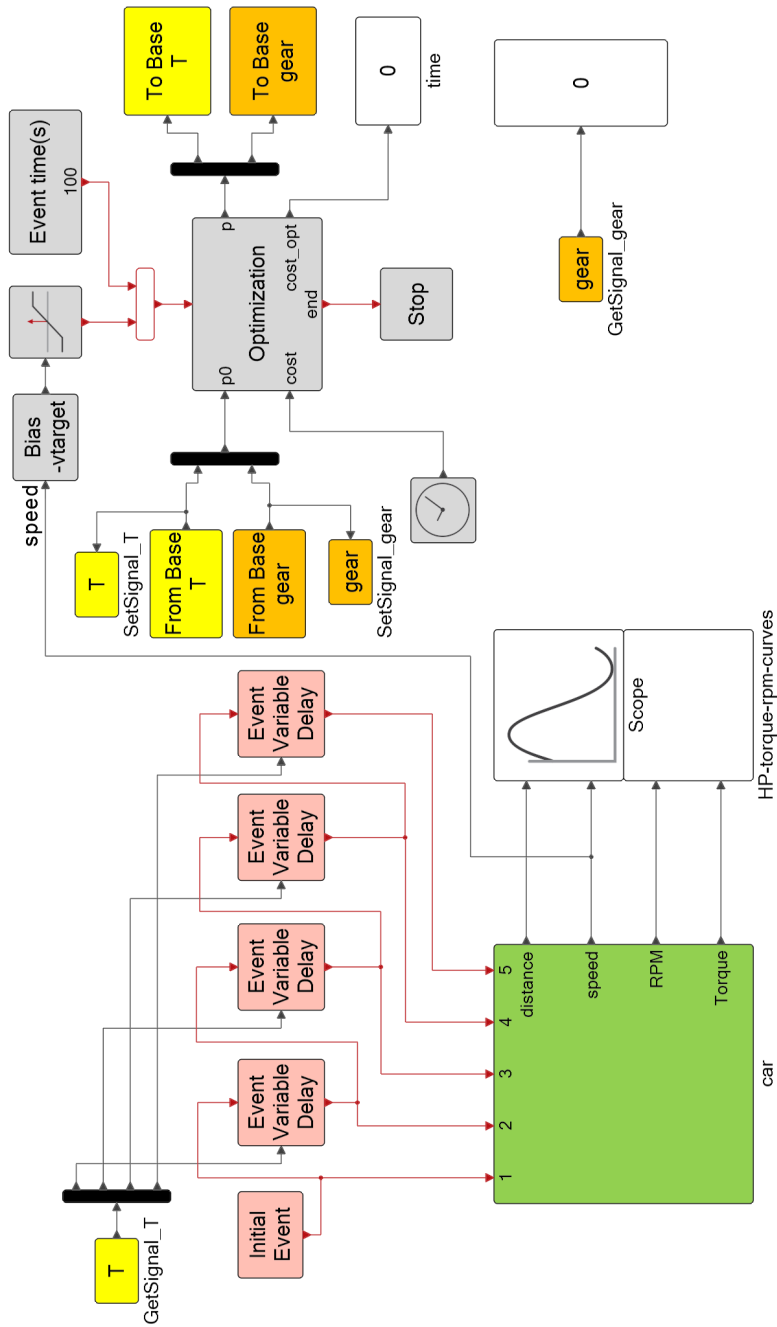


Figure 6.9: Car model for reaching speed in shortest time (car\_min\_time\_speed\_vtarget.scm)

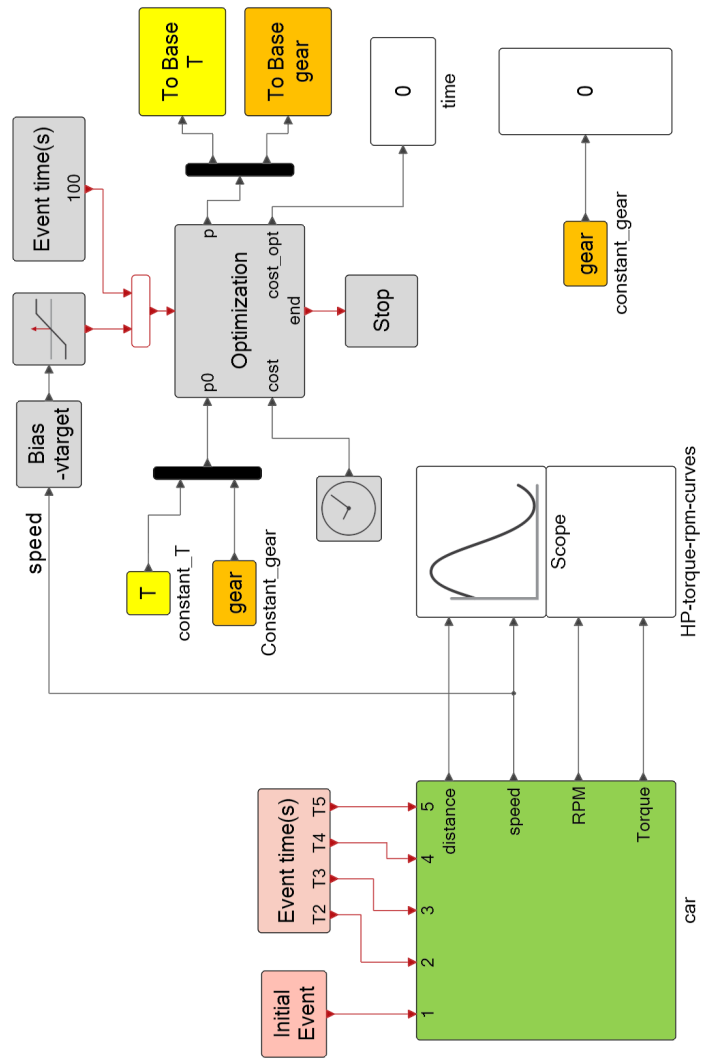


Figure 6.10: Model variant (car\_min\_time\_speed\_vtarget\_2.scm)

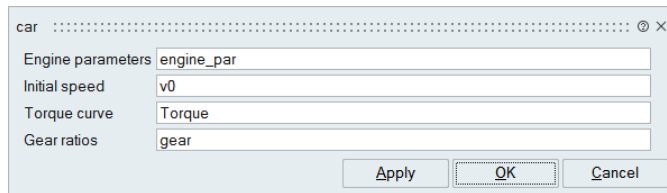


Figure 6.11: Masked "Car" superblock

## 6.4 Script based optimization

Script based optimization provides a powerful mechanism for solving very general optimization problems where the cost and constraints may be obtained from a combination of **Activate** simulation results and an **OML** code. But in many cases, the cost is a simple function of signals generated by one or more **SignalOut** blocks and the optimization problem formulation is straightforward yielding virtually always the same **OML** script. For these simple cases, **Activate** provides a graphical tool for formulating an optimization problem associated with a given model. This tool, which generates transparently the required optimization script, is presented in Section 6.5. But first direct scripting is presented.

### 6.4.1 Direct scripting of optimization code in OML

The full power of **OML** may be used to solve optimization problems associated with **Activate** models. The code in **OML** would be the same as that for solving any optimization problem; the only specificity is that the cost would depend entirely or partially on the result of a **Activate** simulation run. **OML** scripts used for such optimizations may also be included inside the **Activate** model. See Chapter 20 for details.

Running a simulation from **OML** (in batch mode) requires creating a simulation object for the model. This object is needed to activate the graphical outputs of the Scopes present in the model. If graphical outputs are not needed, the simulation object is not needed.

The model evaluator is an **OML** function that is generated specifically for a given model. The evaluator evaluates all model parameters and creates the flat model structure, which is used by the compiler to create the simulation structure. The model evaluator must be created before running the simulator but it can be created only once for a given model. As parameters are changed during simulation runs, the evaluator function is re-evaluated but the evaluator function itself does not need to be recreated.

Finally once the simulation object and the evaluator are created, the model may be simulated.

The **OML** functions corresponding to the above operations are:

- `bdeCreateSimulationObject`: This function creates a simulation object for a given model specified as an scm model file.
- `vssCreateEvaluator`: This function creates an evaluator for the given model.
- `vssRunSimulation`: This function is used to run a **Activate** simulation. The arguments include the evaluation function of the model and various workspaces.

There are various ways to exchange data between **OML** and the **Activate** model. The most direct way to change model parameters from **OML** is to use the `vss_context` workspace. Variables used in a model for defining block parameters are defined in a workspace resulting from the execution of the Initialization script and various diagram Contexts following the semi-global scoping rule. When a simulation is run in batch mode, the workspace `vss_context` is injected in the workspace resulting from the execution of the Initialization script. Variables in `vss_context` may redefine variables defined in the Initialization script. In that case, the original values of the variables in the Initialization script can be considered as default or initial values, which can be modified during batch simulation. Note also that the presence of these default/initial values are needed to make the model self-contained.

To get data back to **OML** from the simulation, the **SignalOut** block may be used. This block returns a Signal object in the working environment. This environment is passed to the batch simulation function as optional argument.



## 6.4.2 Example

Consider again the optimization problem where the objective was to minimize the time to attain 1000m mark. The model is modified (Fig. 6.12) by removing the **Optimization** block and its supporting blocks. Variables  $T$  and  $gear$  are now defined in the Initialization script, and the simulation end time is returned in Signal  $Tf$ .

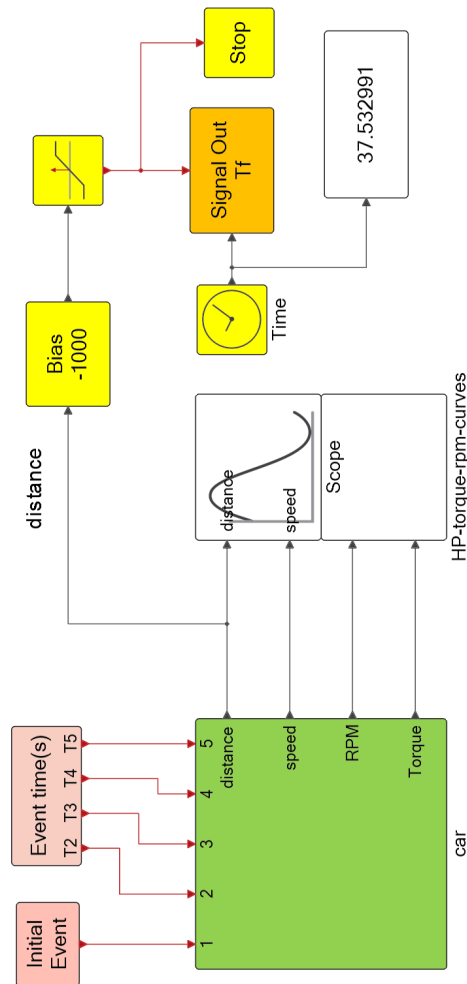


Figure 6.12: Modified model for batch use with OML (car\_min\_time\_distance\_1000\_batch.scm)

The Initialization script includes now the definition of  $T$  and  $gear$ :

This model can be simulated since all the required variables, in particular  $T$  and  $gear$ , are defined. For optimization purpose however the values of these variables must be overwritten. This will be done using the `vss_context` workspace as shown below.

The optimization script may use any of the existing **OML** optimization functions or coded directly in **OML**. The following script, `car_min_time_distance_1000_batch.oml`, uses the `fminuncon` function.

```

function e=eval_cost(p, ev, env)
    vss_context=struct();
    vss_context.T=p(1:4);
    vss_context.gear=p(5:9);
    vssRunSimulation(ev, vss_context, env);
    Tf=getenvvalue(env, 'Tf');
    e=Tf.ch{1}.data;
end

me=omlfilename('fullpath');
p=fileparts(me);
model=[p, '/car_min_time_distance_1000_batch.scm'];
T0=4*ones(4, 1);
gear0=[3.2;1.8;1.3;1;.8];
p0=[T0;gear0];
simobj=bdeCreateSimulationObject(model);
ev=vssCreateEvaluator(model, simobj);
env=getnewenv;
cost=@(p) eval_cost(p, ev, env);
options=optimset('MaxFunEvals', 1000);
p = fminunc(cost, p0, options);
T=p([1:4])
gear=p([5:9])

```

In the above example, the `SignalOut` block is used to return a single value (simulation time) but it can be used to return the whole time trajectory of signals and be used to implement very complex optimization scenarios.

In the following version of the gear ratio optimization problem, a constraint is imposed on the engine RPM. The problem is that of reaching a given speed as fast as possible without revving the engine above a maximum RPM (for example 5500 RPM). The optimization methods used so far, **BobyqaOpt** and `fminunc`, did not allow the specification of such constraints. **Bobyqa** allowed bounds on the optimization variables but in this case, the engine RPM is not directly an optimization variable. To solve such constrained problems, the **OML** function `fmincon` may be used. The function `fmincon`, in addition to a cost value, takes a vector of constraint values. This function minimizes the given cost subject to the constraint being below specified values.

To use `fmincon`, the **Activate** model must be extended to return the engine RPM. This is done inside the signal `omega` as shown in Fig. 6.13

The optimization script is then as follows:

```

function e=ev_cost(p, ev, env)
    vss_context=struct();
    vss_context.T=p([1:4]);
    vss_context.gear=p([5:9]);
    vssRunSimulation(ev, vss_context, env);
    Tf=getenvvalue(env, 'Tf');
    e = Tf.ch{1}.data;
    if isempty(e) e=100; end
end

```

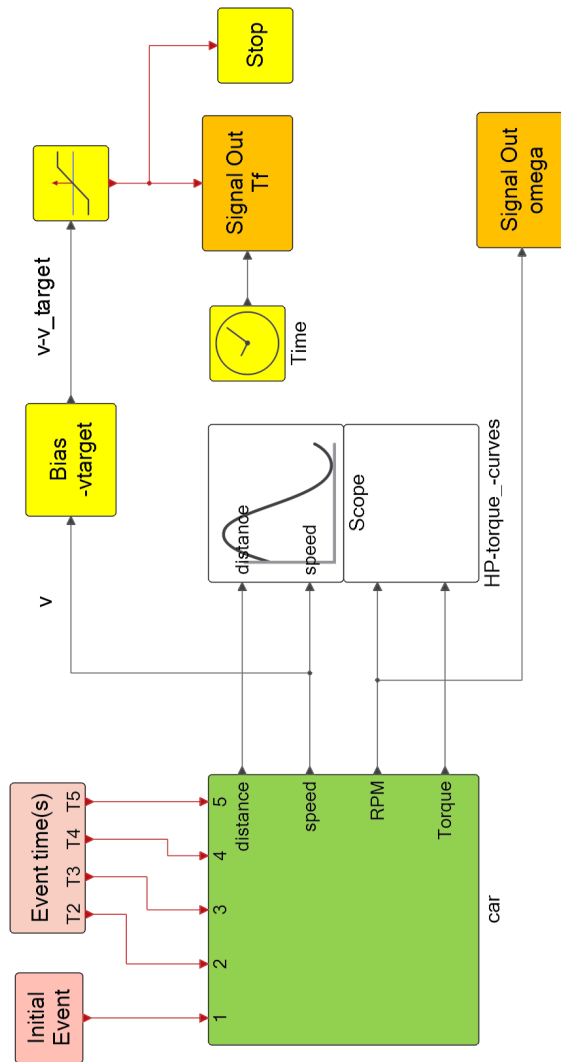


Figure 6.13: Model extended to return the `omega` signal (`car_min_time_speed_vtarget_w_constr_batch.scm`)

```
function c=ev_constr(p,ev,env)
    vss_context=struct();
    vss_context.T=p([1:4]);
    vss_context.gear=p([5:9]);
    vssRunSimulation(ev,vss_context,env);
    omega=getenvvalue(env,'omega');
    w = omega.ch{1}.data;
    c=max(w)-5500;
end

me=omlfilename('fullpath');
p=fileparts(me);
```

```

model=[p,'/car_min_time_speed_vtarget_w_constr_batch.scm'];

T0=5*ones(4,1);gear0=[3.2;1.8;1.3;1;.8];p0=[T0;gear0];
lb=.1*ones(9,1);ub=20*ones(9,1);
simobj=bdeCreateSimulationObject(model);
ev=vssCreateEvaluator(model,simobj);
env=getnewenv;
cost=@(p) ev_cost(p,ev,env);
constr=@(p) ev_constr(p,ev,env);
options=optimset('MaxFunEvals',1000);
p = fmincon(cost,p0,[],[],[],[],lb,ub,constr,options);
T=p([1:4])
gear=p([5:9])

```

After execution of the optimization script, the optimal time evolution of the RPM is obtained as shown in Fig. 6.14.

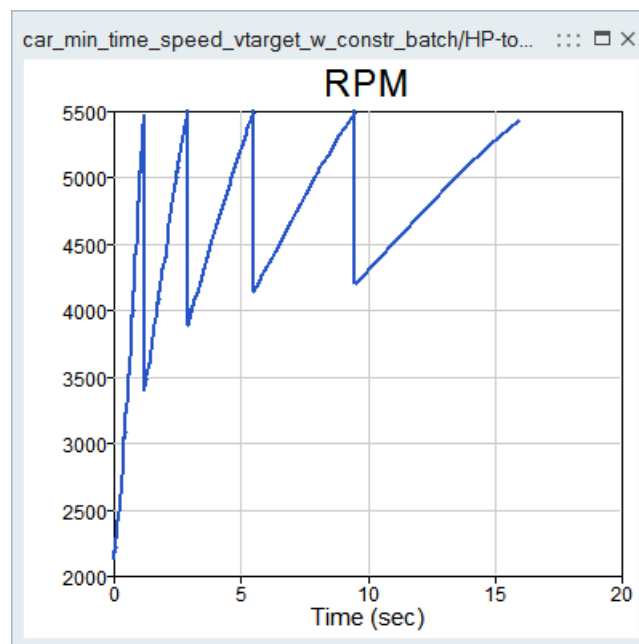


Figure 6.14: RPM vs. time after optimization.

The vehicle reaches 100Km/h in 15.9 seconds. Without the 5500 RPM constraint, the time was 15.76sec.

In this example, the constraint value, i.e., the maximum over the RPM signal was computed inside the **OML** function `ev_constr`. For that, the complete signal time history, `omega`, was returned to **OML**. The maximum value could have also been computed in the model and simply its final value returned to **OML** but that would have required adding a few more blocks to the diagram. In general the computation of the cost function and constraint values could be done partly in the model, partly in the **OML** code.

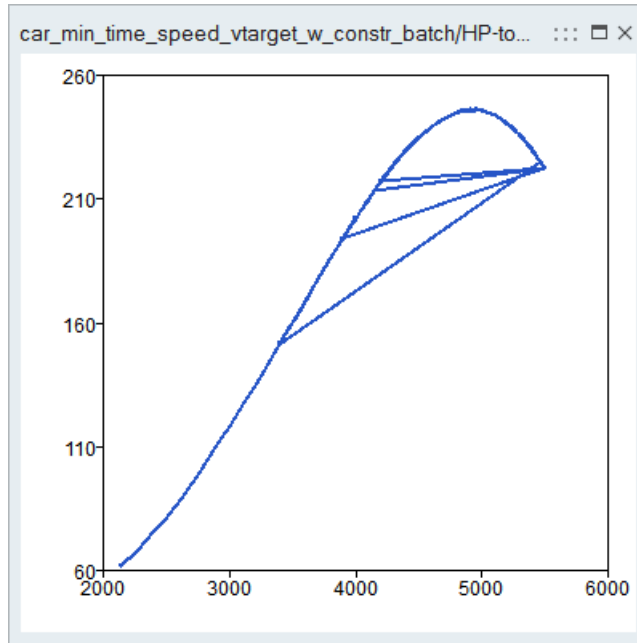


Figure 6.15: Horse power as a function of RPM after optimization.

## 6.5 Activate Graphical optimization tool

The graphical interactive optimization tool generates and runs automatically the **OML** scripts for solving **Activate** optimization problems. The tool can be used to associate optimization criteria to an existing **Activate** model. In particular it provides a simple to use interface for defining optimization variables and defining cost and constraints.

The tool should be seen as a graphical interface for the **OML** optimization functions `fminuncon`, `fmincon` and `ga`. The interface is associated with a **Activate** model, or more specifically with an `scm` file (the distinction is important since the modifications to the model are not taken into account until the model is saved). The information that needs to be provided in the interface is as follows:

- Model path: The association is done by entering the path of the `scm` file containing the **Activate** model.
- Initialization: An **OML** code must be provided. Any variable defined in this code overwrites variables defined in the Initialization script of the model. The initial (guess) values of the optimization variables should not be defined in this code; they will be defined later. Variables needed to define them however may be defined here.
- Number of variables: The number of optimization variables must be given here. Depending on this number, the user is presented with a list of widgets where he should provide the name, the initial value and the range (min and max values) of each variable. The initial value may be defined as an expression using variables defined in the Initialization code. If the variable is a vector or matrix, the min and max values may be given as scalars, otherwise their sizes should match that of the variable.
- Number of constraints: The number of inequality constraints must be specified. This number, which may be defined as an expression using variables defined in the Initialization code, must

match the size of the `_constraint` vector defined in the Optimization script when the optimization problem contains constraints.

- Optimization script: An **OML** code used to define the variable `_cost` and, if the problem has constraints, the vector `_constraint`. The code may assume that all the variables defined in the Initialization code, all the signals associated with **SignalOut** blocks present in the model, in addition to the current values of the optimization variables produced by the optimizer are in its workspace. The optimizer searches the optimization variables within the specified bounds such that all the elements of `_constraint` are negative and the `_cost` is minimal.
- Optimization method: SQP and Genetic algorithm are proposed. More methods will be included in the future. SQP uses the **OML** functions `fmincon` or `fminbnd` functions depending on the presence or absence of constraints. The genetic algorithm uses `ga`.
- Depending on the selected optimization method, different optimization parameters such as Max number of iterations, Convergence tolerance, Constraint tolerance, Population size, etc., must be provided. These parameters are passed to the underlying **OML** optimization routine. See **OML** documentation for details.

The graphical interface provides, at the end of the simulation, the optimal cost, the number of iterations and gives access to the generated **OML** script. The cost value history versus the iteration number may also be plotted.

The solution to the problem is returned in the base environment inside the **OML** table `_optim`. In addition to the optimal values of the optimization variables, the table includes the optimal cost and their histories for all the iterations performed during optimization.

The optimization may be started using the run button on top of the graphical interface once all the required information is provided. At every step, the proposed values for the optimization parameters and the associated costs are displayed. The optimization process may be stopped at any time using the stop button.

The use of the tool is quite intuitive and is illustrated here through an example.

### 6.5.1 Example

The constrained optimization problem considered previously and solved using **OML** scripting is considered again, and formulated and solved with the graphical optimization tool. The problem is to find the best gear ratios to reach a given speed as fast as possible with constraint on the top RPM. The same **Activate** model is used. The top RPM will be parameterized without modifying the model.

The graphical optimization tool corresponding to this problem may be created as shown in Fig. 6.16 and run using the Run button on top.

The Initialization code used in the optimization tool is virtually identical to the model Initialization script (see Fig. 6.17).

But instead of `T` and `gear`, it defines their initial values. Note that all model parameters may be modified directly in the tool without requiring any modifications of the **Activate** model. So the end user does not even have to see the simulation model to run optimizations for different sets of parameters. For example optimization may be performed for different values of target speed and top RPM. All vehicle parameters, including the Torque curve can also be modified.

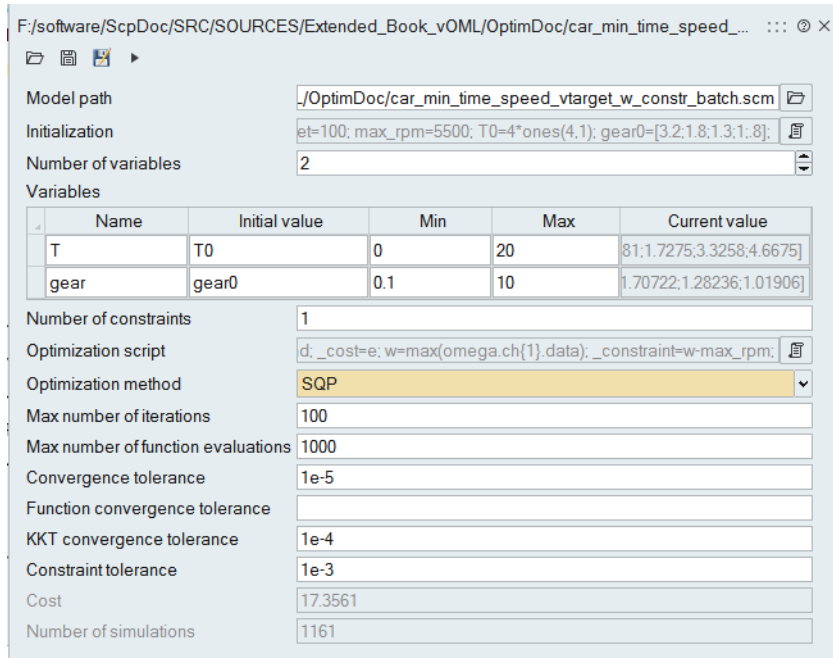


Figure 6.16: Optimization Tool

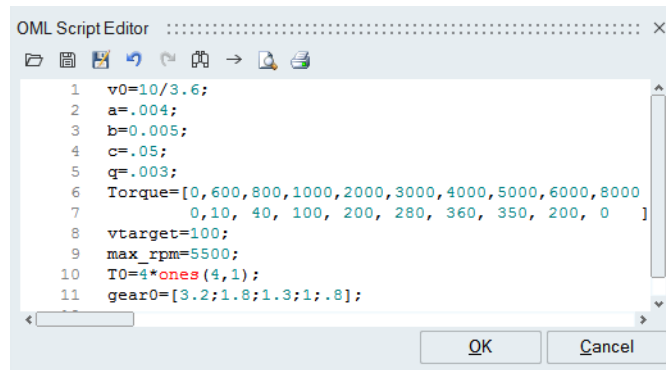


Figure 6.17: Initialization script used in the Optimization Tool

The bounds on the optimization variables `T` and `gear` are set as scalars. The time increment is always positive, so is the gear ratio which is assumed to be arbitrarily greater than 0.1 (0 value would be meaningless).

The number of constraints is set to 1 and the variables `_cost` and `_constraint` are defined as follows (Fig. 6.18):

Note the use of the signals `Tf` and `omega` produced by the **SignalOut** blocks of the model. The `Tf` signal however may be empty since the vehicle does not always reach the target speed before the end of the simulation. If that happens the cost is set to infinity so that the optimization may proceed. The optimization variables could also be used in defining the cost and constraints. Note however that they would have their current values as decided by the optimizer at the current step of the optimization and not necessarily their original values defined in the Initialization code.

Running the simulation gives the same result obtained previously with direct scripting. The optimal

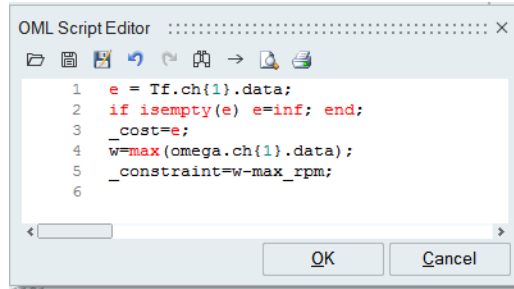


Figure 6.18: Constraint definition in the Optimization Tool

values of  $T$  and  $gear$ , and the optimal cost in addition to other information are available in Table `_optim` subsequent to running the optimization.

## 6.6 Max-min/Min-max optimization problems

Max-min and Min-max optimization problems may be formulated in different ways in **Activate**. For example the min part may be implemented using the **Optimization** block and the max in **OML**. The following example illustrates such an implementation. In this example, the robust version of the gear shifting problem is considered. It is assumed that gear shifting does not occur at prescribed times but they could happen up to a timing error of 0.1 seconds before or after the specified times. The optimization problem consists in optimizing the performance corresponding to the worst case time errors. In particular, the **Activate** model is modified such that the traveled distance is minimized over possible values  $dT$  of the timing error vector. It is then this value which is used as cost to maximize.

The Max-min problem is formulated in the model shown in Fig. 6.19) and is then realized using the **OML** optimization script generated by the Optimization tool (see Fig. 6.20).

Max-min and Min-max optimization problems may also be formulated by using multiple **Optimization** blocks. In general only one such block may be used in a model but more than one block may be used if they are cascaded as illustrated in the diagram of Fig. 6.21 which implements the same Max-min problem considered previously.



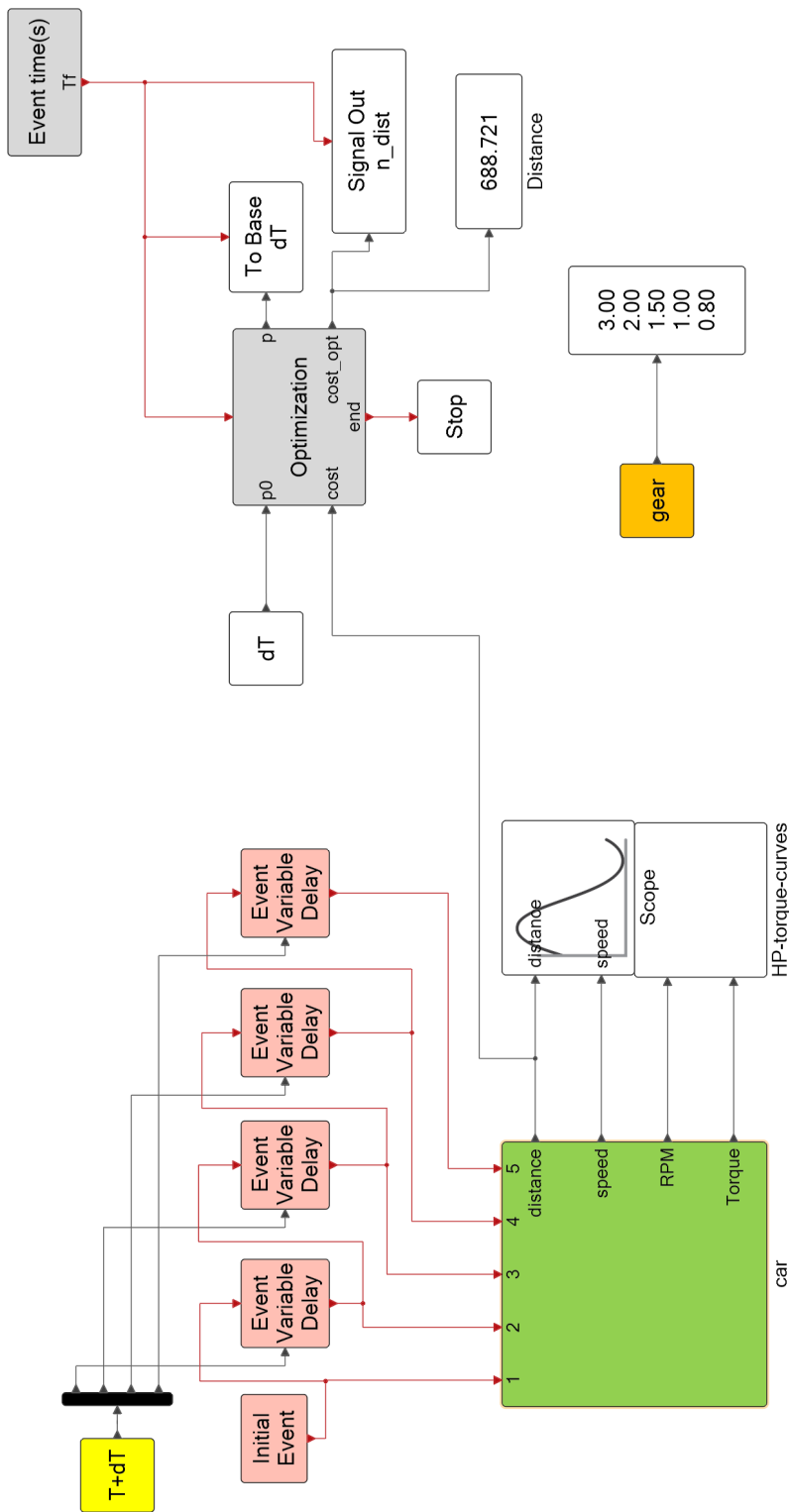


Figure 6.19: Max-min model (car\_max\_dist\_Tf\_robust\_batch.scm)

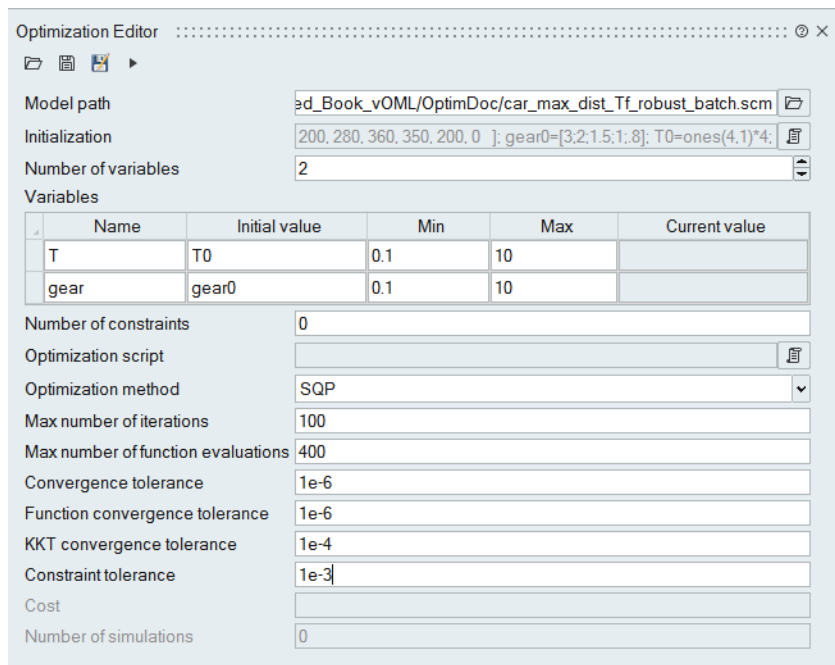


Figure 6.20: Optimization tool for the Max-Min problem for Model in figure 6.19.

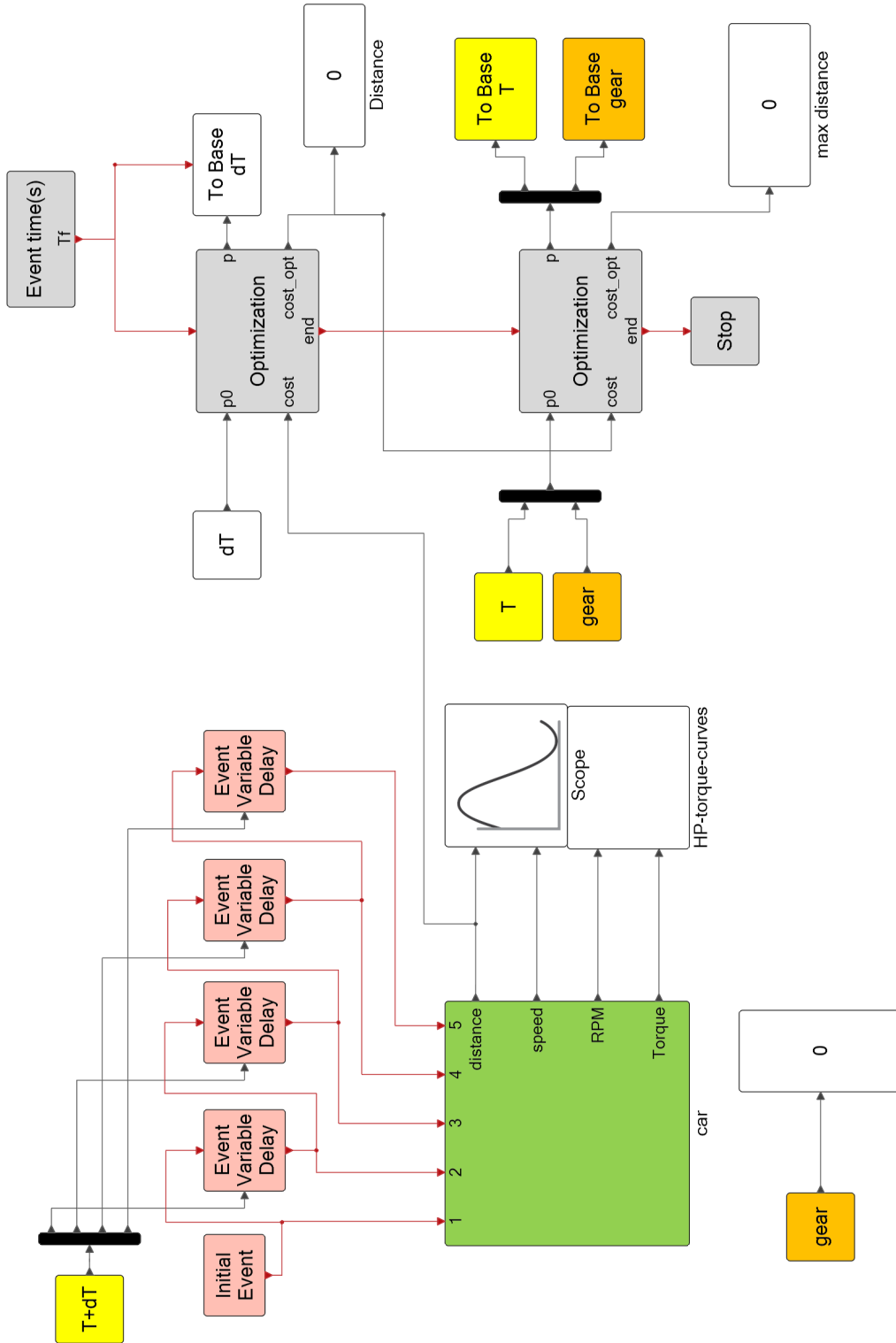


Figure 6.21: Variant of the Max-min model (car\_max\_dist\_Tf\_robust.scm)

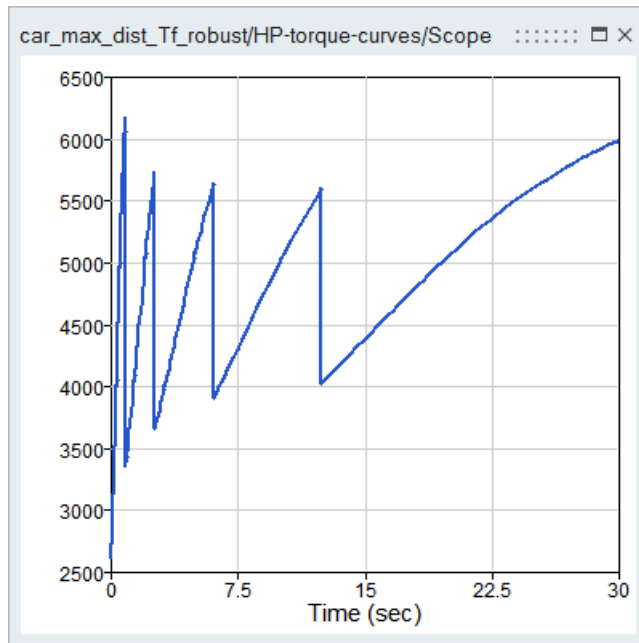


Figure 6.22: RPM as a function of time after optimization in Model in Fig. 6.21.

# Chapter 7

## Linearization

### 7.1 Linearization function

**Activate** provides functions to create linear models from a Super Block by linearization. Linearization consists of constructing an approximate linear model around a given equilibrium (operating) point. Linearized models are constructed in particular for analysis and, controller and filter design.

The equilibrium point for the system

$$\begin{aligned}\dot{x} &= f(x, u) \\ y &= h(x, u)\end{aligned}$$

is defined as the triplet  $(x_e, u_e, y_e)$  satisfying

$$\begin{aligned}0 &= f(x_e, u_e) \\ y_e &= h(x_e, u_e).\end{aligned}$$

The linearization of this system around the equilibrium point  $(x_e, u_e, y_e)$  is a linear state-space system specified in terms of 4 matrices  $A$ ,  $B$ ,  $C$  and  $D$ :

$$\begin{aligned}\dot{\delta x} &= A\delta x + B\delta u \\ \delta y &= C\delta x + D\delta u\end{aligned}$$

where  $\delta x = x - x_e$ ,  $\delta u = u - u_e$ ,  $\delta y = y - y_e$ , and

$$A = \frac{\partial f}{\partial x}(x_e, u_e), \quad B = \frac{\partial f}{\partial u}(x_e, u_e), \quad C = \frac{\partial h}{\partial x}(x_e, u_e), \quad D = \frac{\partial h}{\partial u}(x_e, u_e).$$

The main function to perform linearization in **OML** is `vssLinearizeSuperBlock`: and is used as follows

```
[A,B,C,D] = vssLinearizeSuperBlock(model,sblk,inps,outstf,ctx);
```

The first argument, `model`, representing the model in which linearization is to be performed, can either be a path of an `scm` file or a model object. The latter in general is the current model which can be obtained by the `bdeGetCurrentModel` function.

The second argument is the full name of the Super Block being linearized. The linearization is not necessarily performed with respect to all the inputs and outputs. Some of the inputs, in general corresponding to discrete-time signals, may have to be excluded. Similarly some outputs may need to be excluded. The arguments `inps` and `outs` are vectors of indices representing respectively the inputs and outputs to be considered for linearization.

The time `tf` is the time up to which the simulation is performed (in general to reach an equilibrium point) before linearization is done. The value of `tf` can be set to zero if the model is already in an equilibrium point (possibly thanks to the following argument `ctx` described below).

The final argument, `ctx`, is an external context, used to overwrite some of the variables defined in the Initialization script of the model. This is done in general to put the model in an equilibrium point without modifying it.

The function returns the  $(A, B, C, D)$  matrices of the linearized model.

### 7.1.1 Example: inverted pendulum

The classical problem of controlling an inverted pendulum mounted on a cart is considered. The objective is to control the force applied to the cart in such a way that the pendulum is stabilized pointing upwards. The force moves the cart on a ramp.

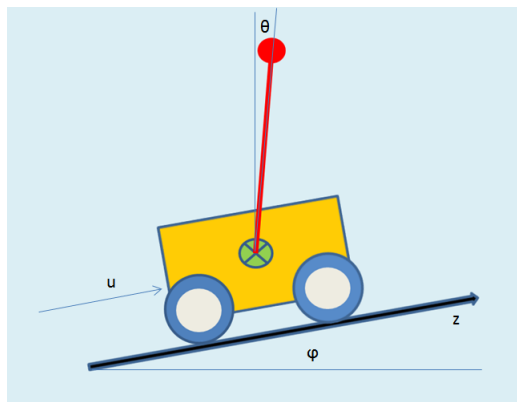


Figure 7.1: Inverted Pendulum on a ramp

Two sensors provide the position of the cart and the angle of the pendulum to the controller.

The equations of motion of the system may be expressed as follows:

$$\begin{cases} (M + m)\ddot{z} + ml\ddot{\theta} \cos(\theta - \phi)/2 - ml\dot{\theta}^2 \sin(\theta - \phi)/2 = u(t) - (M + m)g \sin(\phi) \\ (J + ml^2/4)\ddot{\theta} + ml\dot{z} \cos(\theta - \phi)/2 - mgl \sin(\theta)/2 = 0 \end{cases}$$

$M$  represents the mass of the cart, and  $m$  the mass of the pendulum. The moment of inertia of the pendulum, considered to be a homogeneous bar, is  $J = ml^2/12$ . The cart moves on a rail placed on a ramp with an angle  $\phi$ .

The system variables  $z$  and  $\theta$  represent respectively the position of the cart on the ramp and the angle of the pendulum. The angle zero corresponds to the vertical position, pointing up.

The system may be re-expressed in matrix form as follows:

$$\begin{pmatrix} M + m & ml \cos(\theta - \phi)/2 \\ ml \cos(\theta - \phi)/2 & J + ml^2/4 \end{pmatrix} \begin{pmatrix} \ddot{z} \\ \ddot{\theta} \end{pmatrix} = \begin{pmatrix} ml\dot{z}^2 \sin(\theta - \phi)/2 + u - (M + m)g \sin(\phi) \\ mgl \sin(\theta) \end{pmatrix}$$

The matrix on the left is square and invertible (provided  $m > 0$ ), so  $\ddot{z}$  and  $\ddot{\theta}$  may be computed by using a simple matrix formula. So the cart pendulum system may be implemented in **Activate** using the **MatrixExpression** block as shown in Fig. 7.2.

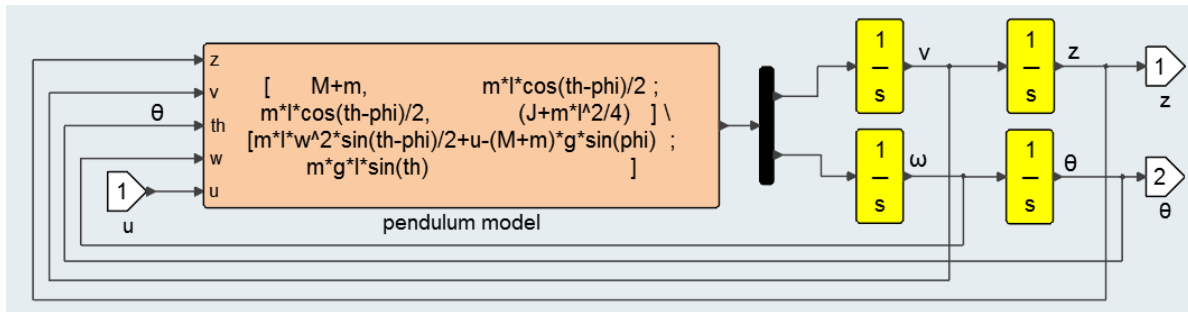


Figure 7.2: Pendulum superblock inside Inverted Pendulum model (inv\_pendulum0.scm).

The parameters of the system are defined in the Super Block Context as follows:

```
m=1;
M=4;
l=.7;
g=9.8;
J=m*l^2/12;
```

The complete model is illustrated in Fig. 7.3. Note that the controller is a state space linear system with parameters  $A_t$ ,  $B_t$ ,  $C_t$ .

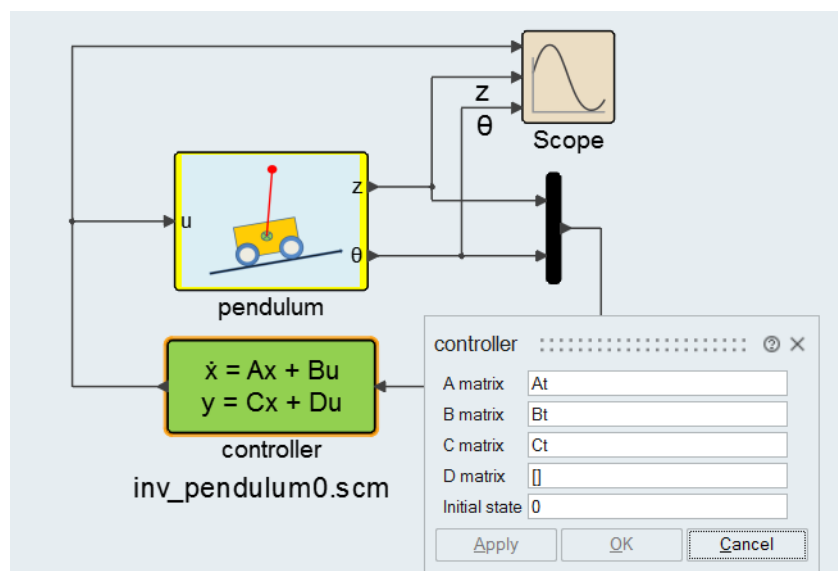


Figure 7.3: Inverted Pendulum model (inv\_pendulum0.scm).

The model is designed in such a way that the simulation can be performed both with and without a linearized model. In the absence of a linearized model, a “zero” controller (controller producing zero as output) is used. This is done in the Context of the top diagram as follows:

```

_syslin=GetFromBase('_syslin', []);
if isempty(_syslin)
    At=[];
    Bt=zeros(0,2);
    Ct=zeros(1,0);
else
    A=_syslin.a;
    B=_syslin.b;
    C=_syslin.c;
    D=_syslin.d;
    n=size(A,1);
    Kc=-place(A,B,-1*ones(1,n));
    Kf=-place(A',C',-2*ones(1,n));
    Kf=Kf';

    At=A+B*Kc+Kf*C+Kf*D*Kc;
    Bt=-Kf;
    Ct=Kc;
end

```

So if a structure called `_syslin` is defined in the base environment, it is used as the linearized model to design an observer-based controller. If not, the controller is set to zero. This way, when the model is linearized, the linearization is performed in the “open-loop” case (i.e., the plant alone).

The linearization can be performed with the following script:

```

clear('_syslin');
% vector of input port indices considered for linearization
inps = 1;
% vector of output port indices considered for linearization
outs = [1,2];
% Modified context
ctx=struct;
ctx.z0 = 0;
ctx.th0 = 0;
ctx.phi = 0;

model=bdeGetCurrentModel;
% Selected Super block to linearize
superblock = 'pendulum';

[A,B,C,D]=vssLinearizeSuperBlock(model,superblock,inps,outs,0,ctx);
_syslin.a=A;
_syslin.b=B;
_syslin.c=C;
_syslin.d=D;

```



The linearization is done around the point  $z = 0, \theta = 0, v = 0$  and  $\omega = 0$ . This is an equilibrium point assuming  $\phi = 0$ . The resulting controller stabilizes the system for small values of  $\phi$  but with a bias in the final position of the cart. For example in the case (Initialization script):

```
phi=0.001;
z0=-0.01;
th0=.002;
```

The simulation result is shown in Fig. 7.4. Note that to obtain this result, first the script must be run so that `_syslin` is made available in the base, then the model simulated.

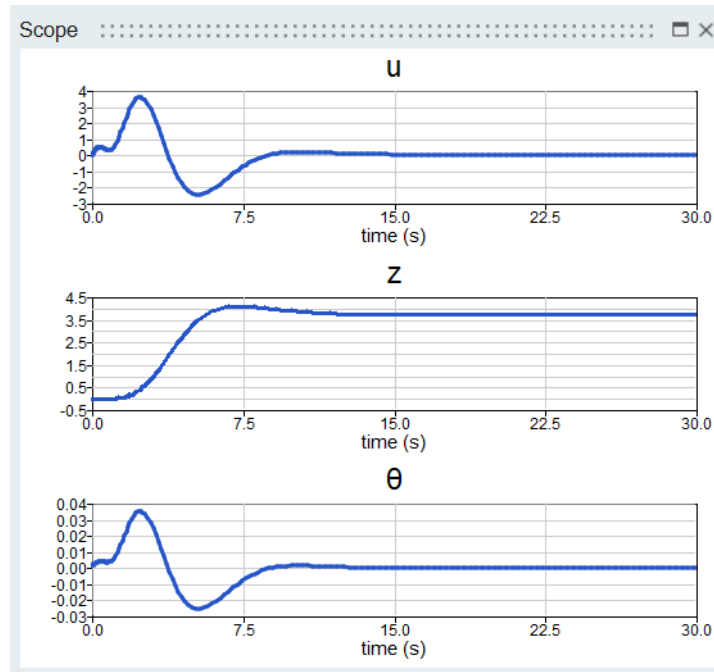


Figure 7.4: Simulation result for (inv\_pendulum0.scm). Note that  $z$  does not converge to zero since  $\phi$  is not zero in this case.

**Changing the model** The final bias in  $z$  can be removed either by taking into account the value of  $\phi$  and linearizing the plant accordingly (this requires the computation of a new equilibrium point and will be addressed in the next section) or by introducing an integral in the feedback loop (of  $z$ ). This can be done by changing the model as illustrated in Figs. 7.5 and 7.6.

The linearization is performed using the following script:

```
clear('_syslin');
% vector of input port indices considered for linearization
inps = 1;
% vector of output port indices considered for linearization
outs = [3,2];
% Modified context
ctx=struct;
ctx.z0 = 0;
ctx.th0 = 0;
```

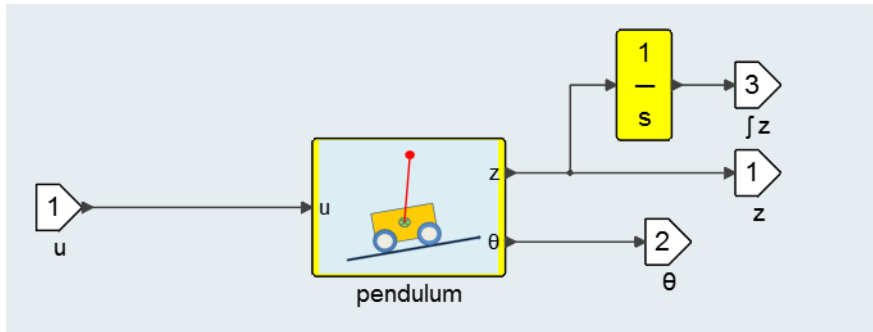


Figure 7.5: Pendulum\_integral superblock inside the Inverted Pendulum model (inv\_pendulum\_integral.scm). The integral of  $z$  is added as an output to be used for feedback control.

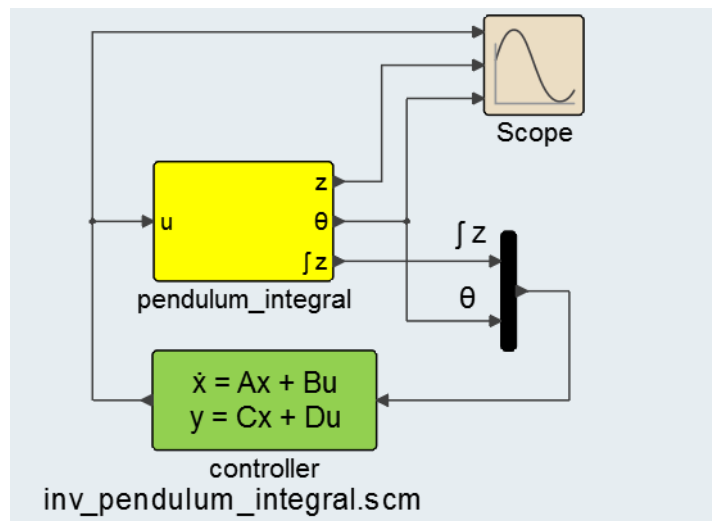


Figure 7.6: Inverted Pendulum model (inv\_pendulum\_integral.scm).

```
ctx.phi = 0;

model=bdeGetCurrentModel;
% Selected Super block to linearize
superblock = 'pendulum_integral';

% Linearization
[A,B,C,D]=vssLinearizeSuperBlock(model,superblock,inps,outs,0,ctx);
_syslin.a=A;
_syslin.b=B;
_syslin.c=C;
_syslin.d=D;
```

Note that only the second and third outputs of the Super Block (in the reverse order) are used in the definition of the Plant to be linearized. The simulation result is shown in Fig. 7.7. The bias is removed thanks to the integral in the loop.

To simplify the use of the function `vssLinearizeSuperBlock` for a given Super Block, the menu `Linearize` is provided. By selecting a Super Block and using the contextual menu `Linearize`, an

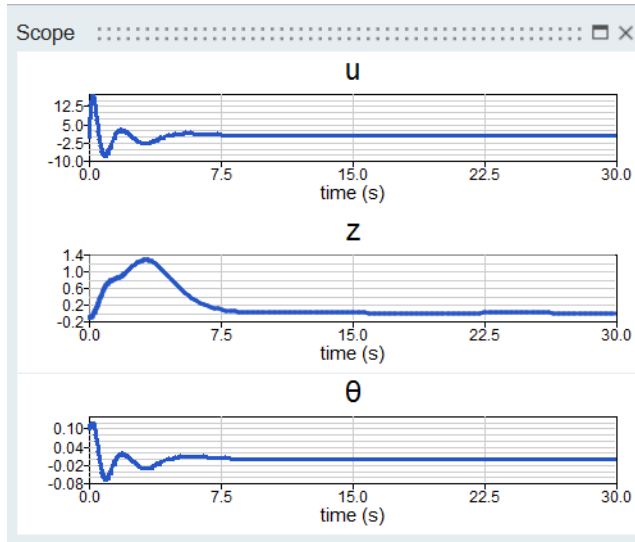


Figure 7.7: Simulation result for (inv\_pendulum\_integral.scm). Note that  $z$  does converge to zero even though  $\phi$  is not zero.

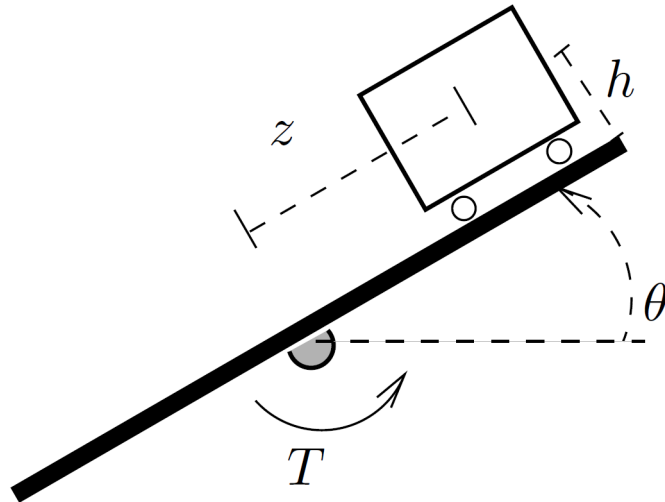
**OML** script is automatically generated containing the proper call to the function `vssLinearizeSuperBlock` for linearizing the selected Super Block. The script contains also information on how to calculate the equilibrium point. This functionality will be discussed later in this section.

The linearization script can be included inside the model using the **ExecOMLScript** block presented in Chapter 20.

The `vssLinearizeSuperBlock` function can also be used inside the Initialization script of the model to integrate controller design within the simulation model. This powerful mechanism is illustrated in the following example.

### 7.1.2 Example: Cart on a beam

A cart of mass  $m$  is placed on a beam. The cart can move freely along the beam without any friction. The beam pivots around a fixed point  $O$ . Its moment of inertial is denoted by  $J$  and its angle of rotation by  $\theta$ . The angle  $\theta = 0$  corresponds to the horizontal position.



The variable  $z$  denotes the projection of the center of mass of the cart on the beam; the zero position corresponds to the point  $O$ . The height of the center of mass of the cart (its distance to the beam) is denoted  $h$ .

The mechanism is controlled by applying a torque  $T$  to the beam around the point  $O$ . The objective of the control is to stabilize the cart on the beam.

The equations of motion for this system can be expressed as follows:

$$\begin{aligned} m\ddot{z} &= mz\dot{\theta}^2 - mg\sin(\theta) \\ (J + mz^2)\ddot{\theta} &= T - 2mz\dot{z}\dot{\theta} - mgz\cos(\theta) + mgh\sin(\theta) \end{aligned}$$

The objective is to design an observer-based controller for the system. This is done by first linearizing the system, then designing an observer-based controller for the linearized model. The measured outputs are considered to be the angle  $\theta$  and the position  $z$ :

$$y = \begin{pmatrix} z \\ \theta \end{pmatrix}.$$

The control input is  $u = T$ . So the system has four states, one input and two outputs.

The system is modeled in Modelica language as follows

```

model cart
  //parameters
  parameter Real h = 1;
  parameter Real m = 2;
  parameter Real J = 10;
  parameter Real th0 = 0;
  parameter Real z0 = 0;
  //input variables
  Real T;
  //output variables
  Real z(start=z0), th(start=th0);
  Real zd(start=0), thd(start=0);
  Real g=9.81;

```

```

equation
  der(th)=thd;
  der(z)=zd;
  der(zd) = z*thd*thd-m*g*sin(th);
  (J+m*z*z)*der(thd) = T-2*m*z*zd*thd-
    m*g*z*cos(th)+m*g*h*sin(th);
end cart;

```

which is implemented using a Custom Modelica block. The system could also be implemented using standard **Activate** blocks.

The **Activate** model is given in Fig. 7.8 and 7.9.

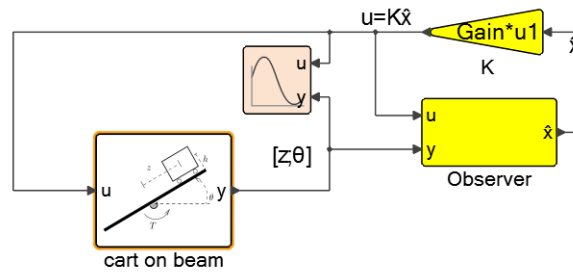


Figure 7.8: The model of the cart on beam system with an observer-based controller.

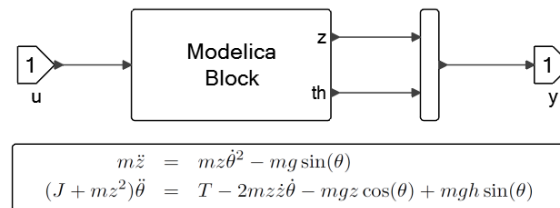


Figure 7.9: Inside the 'cart on beam' Super Block being linearized.

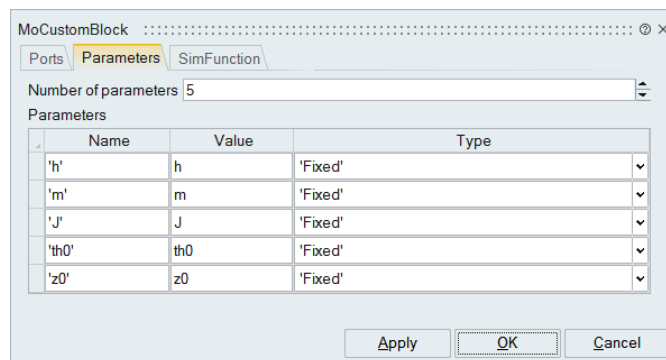


Figure 7.10: The parameters tab of the Custom Modelica block dialog GUI.

The initial position of the cart and the initial angle of the beam,  $z_0$  and  $th_0$ , are not necessarily zero for simulation but must be set to zero for linearization. This is done in the Initialization script of the model

as follows:

```
h=1; m=2; J=10;
nx=4; nu=1; ny=2;
th0=pi/12;
z0=-1;

if ~vssIsInLinearization
    ctx=struct;
    ctx.K=zeros(1,nx);
    ctx.obs=ss([],zeros(0,nu+ny),zeros(nx,0),zeros(nx,nu+ny));
    ctx.th0=0;ctx.z0=0;
    [A,B,C,D]=vssLinearizeSuperBlock(bdeGetCurrentModel,...
    'cart on beam',[1],[1],0,ctx);
    L=-place(A',C',-3*ones(nx,1))';
    obs=ss(A+L*C,[B+L*D,-L],eye(nx),zeros(nx,nu+ny));
    K=-place(A,B,-3*ones(nx,1));
end
```

The variable `vssIsInLinearization` is false in general; it is true only when the Initialization script is executed in the phase of linearization (the model is evaluated and simulated for linearization). As it can be seen in the script, when the model is in the linearization stage, the controller gain  $K$  is set to zero and the observer `obs` similarly to a trivial linear system. This is done through the external context `ctx`. `ctx` is used also to set to zero the initial states for linearization.

When the script is not used in the linearization stage, the observer-based controller is constructed and used for simulation. To construct an observer-based controller for a linear system  $(A, B, C, D)$ , a feedback gain matrix  $K$  is found such that  $A + BK$  is Hurwitz (all eigenvalues have strictly negative real parts) and an observer gain  $L$  is found such that  $A + LC$  is Hurwitz. This is possible as long as the system is stabilizable and detectable. Here the pole placement **OML** function `place` is used to compute  $K$  and  $L$ .

So when this model is simulated, the Super Block 'cart on beam' is linearized, the result of linearization is used to construct the observer-based controller, and the simulation of the resulting model for non zero initial conditions is performed. The simulation result is shown in Fig. 7.11.

**Changing the system** The model used above uses the model of a cart on a beam with the control being the torque applied to the beam. The model however, and in particular the Initialization script does not use any information related to the underlying system defined inside the Super Block that it controls except input, output and state sizes and certain system parameters. So this construction is very generic and can be applied to any other system placed inside the Super Block by bringing small adjustments to the Initialization script.

Consider here the following modification of this example: instead of the control being a torque applied to the beam, it is now a force applied to the cart at the level of its wheels (along the beam). The equations of motion the become:

$$\begin{aligned} m\ddot{z} &= mz\dot{\theta}^2 - mg\sin(\theta) + T \\ (J + mz^2)\ddot{\theta} &= -2mz\dot{z}\dot{\theta} - mgz\cos(\theta) + mgh\sin(\theta) \end{aligned}$$

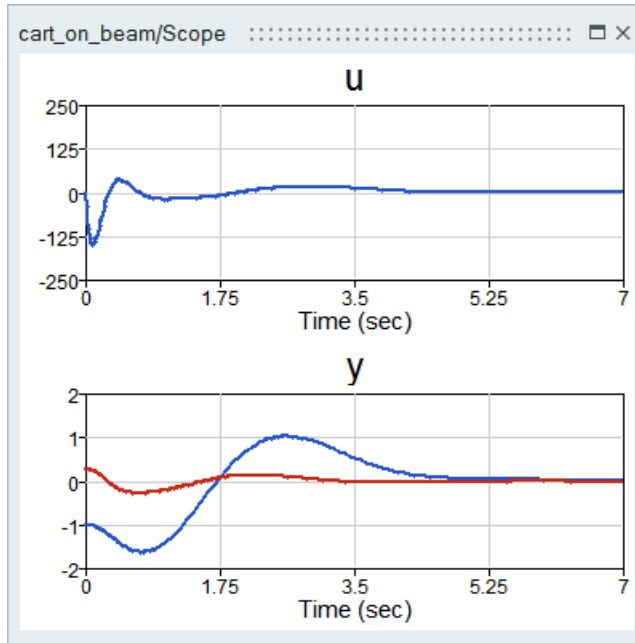


Figure 7.11: The simulation result shows that the system is stabilized.

where  $T$  represents now the applied force. This new system can now be placed inside the Super Block and no other modification is required. This is done in Model `cart_on_beam_force.scm`. The simulation shows that the controller is properly constructed and the system is stabilized.

**Stabilizing the cart off center** Consider now the problem of stabilizing the cart not at the center of the beam but at a different location. A possible approach would be to use the exact same model but introduce a bias in the output of the system before feeding it to the controller,  $y_0$ . See Fig. 7.12. The idea being that the controller will drive  $y - y_0$  to zero thus imposing the desired output.

The variable  $y_0$  is defined in the Initialization script as follows

```
zobj=0.2; y0=[zobj; 0];
```

The value of  $zobj$  corresponds to the location where the cart is to be stabilized. The simulation result is shown in Fig. 7.13. It can be seen that even though the use of bias has moved the location of the cart, the desired value  $zobj$  is not achieved. The reason is that the steady-state gain of the controller is not infinite.

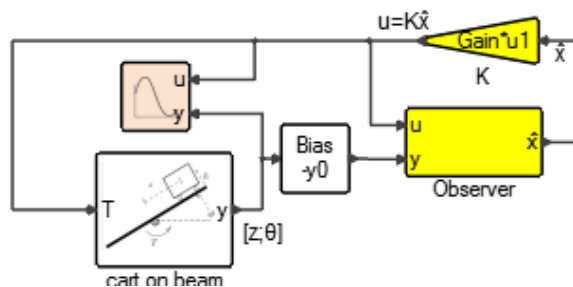


Figure 7.12: The model of the 'cart on beam' system with an observer-based controller.

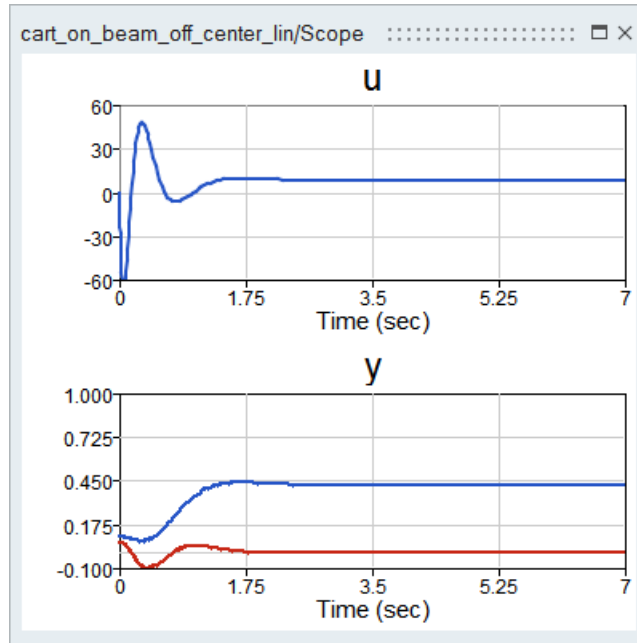


Figure 7.13: The simulation result shows that the cart is stabilized off the center of the beam but not at the desired location.

An alternative solution for stabilizing the cart off center is to linearize the system off center. For that, an equilibrium point must be found. The solution which consists of running the simulation for certain time cannot be used in this case because the system is unstable in open loop. The equilibrium point should then be computed differently.

## 7.2 Computation of the equilibrium point

The function `vssEquilibriumPointSuperBlock` is provided for finding equilibrium points by modifying model parameters defined by the user. The simple usage of this function is as follows

```
[ctx, err]=vssEquilibriumPointSuperBlock(model, sblk, ctx, params);
```

The first three input arguments of the function are identical to that of `vssLinearizeSuperBlock` function seen above. The cell array `params` contains the list of variable names in the `ctx` that can be modified by the function in order to set the state derivatives of all the continuous-time states of the blocks inside the Super Block `sblk` to zero. `ctx` is returned with the value of these variables modified.

In the cart on the beam example, the equilibrium state is known  $x_0=[z_{obj}; 0; 0; 0]$ , so is the equilibrium output  $y_0=[z_{obj}; 0]$ , which is imposed. What is unknown is the equilibrium input  $u_0$ .

The **Activate** model using this function is given in Fig. 7.14 This is done in the Initialization script of the model as follows:

```
h=1; m=2; J=10;
nx=4; nu=1; ny=2;
th0=pi/50;
z0=0.5;
zobj=1; y0=[zobj; 0];
```



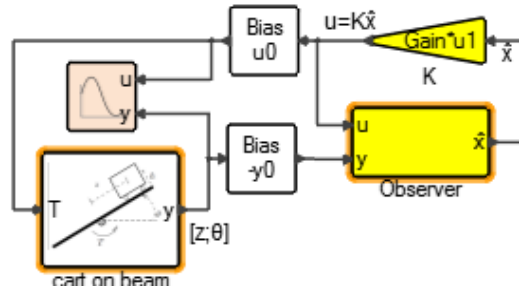


Figure 7.14: The model of the cart on beam stabilized with a controller linearized off center.

```

if ~vssIsInLinearization
    ctx=struct;
    ctx.K=zeros(1,nx);
    ctx.obs=ss([],zeros(0,nu+ny),zeros(nx,0),zeros(nx,nu+ny));
    ctx.th0=0;ctx.z0=zobj;ctx.u0=0;
    model=bdeGetCurrentModel;
    ctx=vssEquilibriumPointSuperBlock(model,...
'cart on beam',ctx,{'u0'});
    [A,B,C,D]=vssLinearizeSuperBlock(model,...
'cart on beam',[1],[1],0,ctx);
    L=-place(A',C',-7*ones(nx,1)');
    obs=ss(A+L*C,[B+L*D,-L],eye(nx),zeros(nx,nu+ny));
    K=-place(A,B,-7*ones(nx,1));
    u0=ctx.u0;
end

```

The simulation results shows that the desired location is achieved. See Fig. 7.15

The `vssEquilibriumPointSuperBlock` function has additional optional arguments:

```
[ctx,err]=vssEquilibriumPointSuperBlock(model,sblk,ctx,params,inps,outs,L,M,verb);
```

The optional argument `inps` is used to set the time-dependency of the inputs of the superblock to consider for the computation of the equilibrium point. This time dependency can also be set directly inside the diagram by checking the time-dependency parameter of the corresponding input port blocks. The input sizes can also be defined by setting the parameters of these blocks. This may be required since in some cases the compiler may not be able to determine port sizes by considering only the diagram inside of the Super Block. `inps` is a vector containing port numbers and its default value is `[]`.

The optional argument `outs` specifies the outputs of the Super Block to take into account in the construction of the output vector `y`. The default is `[]`.

The `L` and `M` matrices specify the vector which is set to zero by `vssEquilibriumPointSuperBlock`. This vector is: `[L*xdot;M*y]`. The default values are 1. If `L` is set to `[]`, `L*xdot` is excluded from the vector. Similarly, if `M` is set to `[]`, `M*y` is excluded from the vector.

If the optional argument `verb` is set to the string `'v'`, the verbose mode is turned on. In this mode,

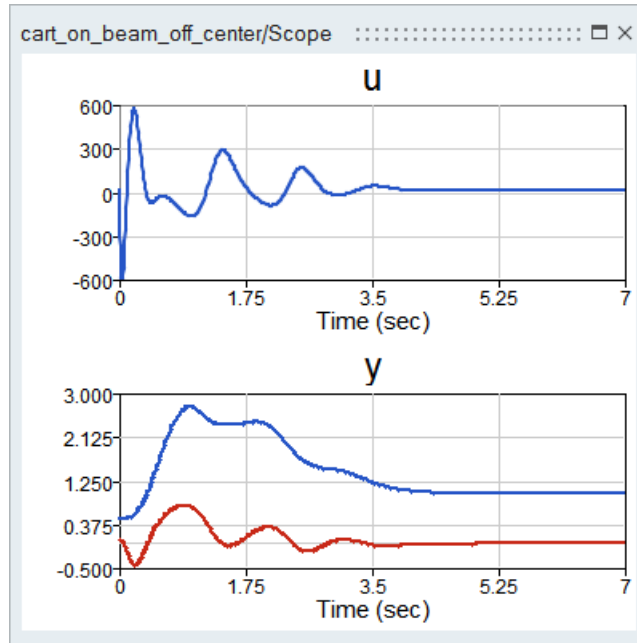


Figure 7.15: The simulation result shows that the objective is achieved.

the values tried by the solver and the corresponding vector  $[L*\dot{x}; M*y]$  are displayed at each iteration. This mode is useful for debugging.

So in general the `vssEquilibriumPointSuperBlock` function can be used to use the specified parameters to set parts or all of the state derivatives  $\dot{x}$  and the outputs  $y$  to zero. A systematic approach to finding the equilibrium point of a system is to construct the corresponding Super Block so that the inputs and output are respectively  $\delta u = u - u_e$  and  $\delta y = y - y_e$ . These modifications can be introduced using Bias blocks. The bias values  $u_e$  and  $y_e$  may be partially imposed, partially free for finding the equilibrium point.

Consider again the cart on the beam example. To obtain the equilibrium point previously, the value of the equilibrium state  $x_e$  was “guessed” from the value of the  $y_e$ , and the only free variable was  $u_e$ . In particular it was noted that the values of  $z_e$  and  $\theta_e$  are equal to the values specified in  $y_e$ .

Here, instead of guessing the values of  $z_e$  and  $\theta_e$ , a systematic approach is used. This is done by creating the Super Block by including bias terms so that the input and output are  $\delta u = u - u_e$  and  $\delta y = y - y_e$ . See Fig. 7.16.

### 7.2.1 Use of infinite steady-state gain controller

It was seen that simply by introducing a bias on the output of the controller designed for stabilizing the cart at the center stabilizes the cart off center. However the location of the stabilization does not correspond to the desired location because of the finite steady-state controller gain. If a controller with infinite gain is used, stabilization at the desired location can be achieved.

A way to obtain infinite steady-state gain is to introduce explicitly an integrator in the system. This is done in the model shown in Fig. 7.17 and 7.18.

The Initialization script in this case is

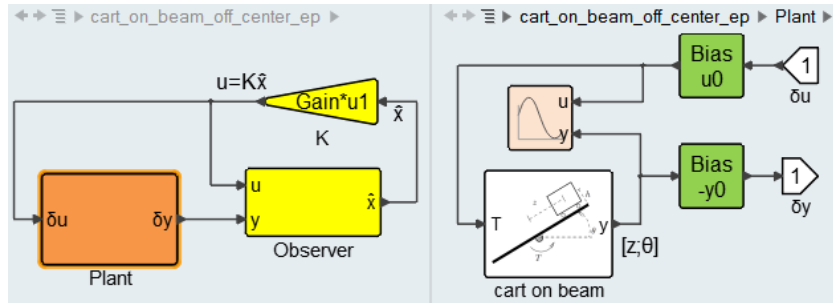


Figure 7.16: The Plant Super Block is used for computing the equilibrium point and linearization. See demo model `cart_on_beam_off_center_ep.scm`.

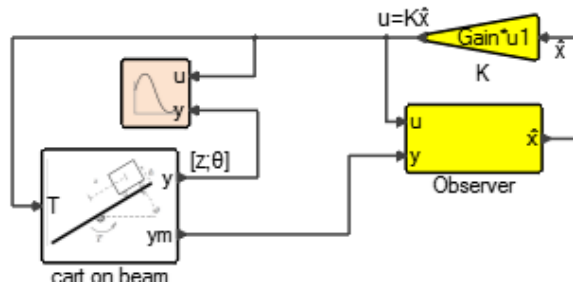


Figure 7.17: The model of the cart using an infinite steady-state gain controller.

```

h=1; m=2; J=10;
nx=5; nu=1; ny=2;
th0=pi/50;
z0=0.01;
zobj=0.025; y0=[zobj; 0];

if ~vssIsInLinearization
    ctx=struct;
    ctx.K=zeros(1, nx);
    ctx.obs=ss([], zeros(0, nu+ny), zeros(nx, 0), zeros(nx, nu+ny));
    ctx.th0=0; ctx.z0=0; ctx.zobj=0;
    model=bdeGetCurrentModel;
    [A, B, C, D]=vssLinearizeSuperBlock(model, ...
        'cart on beam', [1], [2], 0, ctx);
    L=-place(A', C', -5*ones(nx, 1))';
    obs=ss(A+L*C, [B+L*D, -L], eye(nx), zeros(nx, nu+ny));
    K=-place(A, B, -5*ones(nx, 1));
end

```

Note that for the linearization only the second output is taken into account. This is the output which is connected to the controller. The first output corresponding to the actual outputs of the system is used for display only.

The simulation result is shown in Fig. 7.19.

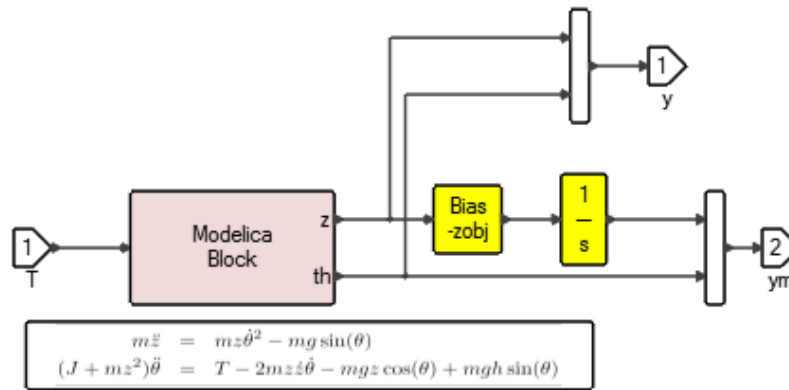


Figure 7.18: Inside the cart on beam Super Block. An integrator is included to force infinite steady-state gain.

The advantage of using this approach is that the value of  $z_{obj}$  is not used in the design process and it can be changed during the simulation. The model in Fig. 7.20 is similar to the previous model except that instead of the desired location being defined by a constant  $z_{obj}$ , it is a time varying signal (a square wave in particular).

For the linearization in this case the first input and the second output are taken into account. The simulation result is shown in Fig. 7.22.

## 7.2.2 Equilibrium point for the inverted pendulum example

Consider again the inverted pendulum problem considered in the previous section. Here the assumption is made that the value of  $\phi$  is known so the Plant can be linearized around the equilibrium point corresponding to the given value of  $\phi$ . In this case the input is not linearized around 0 but a value  $U$ , which must be determined so that the system is in an equilibrium point. The equilibrium state here is zero.

The input bias  $U$  is introduced as shown in Fig. 7.23. The value of  $U$  is not known and must be determined by the computation of the equilibrium point.

The computation of the equilibrium point and linearization is performed with the following script:

```
clear('_syslin');
% vector of input port indices considered for linearization
inps = 1;
% vector of output port indices considered for linearization
outs = 1:2;
% Modified context
ctx=struct;
ctx.z0 = 0;
ctx.th0 = 0;
ctx.U = 0;
```

```
model=bdeGetCurrentModel;
```

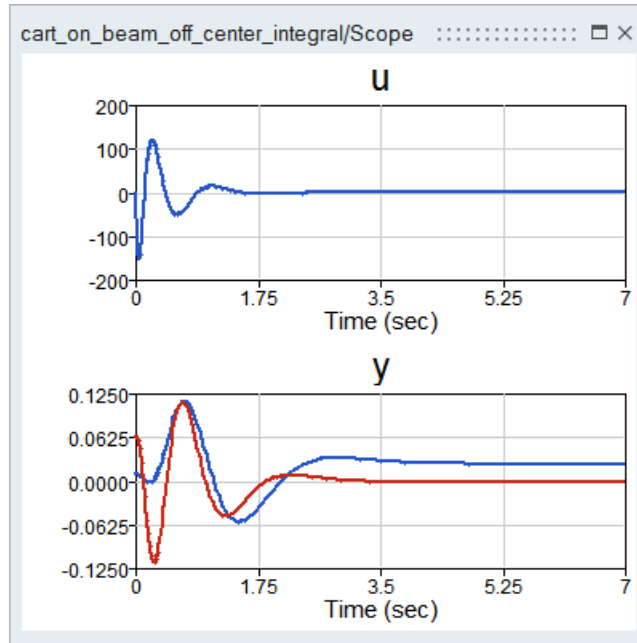


Figure 7.19: The simulation result shows that the system is stabilized at the desired location.

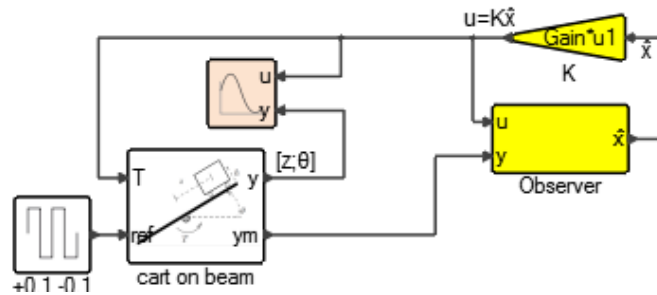


Figure 7.20: The inside of the 'cart on beam' Super Block is shown in Fig. 7.21.

```
% Selected Super block to linearize
superblock = 'pendulum';
% variables in ctx allowed to adjust
vars={'U'};
ctx=vssEquilibriumPointSuperBlock(model,superblock,ctx,vars,ins,outs,1,[],'v');
% Linearization
[A,B,C,D]=vssLinearizeSuperBlock(model,superblock,ins,outs,0,ctx);
_syslin.a=A;
_syslin.b=B;
_syslin.c=C;
_syslin.d=D;
_syslin.u0=ctx.U;
```

The verbose mode is used to monitor the progress of the optimization code computing the equilibrium point.

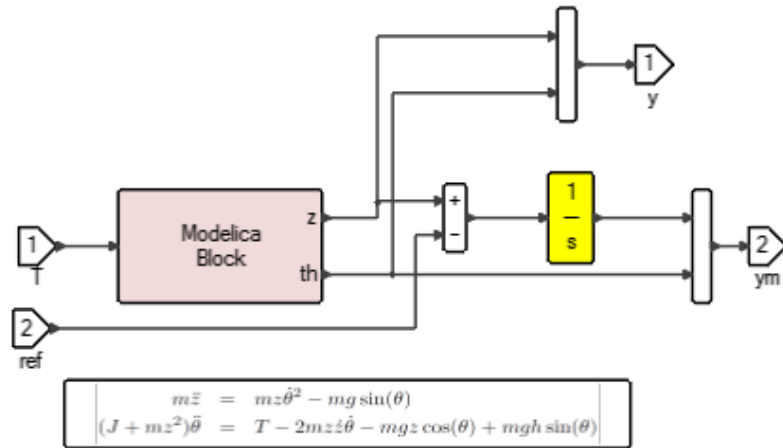


Figure 7.21: Inside the cart on beam Super Block. An integrator is included to force infinite steady-state gain.

The resulting simulation result is shown in Fig.7.24.

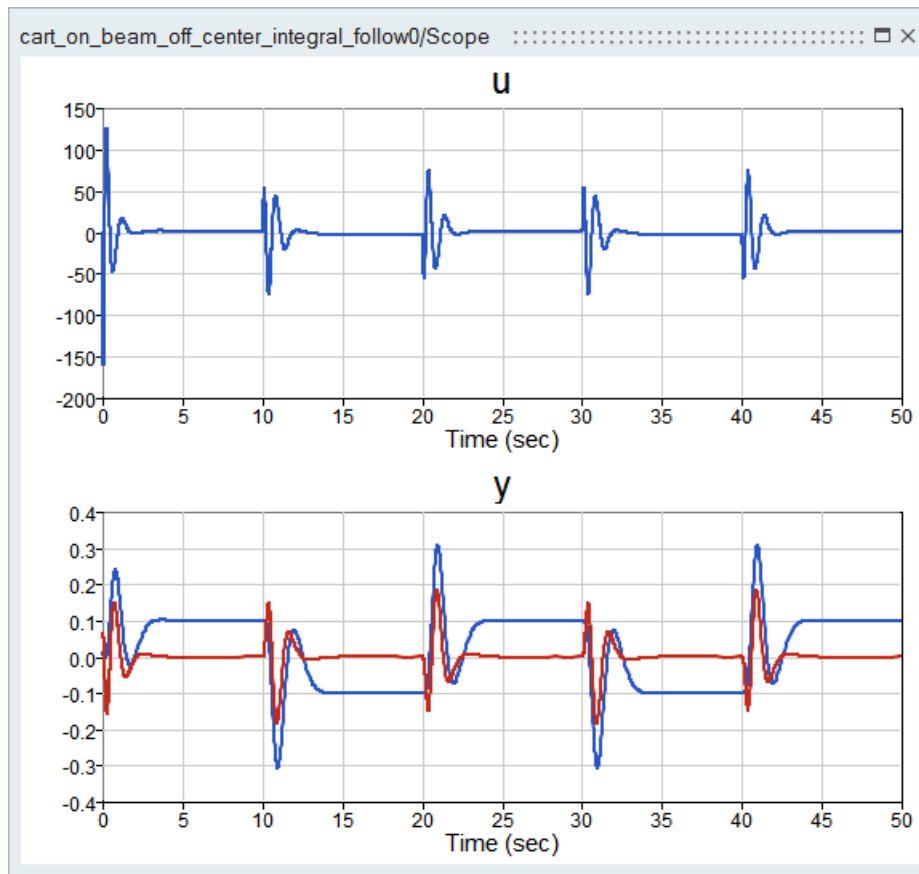


Figure 7.22: The simulation result shows that the location of the cart follows the square wave reference trajectory.

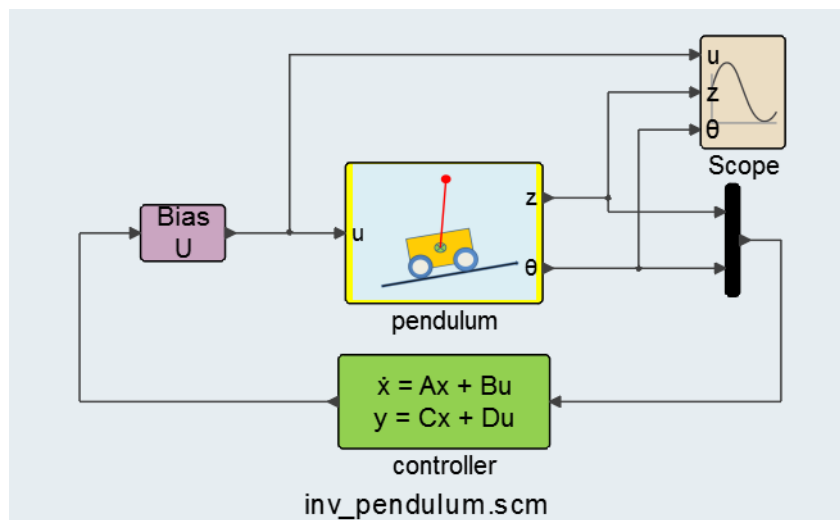


Figure 7.23: Inverted Pendulum model (inv\_pendulum.scm).

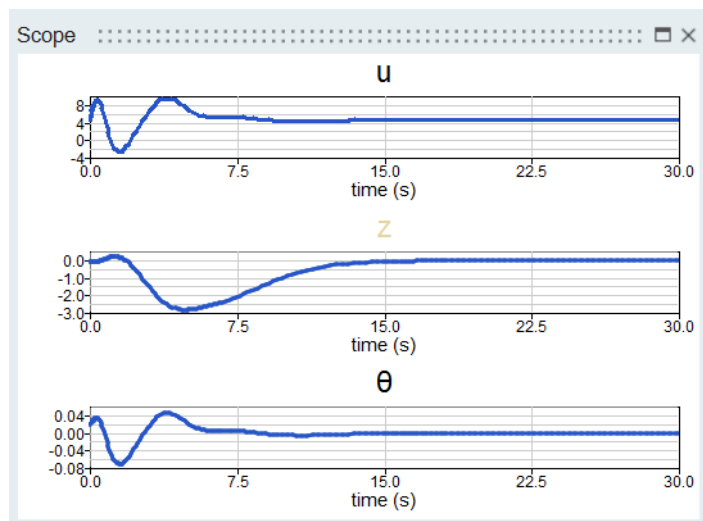


Figure 7.24: Simulation result for (inv\_pendulum.scm). Note that  $z$  converges to zero.



### 7.2.3 3D model of a spacecraft for take off and landing control

The model of the spacecraft is made of two bodies connected by a universal joint. The distance from the joint to the center of mass of the top body,  $(x_0, y_0, z_0)$ , is denoted  $l_0$ , and the distance to the bottom body's center of mass,  $(x_1, y_1, z_1)$ , is denoted  $l_1$ . The top body is the main body of the spacecraft. It has mass  $m_0$  with moments of inertia along the main axes  $I_{0z}$  and  $I_{0x} = I_{0y}$ . The propulsion system is lodged in the second body, which has mass  $m_1$  and inertia  $I_{1z}$  and  $I_{1x} = I_{1y}$ .

The coordinate system used to describe the dynamics of the system is attached to the universal joint. Its center is denoted  $(x, y, z)$ , the joint axis connected to the bottom body is the  $x$  axis of the system and the joint axis connected to the top body is the  $y$  axis of the system. The angle between the  $z$  axis and the main axis of the top body is denoted  $\alpha$  and with the bottom body,  $\beta$ . The two bodies are aligned when  $\alpha$  and  $\beta$  are zero.

The spacecraft is controlled via the propulsion force  $F$  applied orthogonal to the bottom body, at its center, and two torques to modify the angles  $\alpha$  and  $\beta$  at the universal joint. The torques can be produced by motors mounted on the bodies and acting (for example through a belt system) on the universal joint.

**Euler angles** The orientation of the coordinate system attached to the universal joint can be expressed by Euler angles. There are various ways to define Euler angles corresponding to the axis choices and orders in which rotations are applied. The commonly used choice ZXZ (rotate  $\phi$  around Z, then  $\theta$  around Y and finally  $\psi$  around Z) is not appropriate here because it has a singularity corresponding to the vertical position, which obviously is particularly interesting for the spacecraft.

Here the XYZ choice is used. The corresponding rotation matrices are:

$$R_x(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{pmatrix}, R_y(\theta) = \begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix}, R_z(\psi) = \begin{pmatrix} \cos \psi & \sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

resulting in the transformation matrix

$$R_z(\psi)R_y(\theta)R_x(\phi) = \begin{pmatrix} \cos \theta \cos \psi & \cos \theta \sin \psi & -\sin \theta \\ \sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi & \sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi & \sin \phi \cos \theta \\ \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi & \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi & \cos \phi \cos \theta \end{pmatrix}.$$

**Equations of motion** The equations are obtained from the Euler-Lagrange equation and the principle of virtual work. The Lagrangian  $L$  is defined as

$$L = T - V$$

where  $T$  denotes the kinetic energy of the system and  $V$  its potential energy.

The potential energy of the system is straightforward to compute:

$$V = m_0 z_0 + m_1 z_1.$$

Note that the universal joint is assumed to have zero mass. The kinetic energy is made of rotational and linear energies. The angular velocities  $\Omega_0$  and  $\Omega_1$  of the two bodies can be computed as follows:

$$\Omega_0 = R_y(\alpha) \left( \begin{pmatrix} 0 \\ \dot{\alpha} \\ 0 \end{pmatrix} + R_z(\psi) \left( \begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix} + R_y(\theta) \left( \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix} + R_x(\phi) \left( \begin{pmatrix} \dot{\phi} \\ 0 \\ 0 \end{pmatrix} \right) \right) \right) \right),$$

$$\Omega_1 = R_x(\beta) \left( \begin{pmatrix} 0 \\ \dot{\beta} \\ 0 \end{pmatrix} + R_z(\psi) \left( \begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix} + R_y(\theta) \left( \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix} + R_x(\phi) \left( \begin{pmatrix} \dot{\phi} \\ 0 \\ 0 \end{pmatrix} \right) \right) \right) \right)$$

The kinetic energy of the system given by

$$T = \frac{1}{2}(\Omega_0^T I_0 \Omega_0 + \Omega_1^T I_1 \Omega_1 + m_0(\dot{x}_0^2 + \dot{y}_0^2 + \dot{z}_0^2) + m_1(\dot{x}_1^2 + \dot{y}_1^2 + \dot{z}_1^2))$$

where the inertia matrices are defined as follows

$$I_i = \begin{pmatrix} I_{ix} & 0 & 0 \\ 0 & I_{iy} & 0 \\ 0 & 0 & I_{iz} \end{pmatrix}$$

$i = 0, 1$ .

The positions of the centers of masses of the two bodies can be expressed as follows

$$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - l_0 N_0$$

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + l_1 N_1$$

where  $N_i$ 's represent the main body axes of the two bodies:

$$N_0 = R_x(\beta) R_z(\psi) R_y(\theta) R_x(\phi) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

and

$$N_1 = R_y(\alpha) R_z(\psi) R_y(\theta) R_x(\phi) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Thanks to these relations, the system Lagrangian can be expressed in terms of the system state variables  $q = (x, y, z, \phi, \theta, \psi, \alpha, \beta)$ ,  $q_d = (x, y, z, \dot{\phi}, \dot{\theta}, \dot{\psi}, \dot{\alpha}, \dot{\beta})$ ; see Appendix.

The virtual work associated with the torques  $\sigma$  and  $\tau$  and the propulsive force  $F$  can be expressed as

$$W = \int F \left( N_1^T \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} \right) + \sigma d\alpha + \tau d\beta.$$

The equations of motion are then obtained from the Euler-Lagrange equation:

$$\begin{aligned} \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} &= F(-\sin \theta \cos \phi \cos \psi + \sin \phi \sin \psi) \\ \frac{d}{dt} \frac{\partial L}{\partial \dot{y}} - \frac{\partial L}{\partial y} &= F((\cos \psi \sin \phi + \sin \psi \sin \theta \cos \phi) \cos \beta + \sin \beta \cos \phi \cos \theta) \\ \frac{d}{dt} \frac{\partial L}{\partial \dot{z}} - \frac{\partial L}{\partial z} &= F((\cos \psi \sin \phi + \sin \psi \sin \theta \cos \phi) \cos \beta + \sin \beta \cos \phi \cos \theta) \\ \frac{d}{dt} \frac{\partial L}{\partial \dot{\phi}} - \frac{\partial L}{\partial \phi} &= 0 \\ \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} &= 0 \\ \frac{d}{dt} \frac{\partial L}{\partial \dot{\psi}} - \frac{\partial L}{\partial \psi} &= 0 \\ \frac{d}{dt} \frac{\partial L}{\partial \dot{\alpha}} - \frac{\partial L}{\partial \alpha} &= \sigma \\ \frac{d}{dt} \frac{\partial L}{\partial \dot{\beta}} - \frac{\partial L}{\partial \beta} &= \tau \end{aligned}$$

leading to the following second order system

$$M(q)\dot{q}_d + C(q)\dot{q}_d^2 + R(q, q_d) = U(q, u) \quad (7.1)$$

where  $u$  is the control input

$$u = \begin{pmatrix} F \\ \sigma \\ \tau \end{pmatrix}.$$

The expression involved in (7.1) are too complex to be expressed here (hundreds of lines of formulae). They are obtained using the symbolic computational software Maple, which is also used to generate automatically the associated **OML** codes to be used in **Activate** for simulation. They are also used to obtain a linear system by linearization.

### 7.3 Modeling in Activate

The system states are modeled by two integrator blocks representing  $q$  and  $q_d$ . The two integrator blocks are used to implement the dynamics as illustrated in Figure 7.25, the highlighted section.

The integrator blocks in this case realize:

$$\dot{q} = q_d \quad (7.2)$$

$$\dot{q}_d = M^{-1}(q)(-C(q)q_d^2 - R(q, q_d) + U(q, u)). \quad (7.3)$$

The expression on the right hand side of (7.3) is realized using an **OML** Custom block. The **OML** codes associated with matrices  $M$ ,  $C$ , and vectors  $R$  and  $U$ , generated by Maple, are called in this **OML** Custom block.

Using **OML** at run time to perform simulation in this case leads to long simulation times. Maple can be used to generate C code instead and a C Custom block may be used instead for faster simulation if performance becomes an issue.

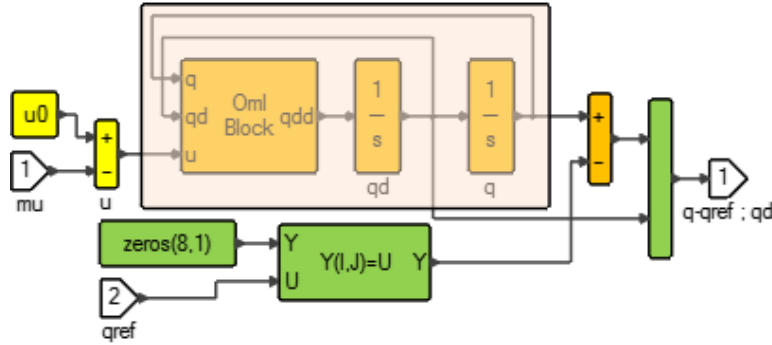


Figure 7.25: **Activate** diagram containing the dynamics of the spacecraft. The system equations are expressed in the **OML** Custom block.

**System linearization** A linear model approximating the behavior of the system around the stationary states of the spacecraft is sought. The stationary points of interest correspond to the state where the spacecraft is pointing straight up (with the two bodies aligned) and not moving. Since the position of the spacecraft, in particular variables  $x$ ,  $y$  and  $z$  do not appear explicitly in the system equations<sup>1</sup>, the linearization can be performed around  $q = 0$ ,  $q_d = 0$  and  $u = u_0$  where the propulsive force  $F_0 = (m_0 + m_1)g$  compensates the force of gravity:

$$u_0 = \begin{pmatrix} F_0 \\ 0 \\ 0 \end{pmatrix}.$$

The approximate linear model resulting from the linearization of the nonlinear system (7.2)-(7.3) can be numerically constructed in **OML** using the **OML** code representing the dynamics of the system. But it can also be constructed using the linearization function `rock3dOML_lin\Spacecraft`.

$$\dot{\xi} = A\xi + B\mu \tag{7.4}$$

where  $\mu = u - u_0$  and

$$\xi = \begin{pmatrix} q \\ q_d \end{pmatrix} - \begin{pmatrix} x^* \\ y^* \\ z^* \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{7.5}$$

## 7.4 State feedback control

The assumption here is that the controller has access to all the states of the system  $q$  and  $q_d$ . A constant state feedback controller is sought. Unfortunately the linear model (7.4) is not controllable. The pair  $(A, B)$  controllable subspace has dimension 14 (where the system state has dimension 16). The uncontrollable modes are zero and seem to correspond to the spacecraft spin around its main axis.

<sup>1</sup>The spacecraft dynamics does not depend on the position of spacecraft in space, in this model.

This spin cannot be controlled by linear control, so brought back to zero if perturbed, but this may not be an issue, especially for a short flight.

For a controllable pair  $(A, B)$ , it is always possible to find a matrix  $K$  such that all eigenvalues of the matrix  $A - BK$  have strictly negative real parts. Such a  $K$  then used as state feedback would make the closed-loop system asymptotically stable. In this case  $(A, B)$  not being controllable, the best that can be achieved is to find  $K$  such that all the controllable modes are moved so that they have negative real parts but the uncontrollable modes would remain in place. To find  $K$  in this case, the system is decomposed into controllable uncontrollable parts and a state feedback matrix  $K_1$  is found for the controllable part, which is then used to construct  $K$ . In particular an orthogonal matrix  $U$  is used as a change of coordinate matrix such that

$$\begin{aligned} U^T A U &= \begin{pmatrix} A_1 & A_2 \\ 0 & A_4 \end{pmatrix} \\ U^T B &= \begin{pmatrix} B_1 \\ 0 \end{pmatrix} \end{aligned}$$

where  $(A_1, B_1)$  is controllable. Then  $K_1$  is chosen such that the matrix  $A_1 - B_1 K_1$  has eigenvalues with strictly negative real parts. Finally  $K$  is constructed as follows

$$K = \begin{pmatrix} K_1 & 0 \end{pmatrix} U^T. \quad (7.6)$$

The **OML** code implementing this technique and pole placement for the construction of the feedback  $K_1$ , placed in the Initialization script of the model, is given below:

```
path=bdeGetModelFilePath(bdeGetCurrentModel());
path=fileparts(path);
addpath(path);

l0=6;l1=5;m0=2;m1=10;g=10;I0x=60;I0z=1;I1x=40;I1z=20;

St=[0,15,15,30,30,50,100];
Sp=[0,0,1,1,1,1,1;0,0,-1,-1,-1,-1,-1;1,1,1,1,0,0,0];

q0=zeros(8,1);X0=[q0;0*q0];
u0=[g*(m0+m1);0;0];
if ~vssIsInLinearization
    ctx=struct; ctx.K=zeros(3,16);
    model=bdeGetCurrentModel;
    [A,B,C,D]=vssLinearizeSuperBlock(model,...
    'Spacecraft',[1],[1],0,ctx);
    A=C*A*inv(C);B=C*B;C=C*inv(C);
    B=-B;
    %[A,B]=fb(X0,u0,[l0;l1;m0;m1;g;I0x;I0z;I1x;I1z]);
    [nc,U]=contr(A,B);

    A1=U'*A*U; B1=U'*B;
    nc=nc(1);A1=A1([1:nc],[1:nc]);
    B1=B1([1:nc],:);
    K1=place(A1,B1,-ones(nc,1)*2);
```

```

K=[K1,zeros(3,16-nc)]*U';
end

```

The controllable modes are moved to  $-2$ .

Linear state feedback  $K$  is used as follows:

$$u = u_0 + K \left( \begin{pmatrix} q \\ q_d \end{pmatrix} - X_{\text{ref}} \right)$$

with  $X_{\text{ref}} = (x^*, y^*, z^*, 0, \dots, 0)$  where  $(x^*, y^*, z^*)$  designates the position where the spacecraft is to be stabilized. The reference signals are used to modify the spacecraft model such that the inputs and outputs are zero at equilibrium point.

The top level diagram of the model is illustrated in Figure 7.26. It shows the implementation of the control law.

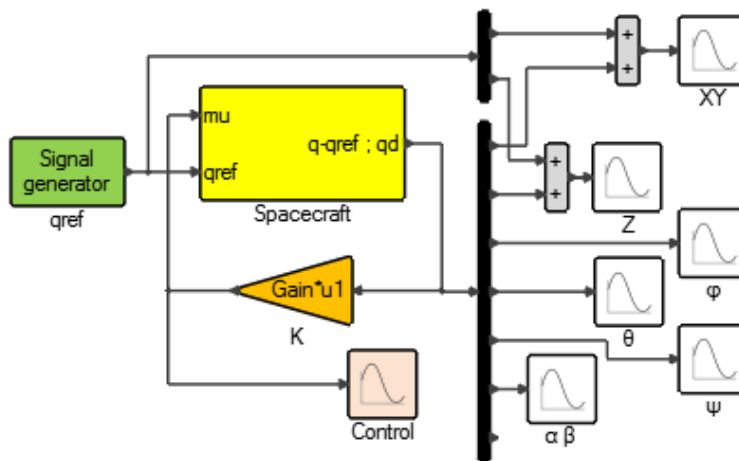


Figure 7.26: **Activate** top diagram shows the complete model.

**Simulation results** The performance of the control law is assessed by running simulations. In the following test simulation, the reference position  $(x^*, y^*, z^*)$  is assumed to be initially  $(0, 0, 1)$  then at  $t = 15$ ,  $(1, -1, 1)$  followed by  $(1, -1, 0)$  at  $t = 30$ . The spacecraft is initially at point  $(0, 0, 0)$  so the objective is to lift the spacecraft up one meter, then move it sideways and then land it.

The simulation results are shown in Figures 7.27 through 7.4.

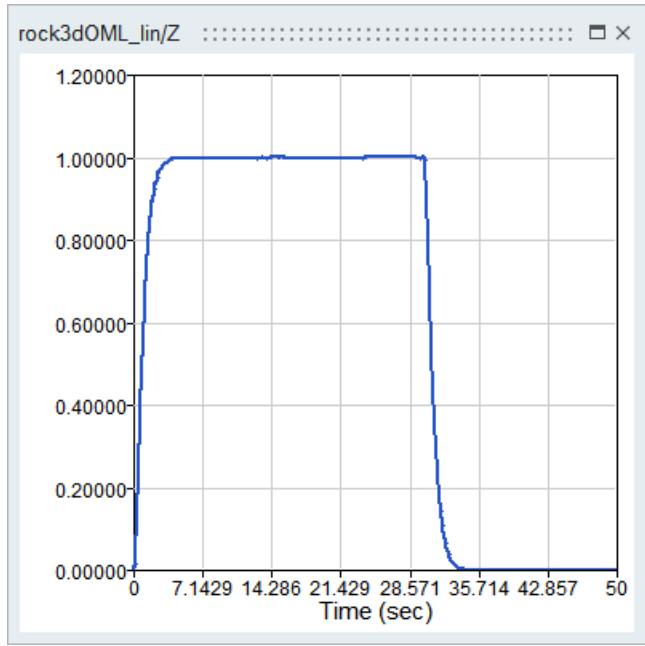


Figure 7.27: The altitude  $z$  follows the reference trajectory by going from 0 to 1 and then back to 0.

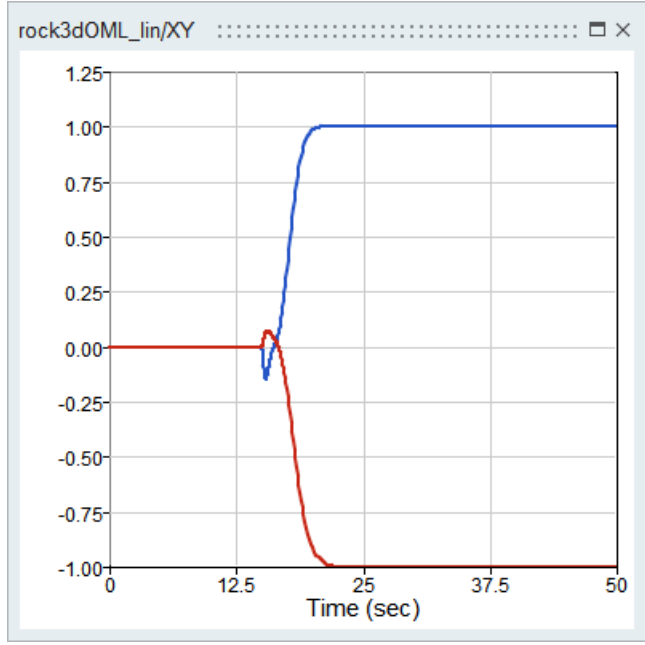


Figure 7.28: The XY position of the spacecraft follows the reference by going from (0, 0) to (-1, 1).

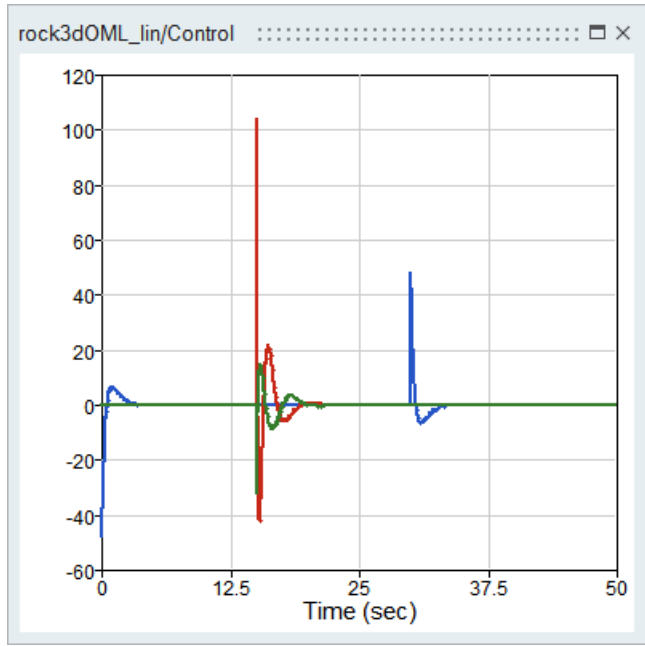


Figure 7.29: The control signals are illustrated here:  $F - F_0$ ,  $\sigma$  and  $\tau$ .

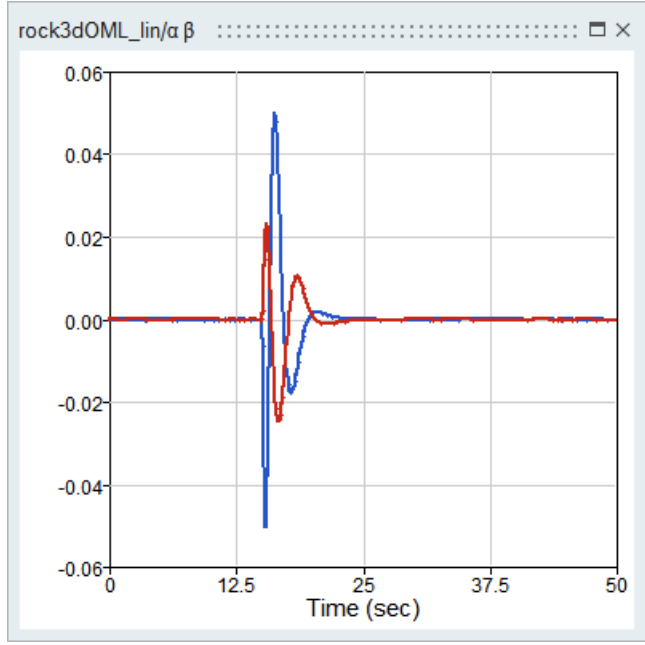


Figure 7.30: The angles  $\alpha$  and  $\beta$  corresponding to the universal joint.



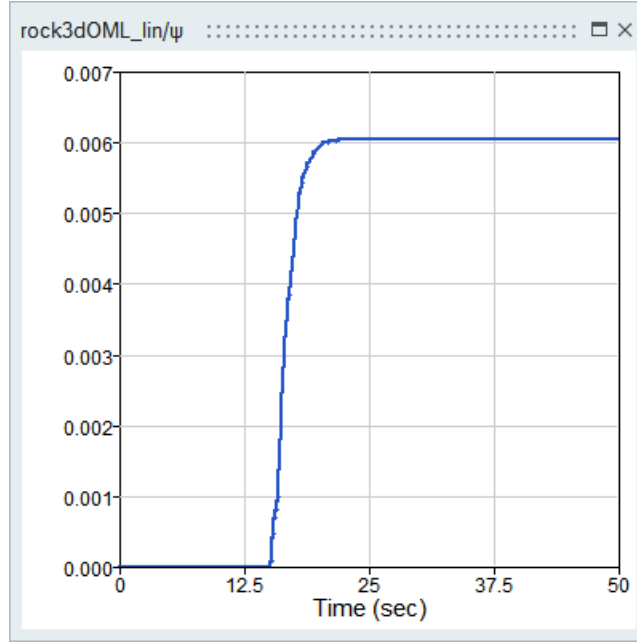


Figure 7.31: The simulation shows a drift in the  $\psi$  signal, which is not controllable and thus cannot be brought to zero. This however is not an issue.

## Appendix

$$\begin{aligned}
 L = & \left( (-0.5 \dot{\alpha}^2 l_1^2 m_1 + 0.5 \dot{\beta}^2 l_0^2 m_0) \cos(\phi)^2 + (0.5 I_{0x} - 0.5 I_{0z}) \dot{\phi}^2 \cos(\alpha)^2 + (0.5 I_{1z} \right. \\
 & - 0.5 I_{1x}) \dot{\phi}^2 \cos(\beta)^2 + (-0.5 I_{0x} + 0.5 I_{0z} - 0.5 I_{1z} + 0.5 I_{1x}) \dot{\phi}^2 \cos(\theta)^2 + (\dot{\alpha}^2 l_1^2 \\
 & m_1 - \dot{\beta}^2 l_0^2 m_0) \cos(\phi)^2 + (-0.5 I_{0x} + 0.5 I_{0z}) \dot{\theta}^2 \cos(\alpha)^2 + (-0.5 I_{1z} + 0.5 I_{1x}) \\
 & \dot{\theta}^2 \cos(\beta)^2 - 0.5 \dot{\alpha}^2 l_1^2 m_1 + (-0.5 I_{0z} - 0.5 I_{1x} + 0.5 I_{1z} + 0.5 I_{0x}) \dot{\theta}^2 \\
 & + 0.5 \dot{\beta}^2 l_0^2 m_0) \cos(\psi)^2 + \left( (\cos(\phi))^2 \sin(\theta) \dot{\beta} l_0^2 m_0 \dot{\psi} + (-\cos(\alpha) l_1 m_1 \dot{\theta} \right. \\
 & \dot{x} - \sin(\alpha) l_1 m_1 \dot{\theta} \dot{z} - \sin(\phi) \dot{\alpha} l_1^2 m_1 \dot{\psi} + l_0 m_0 \dot{\theta} \dot{x}) \\
 & \cos(\phi) + (I_{0x} - I_{0z}) \dot{\theta} \dot{\phi} \cos(\alpha)^2 + (-I_{1x} + I_{1z}) \dot{\theta} \dot{\phi} \cos(\beta)^2 \\
 & + (-I_{1z} + I_{0z} - I_{0x} + I_{1x}) \dot{\theta} \dot{\phi} \sin(\psi) + (I_{0z} - I_{0x}) \dot{\phi}^2 \sin(\alpha) \cos(\alpha) \\
 & \sin(\theta) + (I_{0z} - I_{0x}) \dot{\psi} \dot{\phi} \sin(\alpha) \cos(\alpha) + \dot{\phi} \dot{\beta} (l_0^2 m_0 + I_{1x}) \\
 & \cos(\theta) + \cos(\phi)^2 \dot{\alpha} l_1^2 m_1 \dot{\theta} + \left( (-\dot{\alpha}^2 l_1^2 m_1 + \dot{\beta}^2 l_0^2 m_0) \sin(\phi) \right. \\
 & \sin(\theta) \sin(\psi) + (-\cos(\alpha) \dot{\alpha} l_1 m_1 \dot{z} - \cos(\beta) l_0 m_0 \dot{\psi} \dot{y} + \sin(\phi) \\
 & \dot{\beta} l_0^2 m_0 \dot{\theta} + m_1 l_1 (\dot{x} \dot{\alpha} + g) \sin(\alpha) + \dot{\psi} (z l_0 m_0 \sin(\beta) + m_1 \\
 & \dot{y} l_1)) \sin(\theta) - \cos(\beta) l_0 m_0 \dot{\phi} \dot{y} + \dot{\phi} (z l_0 m_0 \sin(\beta) + m_1 \dot{y} l_1) \\
 & \cos(\phi) + (\cos(\alpha) \sin(\phi) l_1 m_1 \dot{\phi} \dot{x} + (I_{1x} - I_{1z}) \dot{\theta} \dot{\phi} \sin(\beta) \cos(\beta) \\
 & + (\sin(\alpha) l_1 m_1 \dot{\phi} \dot{z} - l_0 m_0 \dot{\phi} \dot{x}) \sin(\theta) + \cos(\alpha) \sin(\phi) l_1 m_1 \\
 & \dot{\psi} \dot{x} + (\sin(\phi) \dot{\beta} l_0 m_0 \dot{z} + (I_{1x} - I_{1z}) \dot{\psi} \dot{\theta} \sin(\beta)) \cos(\beta) \\
 & + (\sin(\alpha) l_1 m_1 \dot{\psi} \dot{z} + (-g + \dot{\beta} \dot{y}) l_0 m_0 \sin(\beta) - l_0 m_0 \dot{\psi} \dot{x}) \sin(\phi) \\
 & + I_{0x} \dot{\alpha} \dot{\theta}) \cos(\psi) + \left( (-0.5 \dot{\psi}^2 + 0.5 \dot{\alpha}^2) l_1^2 m_1 - 0.5 l_0^2 m_0 \dot{\psi}^2 \right) \\
 & \cos(\phi)^2 + (0.5 I_{0x} - 0.5 I_{0z}) \dot{\phi}^2 \cos(\alpha)^2 + (I_{1x} - I_{1z}) \dot{\phi}^2 \cos(\beta)^2 + (0.5 I_{1z} - 0.5 I_{1x}) \\
 & \dot{\phi}^2 \cos(\theta)^2 + (-\sin(\psi) \cos(\phi)^2 \sin(\theta) \dot{\alpha} l_1^2 m_1 \dot{\psi} + (-\cos(\beta) l_0 m_0 \\
 & \dot{\theta} \dot{y} - \sin(\phi) \dot{\beta} l_0^2 m_0 \dot{\psi} + \dot{\theta} (z l_0 m_0 \sin(\beta) + m_1 \dot{y} l_1)) \\
 & \sin(\psi) + (-g - \dot{x} \dot{\alpha}) l_1 m_1 \cos(\alpha) + (g - \dot{\beta} \dot{y}) l_0 m_0 \cos(\beta) + (-l_1^2 m_1 \\
 & \dot{\psi} \dot{\theta} - l_0^2 m_0 \dot{\psi} \dot{\theta}) \sin(\phi) + \sin(\beta) \dot{\beta} l_0 m_0 \dot{z} - \sin(\alpha) \\
 & \dot{\alpha} l_1 m_1 \dot{z}) \cos(\phi) + \left( (-I_{1x} + I_{1z}) \dot{\phi}^2 \sin(\beta) \cos(\beta) \sin(\theta) + (-I_{1x} + I_{1z}) \right. \\
 & \left. \dot{\psi} \dot{\phi} \sin(\beta) \cos(\beta) + (-\dot{\alpha} l_1^2 m_1 - I_{0x} \dot{\alpha}) \dot{\phi} \right) \sin(\psi)
 \end{aligned}$$



# Chapter 8

## Altair Activate Libraries

In **Activate**, libraries are self-contained repositories that essentially provide the application with new blocks. But, beyond just blocks, a library provides also (**OML**) functions that the user can use within his scripts.

### 8.1 Library structure

A library in **Activate** can be described by the following structure:

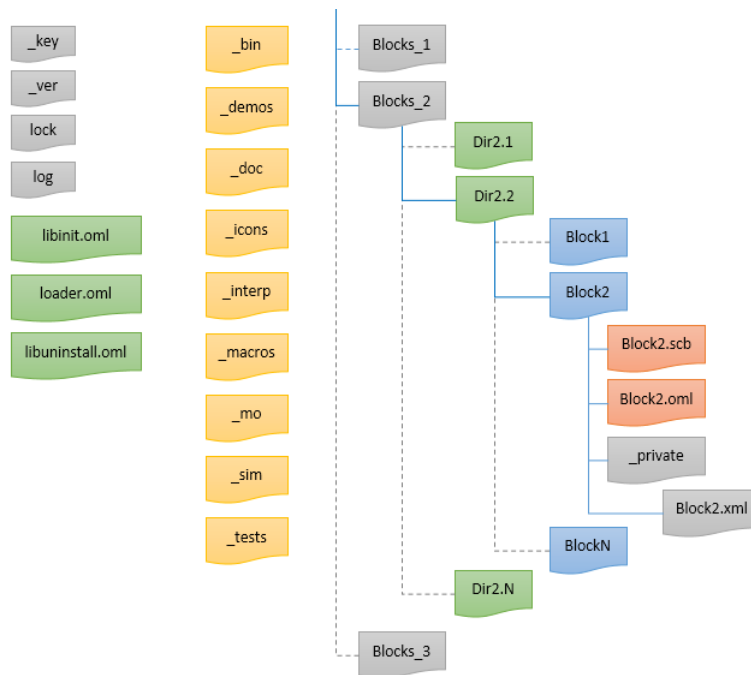


Figure 8.1: Representation of a typical library in Activate

- key: The library is identified by its key. The key is generated automatically by **Activate** in the file `_key`. This file is updated every time the library is edited after a release; a new version of

the library is created. The first line of the `_key` file defines the current version ID, the other lines defines the IDs of the old versions. This file should not be manually edited, as this could alter the library.

- loader: The blocks of the library are loaded into **Activate** through the `loader.oml` file. This file is automatically generated, it must not be edited.
- libinit: The library can add to the **Activate** environment palettes, menus and functions. The user can edit the `libinit.oml` file to perform such actions.
- palette(s): The palettes of the library can be any `scm` file loaded as palette in the `libinit` file. The palette browser shows only the blocks. The links and annotations are discarded.
- block : A block in **Activate** is defined by the interface file (**SCB**) and the simulation functions. The block interface files can be organized hierarchically under the condition that the folder that contains the **SCB** file must have the same name as the **SCB**. The Hierarchical path defines the reference path of the block, it is relative to the library top level. The simulation functions of the block must be inside the `_sim` folder of the library. The relative paths of the simulation functions must be equivalent to the ones defined in the **SCB** file.
- icons: Icons of the blocks should be defined inside the `_icons` folder of the library.
- documentation: The block documentation is generated inside `_doc` folder.

The image below can summarize the library structure:

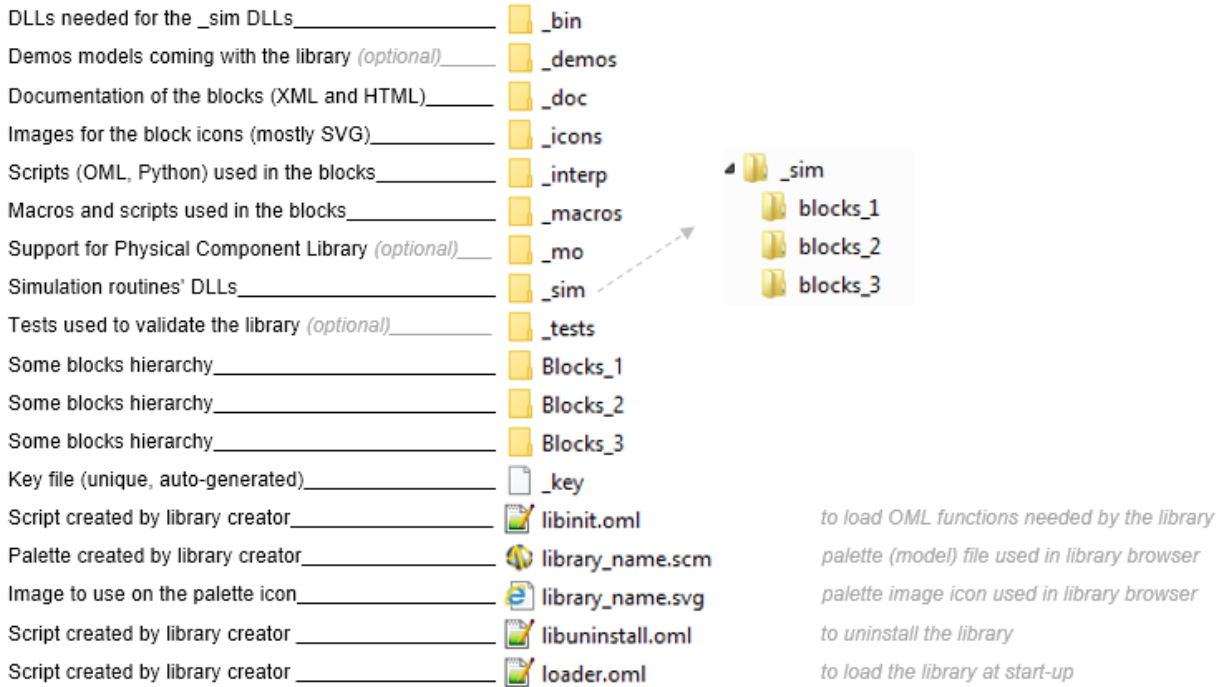


Figure 8.2: Description of a library folder in Activate

## 8.2 Block structure

In a **Activate** library a block is defined by several files.

- **SCB**: The file describes the graphics part of the block. It contains also the parameters definition and the evaluation part of the block. This file is generated by selecting a block and using the menu **File/Save Selected**. The block can also be a masked Super Block, or an atom block. Creating an atom block can be done through the menu **Tools/Block Builder**.
- **OML**: The file is generated from the **SCB**, it should not be edited. It is used during the evaluation phase of the simulation.
- **XML**: The file is used for the documentation, the skeleton of that file is generated automatically from the **SCB** file. This file should be completed by the block creator.
- **HTML**: The file is generated from the **XML** file, it is pushed under `_doc` folder.

As an example, the image below illustrates the structure of the constant block in the Activate library.

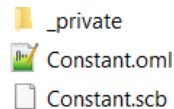


Figure 8.3: Constant block folder content

A block can have one or many simulation functions, they are considered to be C entry point functions. They are defined under the `_sim` folder of the library. A library creator can decide to either ship the C code or not, but it is mandatory to ship the static and the shared libraries. If a simulation function depends on a third party library, the shared and static library of that third party must be delivered under the `_bin` folder of the library.

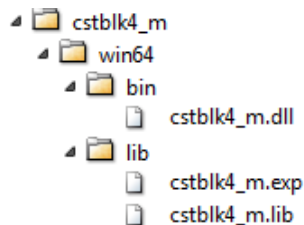


Figure 8.4: Constant block: View of the simulation folder content

## 8.3 Palette

The palette is the way to expose the blocks of a library to the user. It is very similar to a model. It can be edited by **Activate** as any other model. A non masked Super Block in a palette is used to create hierarchy in the palette browser. A masked Super Block is seen in the palette browser as a block. Since the **Block Diagram Editor** manipulates palettes as models, the extension of the palette file is **scm**.

A library can have several palettes, and in contrast, some libraries may not need to expose any block, in which case the library does not have any palette.

Since the palettes are the interface of the library, the user can choose the blocks to be exposed. Some of the blocks that are considered as “support” blocks, may not be place into a palette. However, users can still access such blocks through the menu **File/Libraries/Insert Block From Library**.

A palette does not need to be associated with any library. The user can create his own palette based on the existing blocks. He can, for example, create his favorite palette to host his most used blocks. In that case, the user will create a model and use the menu **File/Libraries/Save as Palette**. This palette is then loaded and exposed under the Personal page of the Library Browser.

When a library is Uninsulated, files are removed. The palettes loaded by that library should be removed from the palette browser. To remove Personal palettes, the user can use the menu **File/Libraries/Remove Palette**.

## 8.4 Library creation

**Activate** offers two ways from creating and editing a library:

- Library Manager: **Activate** provides a **GUI** to help the user create and edit his libraries.
- **OML** functions: **Activate** provides **OML** functions to allow the user to create and edit his libraries.

Note that the Library Manager provides an easy but limited way to edit the library. For advanced users, the use of (**OML**) functions is recommended.

### 8.4.1 Library Manager

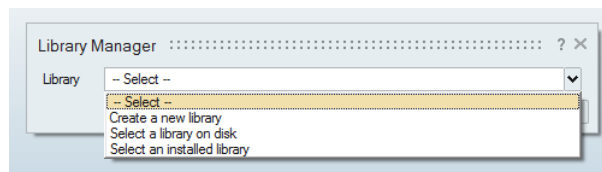


Figure 8.5: Library Manager

The Library Manager allows the user to:

- create a new library: The user must define the name and the path of the library. Note that upon creation, the library is also installed.

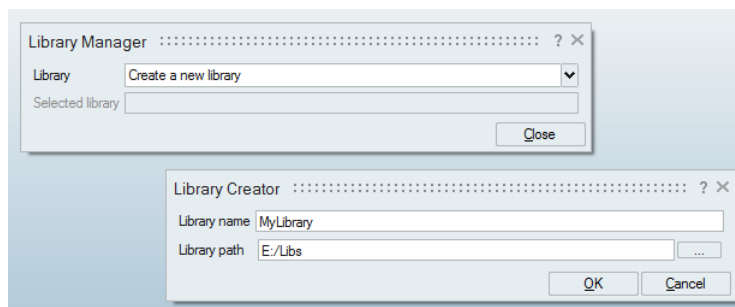


Figure 8.6: Create Library

- edit an installed library: The user can select one of the installed libraries.
- edit a library on disk: The user can select the library (a folder) on his disk. The library will be installed.

Once the library is installed, the Library Manager offers to the user the possibility to add or replace blocks in the library. The user can then select a masked Super Block and click the button **Add/Replace**.

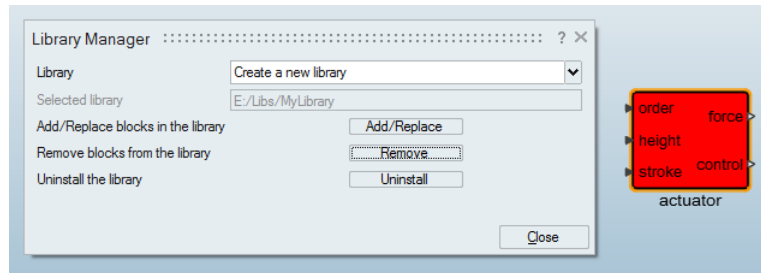


Figure 8.7: Add or Replace action in Library

The block is added to the library, the loader is updated and the palette is regenerated.

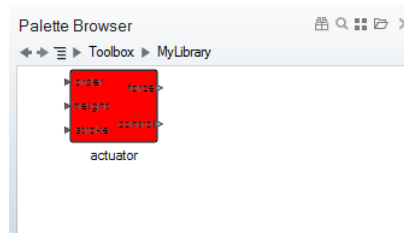


Figure 8.8: Block added to the Library

The user can also remove blocks from the library using the Library Manager.

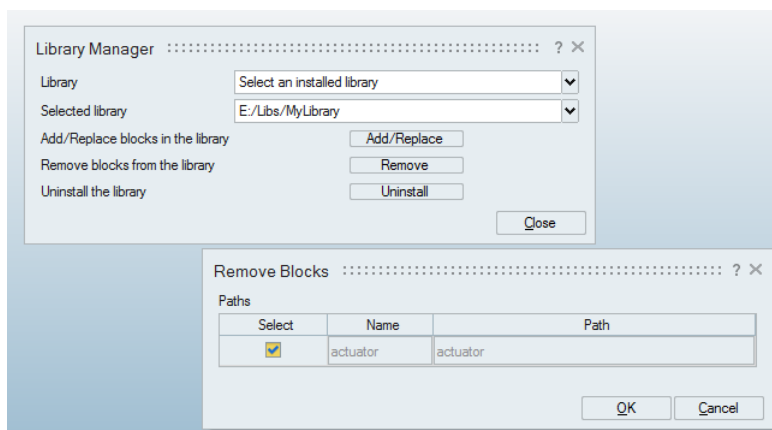


Figure 8.9: Block removal in a library

The Library Manager allows the user to lock his library. When the library is locked it is ready to be released. To do so, the user must uninstall his library through the button **Uninstall**, and then click the

button **Lock**. When the library is locked, the user cannot add or remove any block from it.

The Library Manager gives the possibility to the user to edit a locked library through the button **Unlock**. This feature will create a new version of the library, a new key is created in the `_key` file, the blocks are updated and the palette is regenerated, it references the blocks with the new key.

```
4364825b-cfc2-48b9-b8de-5e236bee2e56  
79b2cae8-905e-4851-acfa-77fe58b4ddb3
```

Figure 8.10: Example of library keys

```
<block name="actuator" type="block">  
  <template name="4364825b-cfc2-48b9-b8de-5e236bee2e56/actuator"/>
```

Figure 8.11: Library key used (referenced) in a block

The models and the scripts that use the old version of the library will still work. The user must make sure that his library is backward compatible. He cannot remove any block from the library. He cannot change the hierarchical organization of the blocks. He cannot remove any exposed **OML** function. For a given block, he cannot remove any parameter.

## 8.4.2 OML functions

**Activate** provides **OML** functions to manipulate explicitly the libraries.

### Create library

This function creates a skeleton of the library on the disk. It generates a new key and assigns that key to the library. The arguments of that function are the name of the library and the path where the library must be created.

```
vssCreateEmptyLibrary('mylib', 'x:/Libraries');
```

### Adding blocks

The user can use the menu **Tools/Block Builder** to create his blocks. The block will be created and inserted in the current model. To add the block to the library the user can select the created block and call the following functions:

```
blk=bdeGetSelectedBlock(bdeGetCurrentDiagram());  
vssAddBlockToLibrary(blk, 'x:/Libraries/mylib/Level1/MyBlock');
```

The user can also use the **Block Diagram Editor** APIs to create the block.

### Refresh library

After populating the library with the blocks, the refresh function generates the **OML** functions used by the evaluator, the **XML** files used to generate the documentation, and the libraries used by the simulator.



This function generates also the loader of the library. The argument of that function is the path of the library.

```
vssRefreshLibrary('x:/Libraries/mylib');
```

The library is now ready to be used.

**Note:** Every time the user adds a block to his library, he must refresh the library and execute the loader so that the new block is taken into account.

## Generate palette

**Activate** offers several methods to create a palette.

- The user can generate the palette automatically from the library; In this case the palette contains all the blocks of the library. He can then edit the palette to remove or add blocks. The palettes of a library must contain only blocks from the library or from the System (Activate) library. In this case he can use the function

```
vssGeneratePalette('x:/Libraries/mylib');
```

- The user can generate the palette manually. In this case, he can use the **Block Diagram Editor** since a palette is in essence a model. Steps are: open a new model, add new block(s) to the model through the menu **File/Libraries/Insert Block From Library** and finally save the model in the library.

## Add palette

Once the palettes are generated we must edit the libinit.oml file by adding the AddPalette function. That function adds the palettes of the library to the toolbox page.

```
vssAddPalette([libPath, '/mylib.scm']);
```

where libPath is the path of the library.

**Note:** to make the library movable, use the variable libPath defined in libinit.oml instead of hard coding the path of the library.

## Install library

This function installs the library in the **Activate** environment. It does not change the location of the library on the disk.

```
vssInstallLibrary('x:/Libraries/mylib');
```

The library is now installed, the palettes are added to the palette browser and the user can start using the library.

## Uninstall library

The user can uninstall the library. The palettes that come with the library will be removed from the Library Browser. Once a library is uninstalled, users cannot run any model that uses blocks from that

library.

```
vssUninstallLibrary('x:/Libraries/mylib');
```

### Document library

The user can start documenting his blocks. He has to edit the **XML** file corresponding to each block. The fields that are already filled must not be changed. He can give for example more information about the block in the description section. Once the blocks are documented, he can use the following function to update the **HTML** pages.

```
vssGenerateDocumentation('x:/Libraries/mylib');
```

### Lock library

The library is now ready to be delivered. The user **must** lock the library before releasing. Once locked user cannot add any block or function to the library. The library is now read-only.

```
vssLockLibrary('x:/Libraries/mylib');
```

### Unlock library

As a library creator, user can still edit his library after it has been locked. In this case he must, first, unlock the library.

A new key is generated indicating that this is a new version of the library. The new version of the library must be backward compatible. He cannot remove any block from the library. He cannot change the hierarchical level of the blocks. He cannot remove any exposed **OML** function. For a given block, he cannot remove any parameter.

```
vssUnlockLibrary('x:/Libraries/mylib');
```

## Chapter 9

# Altair Activate blocks

An **Activate** Model is an interconnection of blocks. Most **Activate** users do not need to develop new blocks because **Activate** provides a large number of blocks through different palettes. Advanced users however may need to develop new specialized blocks. These blocks may be placed in new libraries and distributed to other users.

Blocks in **Activate** may be constructed based on other blocks. A masked Super Block for example behaves almost like any other block in the Block Diagram Editor (BDE); the only difference is that the diagram inside the Super Block can be visualized and edited only using the “Enter Masked Super Block” contextual menu item<sup>1</sup>.

A block may also be defined through a diagram inside, similarly to a Super Block, but without any associated graphical information. These blocks are called *programmable super blocks*. The content of the block is defined via **OML** scripts using special APIs instantiating and connecting blocks to define the inside diagram. At the BDE level, programmable super blocks are indistinguishable from basic blocks. In the absence of graphical information concerning the inside diagram, the “Enter Masked Super Block” operation is not available. In fact this operation cannot apply even if graphical information were available since the content of a programmable super block may depend on block parameters. For example a block parameter may define the number of times a structure is repeated in the diagram. The parameter values not necessarily being available in the BDE, the diagram structure is not even known.

A basic block defines its behavior directly through a simulation function. Simulation functions are defined as C functions in most cases but they can also be defined in **OML**. Simulation functions can be developed first for **CCustomBlock** or **OMLCustomBlock** and tested before being used in the definition of a new basic block.

The construction of simulation functions for basic blocks is discussed in the next section. Once the simulation function is developed, the construction of a new block is greatly simplified using the **Activate** tool *Block builder* tool. Block builder is presented in Section 9.2.

### 9.1 Block simulation function

A block communicates with other blocks via the regular input and output ports and activation input and output ports. Signals associated with regular input and output ports can be of type matrix of real,

---

<sup>1</sup>In case of a library block, the block must be first inlined to enter the Super Block

complex or integer data types. The activation input and output ports, on the other hand, transmit only *events* (which are also called *activations*).

A block can have several regular and activation input and output ports. A block can also have continuous-time, discrete-time states, and zero-crossing surfaces. Of course a block does not need to have all of these elements.

The behavior of an **Activate** block is basically governed by the way it is activated. The block is activated when it receives activation on its activation input ports. If the block does not have input activation, it can be activated by inheritance. In this case, the activation input ports are added by the **Activate** compiler to the block. The block can also be defined as being *Always active*.

A block can be activated in many different ways. In the following a number of scenarios are presented.

**Discrete-time activation, or events** When a block is activated by an event on its activation input port, it can update its regular outputs:

$$y(t_e) = f_{\text{output}}(t_e, x(t_e^-), z(t_e^-), u(t_e), n_{\text{evprt}}, m, \text{phase}, p)$$

where  $t_e$  is the activation or event time,  $x(t_e^-)$  and  $z(t_e^-)$  represent continuous-time and discrete-time states just before the occurrence of the event.  $y$  and  $u$  are output and input of the block.  $n_{\text{evprt}}$  is a positive integer indicating by which activation input port the block has been activated.  $m$ ,  $\text{phase}$ ,  $p$  are `mode`, `simulation phase`, and parameters of the block respectively. `mode` and `simulation phase` will be discussed later.

If the block has output activation ports, it can program events on them by defining the delay time (which can be zero) on each activation output port:

$$t_{\text{evo}} = f_{\text{event}}(t_e, x(t_e^-), z(t_e^-), u(t_e), n_{\text{evprt}}, m, \text{phase}, p).$$

$t_{\text{evo}}$  is a vector with the same size that the number of activation output ports. Once programmed, if  $t_{\text{evo}}[i]$  is positive, an event is programmed at time  $t_e + t_{\text{evo}}[i]$  at activation output port  $i + 1$ .

The continuous-time and discrete-time states of the block, if present, can also be updated at event times.

$$[x(t_e), z(t_e)] = f_{\text{discrete}}(t_e, x(t_e^-), z(t_e^-), u(t_e), n_{\text{evprt}}, m, \text{phase}, p).$$

**Continuous-time activation** If the source of the activation is *always active*, then activation occurs over time interval and not at specific time instants similar to discrete-time events. In Continuous-time activation, the output computation can be represented with

$$y(t) = f_{\text{output}}(t, x(t), z(t), u(t), m, \text{phase}, p)$$

During a continuous-time activation, no state update and no event-programming is performed. Discrete-time states  $z$ , `mode` variables and `simulation phase` stay unchanged. On the other hand, the continuous-time state follows the differential equation.

$$\dot{x}(t) = f_{\text{continuous}}(t, x(t), z(t), u(t), m, \text{phase}, p)$$

**Zero-crossing function and mode variables** **Activate** provides several numerical solvers, including fixed step-size and variable step-size solvers such as `Lsoda` and `Radau`. Variable step-solvers adjust the integration step size during the integration to provide a faster simulation without compromising the precision in the solution.

If the differential equation represented by the diagram is not smooth enough, the variable-step solvers reduce the step-size to cope with the non-smoothness or even discontinuity. If  $f_{\text{output}}$  or  $f_{\text{continuous}}$  is not smooth (continuously differentiable) at some points, then `mode` variable is used in such way as to make sure the numerical solver never encounters these discontinuity points inside the integration interval. Consider the following example:

$$y = \begin{cases} u & \text{if } u \geq 0 \\ -u & \text{otherwise} \end{cases}$$

This function implements the absolute value function and is not differentiable at  $u = 0$ . In this case, a `mode` variable can be defined to specify at the start of the integration period whether  $u$  is positive. To make sure the integration stops when sign of  $u$  changes, a zero-crossing surface is introduced at zero, here the zero-crossing surface is  $u = 0$ . During the integration period (which could end because of the zero-crossing), the output  $y$  is computed as follows:

$$y = \begin{cases} u & \text{if } m = 1 \\ -u & \text{otherwise} \end{cases}$$

Once the zero-crossing is detected, the numerical solver stops and the `mode` is recomputed; then the integration continues. The computation of the `mode` and the zero-crossing surface are performed by the block. The block can be called in different situations or `phases`. In some `phases`, such as during the numerical integration, `mode` variables are fixed and in other `phases`, `modes` are updated. For example when an event activates the block, `mode` variables can be updated. The `phase` variable is not directly accessible to the user. The user can check if `mode` variables are fixed or not by using the macro `isModeFixed(block)`.

When a variable step solver integrates a model, it may happen that a time interval be repeated several times till the solver accepts the step and starts another step. In most situations users need not worry about these intermediate repeated calls, but in some cases, e.g., when the output of the block is displayed, these calls should be ignored. In order to filter these calls, the macro `isinTryPhase(block)` is used. This macro uses the `phase` variable to find out whether the call should be ignored. It is important to indicate that `mode` variables are used only when the numerical solver is variable-step.

Zero-crossing functions can also be used to generate discrete-time events. Once a zero-crossing surface function inside a block crosses zero, the block is activated with a  $n_{\text{evprt}} = -1$ , i.e., at this event time  $t_e$ ,  $f_{\text{discrete}}$  is called with  $n_{\text{evprt}} = -1$ . This type of activation is used, e.g., in modeling a simple bouncing ball that will be presented later.

### 9.1.1 Simulation function implementation

The *simulation function* is normally written in `C`, but can also be written in **OML**. This function defines the behavior of the block during the simulation.

The simulation function is called by the simulator with two arguments: a `C` structure containing block information (`vss_block`) and a integer value called `flag`. The calling sequence is as follows:

```
MyNewBlock(vss_block *block, int flag)
```

The calling arguments will be discussed in the following sections.

#### vss\_block structure

The vss\_block structure is a hierarchical C structure providing access to the block parameters and variables.

```
struct vss_block {
    ....
    BlockInputX input;
    BlockOutput output;
    BlockEventInput evinput;
    BlockEventOutput evoutput;
    SimFunction sim;
    BlockState state;
    BlockDState dstate;
    BlockODState odstate;
    BlockConstraint constraint;
    BlockRpar rpar;
    BlockIpar ipar;
    BlockOpar opar;
    BlockZcross zcross;
    BlockMode mode;
    SCSPOINTER_COP *work;
    SCSINT_COP ajac;
    BlockName name;
    SCSINT_COP dept;
    SCSSTRING_COP blocktype;

    SCSPOINTER_COP simulatorVars;
    .....
};
```

Note that vss\_block is a hierarchical C structure. For example, BlockOutput is another structure defined as:

```
typedef struct {
    ....
    SCSINT_COP nout;
    SCSINT_COP outdim;
    SCSINT_COP *outsz;
    SCSINT_COP *outtyp;
    SCSPOINTER_COP *outptr;
    ....
}BlockOutput;
```

In the above structure, the standard C data-types, are redefined, for example:

```
#define SCSREAL_COP double
```

The complete list of these data-types are given in [9.1.1](#).

The block structure is quite complex; in order to facilitate accessing the embedded data, a large number of macros are provided. All access to this structure must be made through these macros. Most commonly used macros are presented in Sections 9.1.3 and 9.1.4. As an example, in order to access the data-type of the second block output, the user can use the following macro:

```
int *odt=GetOutType(block,2);
```

## Flag

Flag is an integer (or more precisely an enumeration) value indicating the job for which the block has been called by the simulator. Blocks may be called with different flags to perform different operations. For example the block can be called with `flag= VssFlag_OutputUpdate (= 1)`, to update its outputs or can be called with `flag= VssFlag_Derivatives (= 0)`, to provide the state derivatives.

The list of all flags and their meanings are provided below.

- `VssFlag_Initialize`: The block is called for the first time at the beginning of the simulation with this flag where initialization issues can be done. Operations such as opening data files, initializing a TCP/IP socket, or dynamic memory allocation can be done in this flag call. Although continuous-time and discrete-time states have already been initialized, they can be reinitialized (if needed). In this flag, input and output of the block are not available, hence cannot be initialized or read. Each block is called with this flag only once at the beginning of the simulation.
- `VssFlag_Terminate`: Once the simulation is finished, or stopped due to an error or on the user request, the simulator calls the blocks once at final time with this flag. This flag is useful, e.g., for closing a file or a socket that has been opened in flag `VssFlag_Initialize`.
- `VssFlag_OutputUpdate`: When the block is called with this flag, the simulator is requesting the output(s) of the block. The block should compute the outputs as a function of time, inputs, `nevprt`<sup>2</sup>, and internal states. These values should be obtained from the block structure (the first argument of the simulation function). The computed output should be written in the block structure using appropriate macros. Note that if the block contains `modes`<sup>3</sup>, then the output may be computed as a function of whether `modes` are fixed or relaxed; in some cases the call should be ignored. In this case `isModeFixed(block)` and `isinTryPhase(block)` macros may be used.
- `VssFlag_StateUpdate`: When the simulator calls the blocks with this flag, it means that an event has activated the block (`nevprt>0`) and the block can update its discrete-time and continuous-time states. The activation event may also be due to a zero-crossing event happened inside the block (an internal event), in which case (`nevprt=-1`). When this happens, the `jroot`<sup>4</sup> vector indicates the surface that has crossed and the direction of crossing. If *i*th entry of `jroot` is +1 (respectively -1), the *i*th surface has crossed zero with a positive (respectively negative) slope. If it is zero, *i*-th surface has not crossed zero.
- `VssFlag_Derivatives`: When the block is called with this flag, the simulator requests the block to provide the continuous-time state time derivatives ( $\dot{x}$ )<sup>5</sup>. The simulation function should compute  $\dot{x}$  in the address provided by the block for this purpose. If the block is implicit, i.e., the

---

<sup>2</sup>accessible by macro `GetNevIn(block)`

<sup>3</sup>accessible by macro `GetModePtrs(block)`

<sup>4</sup>accessible by macro `GetJrootPtrs(block)`

<sup>5</sup>accessible by macro `GetDerState(block)`

dynamics of the block is represented by a DAE, the block should compute the residual<sup>6</sup> in the corresponding address.

- `VssFlag_ZeroCrossings`: When the block is called with this flag, the simulator requests the block to provide its zero-crossing surfaces. Variable-step numerical solvers of **Activate** assume model smoothness. This means that the behavior of blocks, which contain or affect continuous-time states of the model must be smooth, or at least piecewise smooth. In case of non-smoothness, the points of discontinuity must be specified in such a way that the solver can, if needed, issue a cold restart when going through such points. An example of such a block is the `ABS` block. The absolute value function is smooth except at zero, where it is not differentiable. In this case, we say that absolute value block has two `modes`: one where the output equals the input, and one where the output is the negative of the input. A block may have many different modes. The point of non-smoothness or discontinuity is represented by a zero-crossing, and the `mode` is updated after such a crossing occurs. The `VssFlag_ZeroCrossings` flag is used both for evaluating the zero-crossing function and returning the zero-crossing vector<sup>7</sup> and setting the `mode` variables. Mode setting is done only when `isModeFixed(block)` is `False`. Note that the block may have more zero-crossing than modes. For example, the `Zero-cross` block which generates an event when its input crosses zero, has one zero-crossing surface, but no mode.
- `VssFlag_EventScheduling`: When the block is called with this flag, it means that the block can program a new event at one or more of its output activation ports. After this call, the simulator updates its event scheduler. An event can also be programmed after an internal zero-crossing. In this case, the block can use the `jroot` vector.
- `VssFlag_Reinitialize`: When the simulator calls the block with this flag, the block is allowed to reset its continuous-time state as a function of its input values.
- `VssFlag_ReinitializeImplicit`: In implicit models, after event leading to restarting the solver, the implicit blocks are called with this flag allowing blocks updating states and state derivatives.
- `VssFlag_Projection`: If a block dynamics is represented by an ODE with algebraic constraints, the simulator calls the block with this flag to either compute the projection update or the residual of constraints. The type of constraint is either `CONSTRAINT` or `PROJECTION`. If the model contains blocks with constraints, the number of constraints should be given by this macro `GetNConstraint(block)`. Note that only the `CPODE` solver can be used to integrate these type of models.
- `VssFlag_Jacobians`: When the block is called with this flag, the simulator requests the block to provide the analytical Jacobian matrix of the model. The block is called with this flag only if the block informs the simulator by setting `SetAjac(block, 1)` in `flag=VssFlag_Initialize`.
- `VssFlag_GotoPause`: Before stopping the simulation and going to pause, the block is called for the last time to perform necessary jobs if needed, for example flushing temporary data into file.
- `VssFlag_ReturnFromPause`: After returning from the pause, the blocks are called with this flag before resuming the simulation.

---

<sup>6</sup>accessible by macro `GetResState(block)`

<sup>7</sup>accessible by macro `GetGPtrs(block)`



## Flag calling sequence in blocks

The simulator invokes the blocks with specified `flag` to perform a job or a model update. In order to instantiate a block in the simulator before any subsequent call, the simulator calls all blocks with `flag= VssFlag_Initialize`. Just before finishing the simulation, all blocks are called with `flag= VssFlag_Terminate` to inform the block that this is the last call from the simulator. A simple flowchart has been given in Fig.9.1.

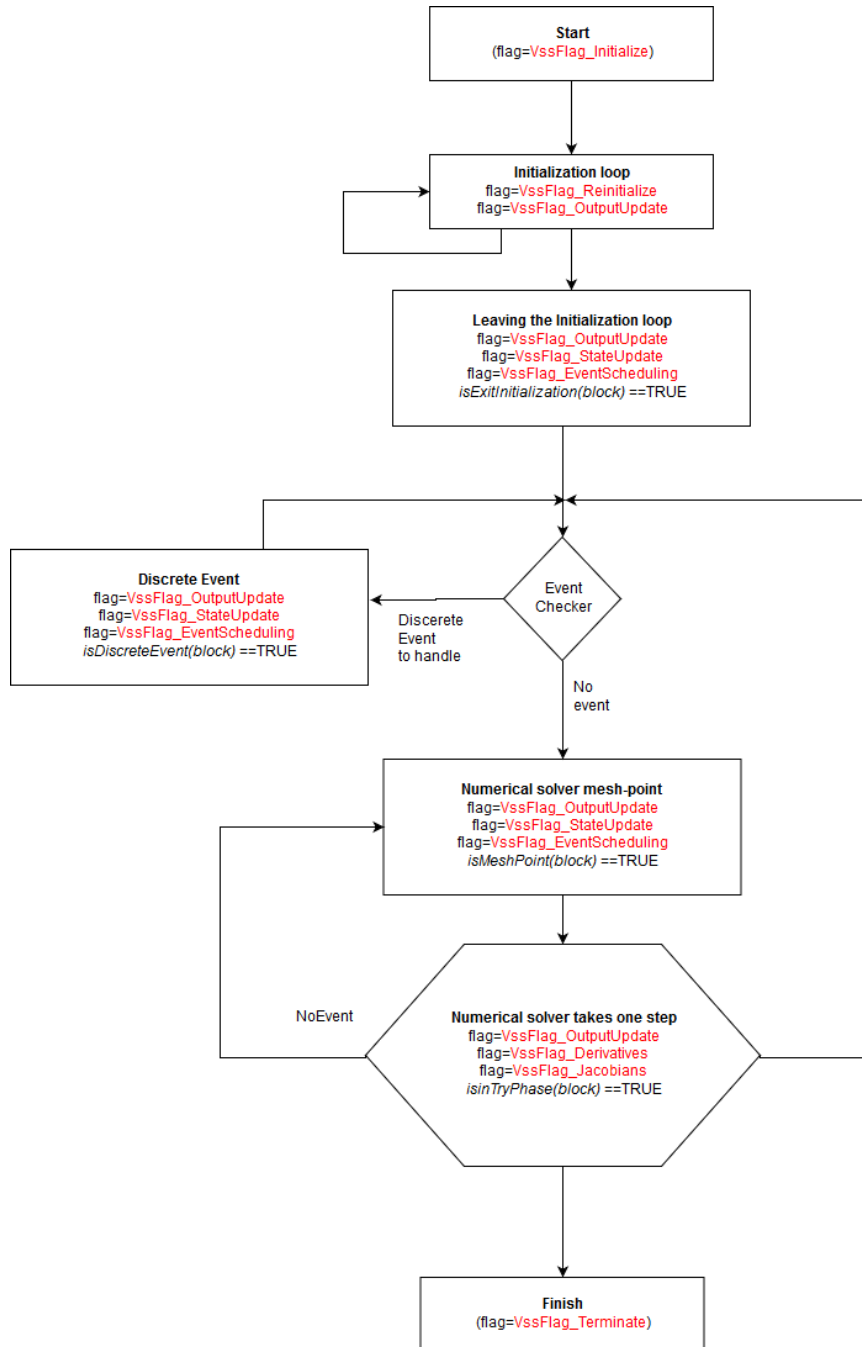


Figure 9.1: Simple flowchart indicating different stages of a simulation and different flag by which a block can be called.

## Data-type supported in blocks

Signals in **Activate** can be matrices of various data-types. The data-type of input and output of a block may either be defined and fixed by the block or be inherited from data-type of other connecting blocks.

In general a block supporting multiple data types should provide a different simulation function for each data type. But in some cases it is possible to avoid this by using a single simulation function. For that, the data types and sizes of the parameters, states, inputs and outputs are made available inside the simulation function via specific macros. For example `GetInType(block, n)` returns the data-type of  $n^{\text{th}}$  input port. The returned data-type can be one of these enumeration values.

- `SCSREAL_N`: double data-type ID
- `SCSCOMPLEX_N`: complex data-type ID
- `SCSINT32_N`: int32 data-type ID
- `SCSINT16_N`: int16 data-type ID
- `SCSINT8_N`: int8 data-type ID
- `SCSUINT32_N`: unsigned int32 data-type ID
- `SCSUINT16_N`: unsigned int16 data-type ID
- `SCSUINT8_N`: unsigned int8 data-type ID
- `SCSBOOL_N`: Boolean data-type ID
- `SCSSTRING_N`: string data-type ID

The corresponding types that can be used in the simulation function are:

- `SCSREAL_COP`: double data-type
- `SCSCOMPLEX_COP`: complex data-type
- `SCSINT32_COP`: int32 data-type
- `SCSINT16_COP`: int16 data-type
- `SCSINT8_COP`: int8 data-type
- `SCSUINT32_COP`: unsigned int32 data-type
- `SCSUINT16_COP`: unsigned int16 data-type
- `SCSUINT8_COP`: unsigned int8 data-type
- `SCSBOOL_COP`: Boolean data-type
- `SCSSTRING_COP`: string data-type

Users are encouraged to use these data-type instead of using directly C data-types in the definition of simulation functions.

## Feedthrough information in blocks

The feedthrough is a block property that specifies the input-output dependency of the block. This property must be specified when a new block is constructed (including for blocks realized by custom blocks). Specifically the feedthrough property is a vector of size equal to the number of block inputs.

The  $i^{th}$  entry of the vector indicates whether or not the block outputs depend "directly" on the  $i^{th}$  input value. Consider a Gain block with gain  $G$ , input  $u$  and output  $y$  (one input, one output) realizing:

$$y = Gu$$

In this case the output depends directly on the input (unless  $G$  is identically zero), so the Gain block in general has feedthrough property (more specifically its input has it). Now consider an integrator block with input  $u$ , output  $y$  and state  $x$ :

$$\begin{cases} \dot{x} = u \\ y = x \end{cases}$$

In this case the output  $y$  does not directly depend on the input  $u$ . The integrator block (its input in particular) does not have feedthrough property. The feedthrough property should be provided correctly otherwise the **Activate** compiler cannot compute proper scheduling. A scheduling error may occur if a true feedthrough property is erroneously set to false. On the other hand, specifying a feedthrough property to be true when it is not can lead to a non-existing algebraic loop error detected by the compiler. The algebraic loop error occurs when the diagram contains a path looping through ports having all feedthrough properties. In that case the blocks cannot be scheduled by the compiler. Even if there are ways to "break" algebraic loops (in particular using the **Loopbreaker** block), it is better not to introduce non-existing ones by properly defining the feedthrough properties of new blocks. For end users, learning about this property is useful for dealing with algebraic loop errors.

## 9.1.2 Examples

In this section in order to illustrate the way a simulation function is written, several examples are given.

### Simplified Gain block

Consider a Gain block that multiplies the input matrix by a matrix gain value and puts the results at the block output. Here is the simulation function of this block.

```
#include "vss_block4.h"

VSS_EXPORT void SimpleGainBlock (vss_block *block,int flag)
{
    if (flag==VssFlag_OutputUpdate) {
        SCSREAL_COP *u=GetRealInPortPtrs (block,1);
        int ru=GetInPortRows (block,1);
        int cu=GetInPortCols (block,1);

        SCSREAL_COP *y=GetRealOutPortPtrs (block,1);
        int ry=GetOutPortRows (block,1);
        int cy=GetOutPortCols (block,1);

        SCSREAL_COP *Gain=GetRealOparPtrs (block,1);
        int rowGain=GetOparSize (block,1,1);
        int colGain=GetOparSize (block,1,2);
```

```

    int i, j, k;

    for (i=0;i<ry;++i){
        for (j=0;j<cy;++j){
            k=i+j*ry;
            y[k]=Gain[k]*u[k];
        }
    }
}

```

Note that the macro `GetRealInPortPtrs(block,1)` returns a pointer to the vector of the first input port. The gain matrix which is a block parameter is accessed by `GetRealOparPtrs(block,1)`. The gain matrix value is of type `Real` (double). The row and column size of the gain matrix are accessed as follows:

```

rowGain=GetOparSize(block,1,1);
colGain=GetOparSize(block,1,2);

```

The output of the block should be computed whenever the output is requested by the simulator, i.e., when the block is called with `flag=VssFlag_OutputUpdate`. At this flag, the input and parameter matrices are read and the output is computed. This block may also be invoked by other flags, but it does nothing.

Note that the dimensions of the matrices are not checked for consistency. The checks should be made at the evaluation or compilation phases.

### Simplified ODE/DAE block

In order to demonstrate the way a simple ODE and DAE systems can be implemented inside an **Activate** block, consider the following initial value problem problem.

$$\begin{cases} \dot{x}_1 = -x_1 + x_2x_1 \\ \dot{x}_2 = 2x_1 - 3x_2 \end{cases}$$

with initial value

$$[x_1, x_2] = [1, 2].$$

This ODE can be implemented as follows.

```

#include "vss_block4.h"

VSS_EXPORT void Simple_ODE_Block (vss_block *block, int flag)
{
    SCSREAL_COP *y=GetRealOutPortPtrs(block,1);
    SCSREAL_COP *xd=GetDerState(block);
    SCSREAL_COP *x=GetState(block);
    int nx=GetNstate(block);
    int i;

    switch(flag)

```

```

{
  case VssFlag_Initialize:
    x[0]=1.0;
    x[1]=2.0;
    break;
  case VssFlag_OutputUpdate:
    for (i=0;i<nx;++i){
      y[i]=x[i];
    }
    break;
  case VssFlag_Derivatives:
    xd[0]=-x[0]+x[1]*x[0];
    xd[1]=-3*x[1]+2*x[0];
    break;
  default:
}
}

```

In this code, first the pointers to state (x) and state derivative (xd) as well as the size of the state (nx) are obtained. Three flags are used, VssFlag\_OutputUpdate for output the state value and VssFlag\_Derivatives for returning the time derivative of the state.

In order to demonstrate the way a DAE is implemented in an **Activate** block, let's reformulate the above ODE as a DAE. We get the following DAE.

$$\begin{cases} 0 = -\dot{x}_1 - x_1 + x_2x_1 \\ 0 = -\dot{x}_2 + 2x_1 - 3x_2 \end{cases}$$

The C code for this DAE can be expressed as follows:

```

#include "vss_block4.h"

VSS_EXPORT void Simple_DAE_Block (vss_block *block, int flag)
{
  SCSREAL_COP *y=GetRealOutPortPtrs(block,1);
  SCSREAL_COP *xd=GetDerState(block);
  SCSREAL_COP *x=GetState(block);
  SCSREAL_COP *res=GetResState(block);
  int nx=GetNstate(block);
  int i;

  switch(flag)
  {
  case VssFlag_Initialize:
    x[0] =1.0;
    x[1] =2.0;
    xd[0]=0.0;
    xd[1]=0.0;
    break;

```

```

    case VssFlag_OutputUpdate:
        for (i=0;i<nx;++i){
            y[i]=x[i];
        }
        break;

    case VssFlag_Derivatives:
        res[0]=-xd[0]-x[0]+x[1]*x[0];
        res[1]=-xd[1]-3*x[1]+2*x[0];
        break;

    default:
}
}

```

### Simplified Counter (inc/dec) block

In this example, we develop a counter block. Whenever an event activates the counter, the counter increments or decrements its value. The initial value of the counter and the direction of the counter are given as block parameters.

```

#include "vss_block4.h"
VSS_EXPORT void Simple_counter_Block (vss_block *block, int flag)
{
    SCSINT8_COP *y=Getint8OutPortPtrs(block,1);
    SCSINT8_COP initial=*Getint8OparPtrs(block,1);
    SCSINT8_COP dir    =*Getint8OparPtrs(block,2);
    SCSINT8_COP mem    = Getint8OzPtrs(block,1);
    int nevprt = GetNevIn(block);

    if (flag==VssFlag_Initialize){
        mem[0]=initial;
    }else if (flag==VssFlag_OutputUpdate) {
        y[0]=mem[0];
    }else if (flag== VssFlag_StateUpdate) {
        if (nevprt ==1){
            if (dir==1)
                mem[0]++;
            else
                mem[0]--;
        }
    }
}
}

```

The initial value of the counter is returned by the macro `Getint8OparPtrs(block,1)`. The block needs a storage to keep the counter value. We chose `OZ` storage, which here is a scalar 8-bit integer. In general `OZ` is initialized before the simulation phase, but for demonstration purposes we reinitialized it in `flag=VssFlag_Initialize`. On every block activation with `flag=VssFlag_StateUpdate`, the

nevprt is checked to verify if the activation is a discrete event. Then based on the block parameter (dir), the counter is incremented or decremented.

### Simplified Counter block with overflow

In the above counter suppose that the block needs to generate an event when the counter overflows/underflows. In this case, an event needs to be programmed in flag=VssFlag\_EventScheduling.

```
#include "vss_block4.h"
VSS_EXPORT void Simple_counter_Block (vss_block *block, int flag)
{
    SCSINT8_COP *y=Getint8OutPortPtrs(block,1);
    SCSINT8_COP initial=*Getint8OparPtrs(block,1);
    SCSINT8_COP dir     =*Getint8OparPtrs(block,2);
    SCSINT8_COP mem      = Getint8OzPtrs(block,1);
    int nevprt = GetNevIn(block);
    double *evout= GetNevOutPtrs(block);

    if (flag==VssFlag_Initialize){
        mem[0]=initial;
    }else if (flag==VssFlag_OutputUpdate) {
        y[0]=mem[0];
    }else if (flag== VssFlag_StateUpdate) {
        if (nevprt ==1){
            if (dir==1)
                mem[0]++;
            else
                mem[0]--;
        }
    } else if (flag==VssFlag_EventScheduling){
        if (nevprt ==1){
            if (dir==1){
                if (mem[0]==127) {
                    evout[0]=0.0;
                }
            }else{
                if (mem[0]==-128){
                    evout[0]=0.0;
                }
            }
        }
    }
}
```

In this counter, when the block is called with flag=VssFlag\_EventScheduling, the content of the counter is checked and based on the counter direction and its value, an immediate discrete event (with zero delay) is generated at its first activation output port.

## Simplified Bouncing ball block

In order to demonstrate the way zero-crossings and `modes` are handled, consider the model of a simple bouncing ball.

$$\begin{cases} \dot{h} = v \\ \dot{v} = -9.81 \end{cases}$$
$$[h, v] = [10, 0]$$

When the ball hits the ground, its vertical speed changes according to

$$v = -0.9v^-$$

The code for modeling this system is:

```
#include "vss_block4.h"
VSS_EXPORT void Simple_BouncingBall_Block (vss_block *block, int flag)
    SCSREAL_COP *h=GetRealOutPortPtrs (block, 1);
    SCSREAL_COP *v=GetRealOutPortPtrs (block, 2);
    SCSREAL_COP *xd=GetDerState (block);
    SCSREAL_COP *x=GetState (block);
    SCSREAL_COP *g=GetGPtrs (block);
    int nevprt = GetNevIn (block);

    switch (flag)
    {
    case VssFlag_OutputUpdate:
        h[0]=x[0];
        v[0]=x[1];
        break;

    case VssFlag_Derivatives:
        xd[0]= x[1];
        xd[1]= -9.81 ;
        break;

    case VssFlag_StateUpdate:
        if (nevprt==-1){
            x[0]= 0.0;
            x[1]= -0.9*x[1];
        }
        break;

    case VssFlag_ZeroCrossings
        g[0]=x[0];
        break;

    default:
    }
```



```
}
```

The model contains two state variables, one for the height of the ball ( $h$ ) and another for the velocity of the ball ( $v$ ). The block has two output ports, the first outputs  $h$  and the second,  $v$ . A zero-crossing surface has been used to check whether the ball has hit the ground. Naturally the height of the ball ( $h$ ) can be used as a zero-crossing surface in the block. When the ball hits the ground, an internal event is generated (with `nevpert== -1`). At this moment, the state variable ( $h$  and  $v$ ) are reset with new values.

## Absolute value block

In order to demonstrate the way `mode` variables are handled in a block, consider a simplified version of the `Abs` block in **Activate**. This block computes the absolute value of its input.

$$y = \begin{cases} u & \text{if } u \geq 0 \\ -u & \text{otherwise} \end{cases}$$

When a variable step-size numerical solver is chosen in **Activate**, if the output of the `Abs` block affects continuous-time state of the model, `mode` variable should be used inside the block to handle correctly the non-smoothness or discontinuity. The absolute value block needs then a zero-crossing surface to detect when the input crosses zero and a `mode` to handle the discontinuity. The zero-crossing surface is defined in `flag=VssFlag_ZeroCrossings`. When the block is called with this flag, the block updates the zero-crossing surface and modes.

In order to update the `mode` variables, the `areModesFixed(block)` macro is checked, if false, `modes` can be updated. If the simulator calls the block with `flag=VssFlag_OutputUpdate` for updating the outputs, the block should make use of the `mode` variable. If the simulator calls the block when `areModesFixed(block)` is true, the outputs depends on the `mode`. The code for the simulation function of the block is:

```
#include <vss_block4.h>
VSS_EXPORT void simple_abs(vss_block *block, int flag)
{
    SCSREAL_COP *u=GetRealInPortPtrs(block,1);
    SCSREAL_COP *y=GetRealOutPortPtrs(block,1);
    SCSREAL_COP *g=GetGPtrs(block);
    SCSINT32_COP *mode=GetModePtrs(block);
    int side;

    switch(flag)
    {
    case VssFlag_OutputUpdate:
        if (areModesFixed(block)) {
            side=mode[0];
        }else {
            if (u[0]<0){ side=2;
            } else{ side=1;
            }
        }
        if (side==1){ y[0]=+u[0];
```

```

        } else{          y[0]=-u[0];
        }
    break;
case VssFlag_ZeroCrossings:
    g[0]=u[0];
    if (!areModesFixed(block)) {
        if(g[0]<0){
            mode[0]=2;
        }else{
            mode[0]=1;
        }
    }
}
}
}

```

### A block to write signals to a file

In this example we demonstrate the way the block input can be written into an ascii file. The input of the block is scalar of data type double. The code is:

```

#include "vss_block4.h"

typedef struct {
    unsigned int index;
    FILE *pFile;
}write2File_t;

#define WS      ((write2File_t *) GetWorkPtrs(block))

VSS_EXPORT void write_to_file(vss_block *block,int flag)
{
    switch(flag)
    {
        /*-----*/
        case VssFlag_Initialize:
        {
            char *fname=Getint8OparPtrs(block,1);
            GetWorkPtrs(block)= (write2File_t*)vss_malloc(block,
                sizeof(write2File_t) );
            if (!WS){SetBlockError(block,1);return;}
            WS->index=0;
            WS->pFile = fopen (fname,"w");
            if(!WS->pFile) {Coserror(block,"Unable to open the file %s\n"
                , fname); return;}
            fprintf (WS->pFile, "input data.....\n");
        }
        break;
        /*-----*/
    }
}

```

```

case VssFlag_Terminate:
{
    if (!WS) return;
    if (WS->pFile){
        fprintf (WS->pFile, "Number of lines: %d",WS->index);
        fclose(WS->pFile);
    }
}
break;
/*-----*/
case VssFlag_OutputUpdate:
{
    SCSREAL_COP *u = GetRealInPortPtrs(block,1);
    if (!isinTryPhase(block)){
        fprintf (WS->pFile, "%g\n", u[0]);
        WS->index++;
    }
}
break;
}
return;
}

```

At the beginning of the simulation, when the block `write_to_file` is called with `flag = VssFlag_Initialize` the file whose name (`fname`) is given as a block parameter is opened for writing. In order to dump the block input data into the file at block calls with `flag = VssFlag_OutputUpdate`, the pointer to file should be kept throughout the simulation. This pointer can either be stored as a static variable in the simulation function or in the block's `Workspace`<sup>8</sup>. The static method is not recommended, since if two instances of this block are used in a model, both share the same static variable that perturbs the operation of the blocks. For this reason, we create a structure to keep the file pointer as well as any other variable that needs to be kept during the simulation. The structure is allocated with `vss_malloc` function of **Activate**. This function is similar to `malloc`, but with the difference that the pointer of the allocated memory is managed and freed by **Activate** simulator. So the user does not need to worry about freeing the memory at the end. **Activate** provides also the function `vss_file_open(block, fname, "w");` instead of `fopen(fname, "w")`. `vss_file_open` does not need `fclose`, it is done automatically by the simulator. Another important point in the above code is using `isinTryPhase (block)` macro. This macro filters the try calls where the input of the block is not a valid value and can be ignored.

## Simplified ODE with Constraint preservation

In some models the states of the dynamic systems need to satisfy invariant or some algebraic relationships. This kind of invariants is usually resulting from mass or energy conservation or other physical laws. If the ODE is being integrated by standard numerical solvers, slight errors in the numerical solution are accumulated and cause the solutions to fail to satisfy solution invariant exactly. In order to add these kind of invariant to the ODE of the block, **Activate** provides the use of `flag =VssFlag_Projection`.

<sup>8</sup>accessible by macro `GetWorkPtrs (block)`

and the CPODE solver. During the simulation the simulator calls the simulation functions of the blocks with constraints with this flag and requests the block to compute the invariant or the projections as a function of newly computed states. This results in a solution that sticks to the constraints. The following example is the standard pendulum model with two invariants. The dynamic model of the pendulum is

$$\begin{cases} \dot{x} &= v_x \\ \dot{y} &= v_y \\ \dot{v}_x &= -x(v_x^2 + v_y^2 - gy)/L^2 \\ \dot{v}_y &= -y(v_x^2 + v_y^2 - gy)/L^2 - g \end{cases}$$

The first invariant, e.g., ensures that the length of the pendulum remains equal to  $L$  throughout the simulation.

$$\begin{cases} x^2 + y^2 &= L^2 \\ xv_x + yv_y &= 0 \end{cases}$$

The code for the simulation function of the pendulum model with constraint preservation follows.

```
#include "vss_block4.h"
#define g 9.81
VSS_EXPORT void CBlockFunction(vss_block *block, int flag)
{
    double *xd=GetDerState(block);
    double *X=GetState(block);
    double *y=GetRealOutPortPtrs(block,1);
    int nx=GetNstate(block);
    int i, ip;

    switch(flag)
    {
        /*-----*/
        case VssFlag_Initialize:
        {
            SetConstraintKind(block,CONSTRAINT);
            SetNConstraint(block,2);
            X[0]=1;
            X[1]=X[2]=X[3]=0;
            return;
        }

        case VssFlag_Projection:
        {
            double x, y, xd, yd;
            double *corr=GetCorrPtr(block);
            x = X[0];
            y = X[1];
            xd = X[2];
            yd = X[3];

            corr[0]= x*x + y*y - 1.0;
```

```

        corr[1]= x*xd + y*yd;
        return;
    }

case VssFlag_Derivatives:
    {
        double x, y, xd, yd, tmp;
        x = X[0];
        y = X[1];
        xd = X[2];
        yd = X[3];
        tmp = xd*xd + yd*yd - g*y;

        xd[0] = xd;
        xd[1] = yd;
        xd[2] = -x*tmp;
        xd[3] = -y*tmp - g;
        return;
    }

case VssFlag_OutputUpdate:
    {
        for (i=0;i<nx;i++)
            y[i] = X[i];
        return;
    }
default:
    return;
}
}

```

## ODE and DAE blocks providing Analytical Jacobian

In order to solve an ODE/DAE, the numerical variable-step solvers need the Jacobian matrix of the ODE/DAE. In **Activate**, the Jacobian matrix can either be computed numerically by the finite difference method or can be provided by blocks. In this section, we show the way the Jacobian of a ODE/DAE can be provided by the block. Consider the following ODE model and its Jacobian matrix is

$$\dot{x} = f(x) = \begin{cases} -x_1 + x_1x_2 \\ 2x_1 - 3x_2 \end{cases}$$

$$J = \frac{\partial f}{\partial x} = \begin{bmatrix} -1 + x_2 & x_1 \\ 2 & -3 \end{bmatrix}$$

The simulation function for this block providing the analytical Jacobian matrix is given below. In flag=VssFlag\_Initialize, the jacobian flag is set to 1 to inform the simulator that the block provides the analytical

Jacobian. The  $J$  vector has been filled with the Jacobian entries in flag `VssFlag_Jacobians`. Note that the  $J$  matrix is filled column-wise.

```
#include "vss_block4.h"

VSS_EXPORT void Simple_ODE_Block (vss_block *block, int flag)
{
    SCSREAL_COP *y=GetRealOutPortPtrs (block,1);
    SCSREAL_COP *xd=GetDerState (block);
    SCSREAL_COP *x=GetState (block);
    SCSREAL_COP *J= GetJacobianPtrs (block);
    int nx=GetNstate (block);
    int i;

    switch(flag)
    {
        case VssFlag_Initialize:
            SetAjac (block,1);
            return;

        case VssFlag_OutputUpdate:
            for (i=0;i<nx;++i){
                y[i]=x[i];
            }
            break;

        case VssFlag_Derivatives:
            xd[0]=-x[0]+x[1]*x[0];
            xd[1]=-3*x[1]+2*x[0];
            break;

        case VssFlag_Jacobians:
            J[0]=-1+x[1];
            J[1]=2;
            J[2]=x[0];
            J[3]=-3;
            break;

        default:
    }
}
```

The Jacobian matrix for a DAE is a little different. For the DAE

$$0 = F(\dot{x}, x)$$

the Jacobian Matrix is defined as follows

$$J = \alpha \frac{\partial F}{\partial x} + \beta \frac{\partial F}{\partial \dot{x}}$$

where  $\alpha^9$  and  $\beta^{10}$  values are given by the simulator. As a result the Jacobian matrix for the above DAE will be

$$J = \begin{bmatrix} -\alpha + \alpha x_2 - \beta & \alpha x_1 \\ 2\alpha & -3\alpha - \beta \end{bmatrix}$$

```
#include "vss_block4.h"

VSS_EXPORT void Simple_ODE_Block (vss_block *block, int flag)
{
    SCSREAL_COP *y=GetRealOutPortPtrs (block,1);
    SCSREAL_COP *xd=GetDerState (block);
    SCSREAL_COP *x=GetState (block);
    SCSREAL_COP *J= GetJacobianPtrs (block);
    SCSREAL_COP *alpha=GetAlphaPt (block);
    SCSREAL_COP *beta =GetBetaPt (block);
    int nx=GetNstate (block);
    int i;

    switch(flag)
    {
        case VssFlag_Initialize:
            SetAjac (block,1);
            return;
        case VssFlag_OutputUpdate:
            for (i=0;i<nx;++i){
                y[i]=x[i];
            }
            break;
        case VssFlag_Derivatives:
            xd[0]=-x[0]+x[1]*x[0];
            xd[1]=-3*x[1]+2*x[0];
            break;
        case VssFlag_Jacobians:
            J[0]=-alpha[0]+alpha[0]*x[1]-beta[0];
            J[1]=-2*alpha[0] ;
            J[2]=alpha[1]*x[0];
            J[3]=-3*alpha[1]-beta[1];
            break;
        default:
    }
}
```

<sup>9</sup>accessible by macro GetAlphaPt (block)

<sup>10</sup>accessible by macro GetBetaPt (block)

## A simple block for programming an initial event

Consider a simple block with one output event port that generates an event at a time instant specified by a block parameter. In order to generate the event, the block should read the parameter value using `evtime= GetRealOparPtrs(block,1)`. Events are only programmed at `flag== VssFlag_EventScheduling`. In order to program this event only once, at the beginning of the simulation, the `isExitInitialization(block)` macro can be used. This macro indicates if the initialization has finished and the simulation is being started. The relative time with respect to the current simulation time and the desired event time is defined and set to the `evout` vector.

```
#include "vss_block4.h"

VSS_EXPORT void EventGenerateBlock(vss_block *block,int flag)
{
    double *evout =GetNevOutPtrs(block);
    double *evtime=GetRealOparPtrs(block,1);
    if(flag==VssFlag_EventScheduling){
        if (isExitInitialization(block)){
            evout[0]=evtime[0]-GetVssInitialTime(block);
        }
    }
}
```

### 9.1.3 Macros for accessing the block structure

The following macros are used to access `vss_block` structure inside the block simulation function. In all macros the `block` argument is of type `vss_block` and `n`, `m` are strictly positive integer values.

- `GetNin(block)`: Get the number of regular input ports of the block.
- `GetInDim(block)`: Get the dimension of the regular input ports.
- `GetInPortPtrs(block,n)`: Get regular input port pointer of port number `n`. `n` must be strictly positive.
- `GetNout(block)`: Get number of regular output port of the block.
- `GetOutDim(block)`: Get the dimension of the regular output ports.
- `GetOutPortPtrs(block,n)`: Get regular output port pointer of port number `n`. `n` must be strictly positive.
- `GetInPortSize(block,n,m)`: Get size of the regular input port number `n` for the dimension `m`. `n` and `m` must be strictly positive.
- `GetInPortSize(block,n,1)`: Get first dimension (row size) of input port number `n`. `n` must be strictly positive.
- `GetInPortSize(block,n,2)`: Get second dimension (column size) of input port number `n`. `n` must be strictly positive.
- `GetInPortRows(block,n)`: Get number of rows (first dimension) of regular input port number `n`. `n` must be strictly positive.



- `GetInPortCols (block, n)`: Get number of columns (second dimension) of regular input port number `n`. `n` must be strictly positive.
- `GetInType (block, n)`: Get data type of regular input port number `n`. `n` must be strictly positive.
- `GetOutPortSize (block, n, m)`: Get size of the regular output port size number `n` for the dimension `m`. `m, n` must be strictly positive.
- `GetOutPortSize (block, n, 1)`: Get first dimension (row size) of output port number `n`. `n` must be strictly positive.
- `GetOutPortSize (block, n, 2)`: Get second dimension (column size) of output port number `n`. `n` must be strictly positive.
- `GetOutPortRows (block, n)`: Get number of rows (first dimension) of regular output port number `n`. `n` must be strictly positive.
- `GetOutPortCols (block, n)`: Get number of columns (second dimension) of regular output port number `n`. `n` must be strictly positive.
- `GetOutType (block, n)`: Get data-type of regular output port number `n`. `n` must be strictly positive.
- `GetRealInPortPtrs (block, n)`: Get pointer of real part of regular input port number `n`. `n` must be strictly positive.
- `GetImagInPortPtrs (block, n)`: Get pointer of imaginary part of regular input port number `n`. `n` must be strictly positive.
- `GetInt8InPortPtrs (block, n)`: Get pointer of int8 typed regular input port number `n`. `n` must be strictly positive.
- `GetInt16InPortPtrs (block, n)`: Get pointer of int16 typed regular input port number `n`. `n` must be strictly positive.
- `GetInt32InPortPtrs (block, n)`: Get pointer of int32 typed regular input port number `n`. `n` must be strictly positive.
- `GetUInt8InPortPtrs (block, n)`: Get pointer of unsigned int8 typed regular input port number `n`. `n` must be strictly positive.
- `GetUInt16InPortPtrs (block, n)`: Get pointer of unsigned int16 typed regular input port number `n`. `n` must be strictly positive.
- `GetUInt32InPortPtrs (block, n)`: Get pointer of unsigned int32 typed regular input port number `n`. `n` must be strictly positive.
- `GetRealOutPortPtrs (block, n)`: Get pointer of real part of regular output port number `n`. `n` must be strictly positive.
- `GetImagOutPortPtrs (block, n)`: Get pointer of imaginary part of regular output port number `n`. `n` must be strictly positive.
- `GetInt8OutPortPtrs (block, n)`: Get pointer of int8 typed regular output port number `n`. `n` must be strictly positive.
- `GetInt16OutPortPtrs (block, n)`: Get pointer of int16 typed regular output port number `n`. `n` must be strictly positive.

- `GetInt32OutPortPtrs (block, n)` : Get pointer of int32 typed regular output port number n. n must be strictly positive.
- `GetUint8OutPortPtrs (block, n)` : Get pointer of unsigned int8 typed regular output port number n. n must be strictly positive.
- `GetUint16OutPortPtrs (block, n)` : Get pointer of unsigned int16 typed regular output port number n. n must be strictly positive.
- `GetUint32OutPortPtrs (block, n)` : Get pointer of unsigned int32 typed regular output port number n. n must be strictly positive.
- `GetNipar (block)` : Get number of integer parameters of the block.
- `GetNrpar (block)` : Get number of real parameters of the block.
- `GetNopar (block)` : Get number of object parameters of the block. This kind of parameters can be of any data type or tables.
- `GetIparPtrs (block)` : Get pointer of the integer parameters vector .
- `GetRparPtrs (block)` : Get pointer of the real parameters vector.
- `GetPtrWorkPtrs (block)` : Get the pointer of pointer of the Work array.
- `GetNstate (block)` : Get number of continuous state. n must be strictly positive.
- `GetState (block)` : Get pointer of the continuous state vector.
- `GetDerState (block)` : Get the pointer to the vector of the derivative of continuous state.
- `GetResState (block)` : Get pointer of the residual continuous state vector.
- `GetJacobianPtrs (block)` : Get pointer on the Jacobian matrix.
- `GetXpropPtrs (block)` : Get pointer of continuous state properties vector. For implicit blocks, this vector indicates if a state is differential or algebraic
- `GetNdstate (block)` : Get number of real data-type discrete state.
- `GetDstate (block)` : Get pointer of the discrete state vector.
- `GetNevIn (block)` : Get the input event number. This number is coded as binary. For example if first than third events are synchronously activated, this macro returns 5.
- `GetNevin (block)` : Get number of event input ports.
- `GetNevOut (block)` : Get number of event output ports.
- `GetNevOutPtrs (block)` : Get pointer of event output vector. The block fills this vector with the time of the next coming event. The time is relative, e.g., for programming an event instantly the vector element is 0.0.
- `GetEvOutVal (block, n)` : Get the content of the event vector, negative element means no event is programmed. n must be strictly positive.
- `GetOparType (block, n)` : Get data-type of object parameters number n. n must be strictly positive.
- `GetOparSizePtrs (block)` : Get size of object parameters
- `GetOparSize (block, n, m)` : Get size of object parameters number n for the dimension m. m and n must be strictly positive.

- `GetOparSize (block, n, 1)`: Get first dimension (row size) of object parameters number  $n$ .  $n$  must be strictly positive.
- `GetOparSize (block, n, 2)`: Get second dimension (column size) of object parameters number  $n$ .  $n$  must be strictly positive.
- `GetOparPtrs (block, n)`: Get pointer of object parameters number  $n$ .
- `GetRealOparPtrs (block, n)`: Get pointer of real object parameters number  $n$ .
- `GetImagOparPtrs (block, n)`: Get pointer of imaginary part of object parameters number  $n$ .
- `GetInt8OparPtrs (block, n)`: Get pointer of int8 typed object parameters number  $n$ .
- `GetInt16OparPtrs (block, n)`: Get pointer of int16 typed object parameters number  $n$ .
- `GetInt32OparPtrs (block, n)`: Get pointer of int32 typed object parameters number  $n$ .
- `Getuint8OparPtrs (block, n)`: Get pointer of unsigned int8 typed object parameters number  $n$ .
- `Getuint16OparPtrs (block, n)`: Get pointer of unsigned int16 typed object parameters number  $n$ .
- `Getuint32OparPtrs (block, n)`: Get pointer of unsigned int32 typed object parameters number  $n$ .
- `GetStringOparPtrs (block, n)`: Get pointer of unsigned int32 typed object parameters number  $n$ .
- `GetNoz (block)`: Get number of object discrete state.
- `GetOzType (block, n)`: Get type of object discrete state number  $n$ .
- `GetOzTypePtrs (block)`: Get type of object state.
- `GetOzSizePtrs (block)`: Get size of object state.
- `GetOz (block)`: Get object state
- `GetOzSize (block, n, m)`: Get size of object state number  $n$  for the dimension  $m$ .
- `GetOzSize (block, n, 1)`: Get first dimension (row size) of object state number  $n$ .
- `GetOzSize (block, n, 2)`: Get second dimension (column size) of object state number  $n$ .
- `GetOzPtrs (block, n)`: Get pointer of object state number  $n$ .
- `GetRealOzPtrs (block, n)`: Get pointer of real object state number  $n$ .
- `GetImagOzPtrs (block, n)`: Get pointer of imaginary part of object state number  $n$ .
- `GetInt8OzPtrs (block, n)`: Get pointer of int8 typed object state number  $n$ .
- `GetInt16OzPtrs (block, n)`: Get pointer of int16 typed object state number  $n$ .
- `GetInt32OzPtrs (block, n)`: Get pointer of int32 typed object state number  $n$ .
- `Getuint8OzPtrs (block, n)`: Get pointer of unsigned int8 typed object state number  $n$ .
- `Getuint16OzPtrs (block, n)`: Get pointer of unsigned int16 typed object state number  $n$ .
- `Getuint32OzPtrs (block, n)`: Get pointer of unsigned int32 typed object state number  $n$ .

- `GetSizeOfOz (block, n)`: Get the data size of each element of the block discrete OZ state. As an example, if the data type of OZ is Double, this macro returns `sizeof(Double)`.
- `GetSizeOfOpar (block, n)`: Get the data size of each element of the block Opar parameter. As an example, if the data type is Double, this macro returns `sizeof(Double)`.
- `GetSizeOfOut (block, n)`: Get the sizeof of the regular output port number n. Get the data size of each element of the block output. As an example, if the data type is Double, this macro returns `sizeof(Double)`.
- `GetSizeOfIn (block, n)`: Get the data size of each element of the block input. As an example, if the data type is Double, this macro returns `sizeof(Double)`.
- `GetNg (block)`: Get number of zero crossing surface.
- `GetGPtrs (block)`: Get pointer of the zero crossing vector. This vector should be filled by the block and contain the zero-crossing surfaces.
- `GetJrootPtrs (block)`: Get pointer of the direction of the zero crossing vector
- `GetNmode (block)`: Get number of modes
- `GetWorkPtrs (block)`: Get the pointer of the Work array.
- `GetModePtrs (block)`: Get pointer of the mode vector
- `GetLabelPtrs (block)`: Get pointer of the block label
- `GetBoolInPortPtrs (block, n)`: Get pointer of boolean typed regular input port number n.
- `GetBoolOutPortPtrs (block, n)`: Get pointer of boolean typed regular output port number n.
- `GetImplicitFlag (block)`: check if the block is implicit (DAE)
- `GetBlockType (block)`: Get Block Type.
- `GetBlockNamePtrs (block)`: Get Block Name structure pointer ( II)
- `GetBlockName (block)`: Get the block *instance* name
- `GetBlockNSubNames (block)`: list of block inside a virtual block
- `GetBlockSubName (block, n)`: Get the block number n from the list of blocks inside a virtual block
- `GetDept (block)`: check if a block is time dependent
- `GetInher (block)`: Get the inheritance method, 0)no inheritance 1)inheritance from a single event input 2)inheritance from multiple event inputs
- `GetAjac (block)`: check if the block provides the Analytical Jacobian
- `GetCorrPtr (block)`: Get pointer of projection corrector vector
- `GetConstraintKind (block)`: This macro returns the constraint type defined by the block. There are two constraint types can be defined by the block:
  - **CONSTRAINT**: Additional equations defining algebraic equation between states are added to the block dynamical equations. In this case number of additional equations should be given by macro `SetNConstraint (block, n)`.

- PROJECTION: When this option is chosen, states values are readjusted to meet some constraints inside the model.
- `GetNConstraint (block)`: Get number of Constraints, if the constraint type is `CONSTRAINT`
- `SetAjac (block)`: Indicate that the block provides the Analytical Jacobian

#### 9.1.4 Macros for accessing the simulator structure

The following macros used to access some global variables or information such as simulator current time or error tolerances. These variables are available on the simulator structure.

- `GetFinalTime (block)`: Get the final time of the simulation.
- `GetHmax (block)`: Get the (maximum) step-size provided by the user (Note that this value can be *auto*). If the solver is fixed-step, this value will be the step-size of the solver.
- `GetHmaxUsed (block)`: Get the (maximum) step-size actually used by the simulator.
- `GetAtol (block)`: Get the absolute error tolerance used by the simulator.
- `GetRtol (block)`: Get the relative error tolerance used by the simulator.
- `GetVssInitialTime (block)`: Get the start time of the simulation used by the simulator.
- `GetTtol (block)`: Get the error tolerance in time provided by the user (Note that this value can be *auto*).
- `GetTtolUsed (block)`: Get the error tolerance in time actually used by the simulator.
- `GetAtolptr (block)`: Get the absolute error tolerance vector used by the simulator.
- `GetBlockNum (block)`: Get the ordering number of the block defined by the simulator.
- `GetSQRU (block)`: Get the square-root of the machine epsilon used by the simulator.
- `GetNeq (block)`: Get the number of continuous-time state of the model used by the simulator.
- `GetNzero (block)`: Get the number of zero-crossing surfaces of the model used by the simulator.
- `GetInitialX (block)`: Get initial value of continuous-time states of the model used by the simulator.
- `GetInitialXd (block)`: Get initial value of the derivative of continuous-time states of the model used by the simulator.
- `IsHotReStart (block)`: Indicates if the block is called while the solver is just being reinitialized after a discrete event. 0 means that the solver is just being restarted.
- `isColdRestart (block)`: Same as `IsHotReStart`, but 1 means that the solver is just being restarted.
- `DoColdRestart (block)`: Force the solver to reinitialize.
- `GetVssTime (block)`: Get the current time in the simulator.
- `GetSimulationPhase (block)`: Get the phase of the simulation used by the simulator.
- `GetMeshPoint (block)`: Get the last mesh-point time instant taken by the solver.

- `GetLastStepSize(block)`: Get the last step-size successfully taken by the solver.
- `GetNextTryStepSize(block)`: Get the next step-size being tried by the solver.
- `GetSimulatorInitiated(block)`: indicates if the simulator has finished the initialization of the model.
- `isStopRequested(block)`: Indicates if the user has pressed the stop button to terminate the simulation.
- `StopSimulation(block, val)`: Force the simulator to finish the simulation.
- `isZeroCrossingEnabled(block)`: Indicates if the zero-crossing option is enabled in the simulator.
- `isModeEnabled(block)`: Indicates if handling mode is enabled by the simulator.
- `GetXpropPtr(block)`: Get the vector of the xproperty assigned to the block by the simulator. This vector is used to define if a continuous-state is differential or algebraic.
- `GetAlphaPt(block)`: Get the actual value of alpha ( $\alpha$ ) when computing the Jacobian. Note that the Jacobian of the DAE  $0 = F(\dot{x}, x)$  is computed as follows:  $J = \alpha \frac{\partial F(\dot{x}, x)}{\partial x} + \beta \frac{\partial F(\dot{x}, x)}{\partial \dot{x}}$ .
- `GetBetaPt(block)`: Get the actual value of beta ( $\beta$ ) when computing the Jacobian.
- `isinTryPhase(block)`: Indicates if the block is being called by the numerical solver. In this case, the value given to the block is not a definitive value.
- `areModesFixed(block)`: Indicates that if the block is being called with mode values fixed or relaxed (can be changed/updated by the block).
- `isZeroCrossing(block)`: Indicates if the block is being called at a zero-crossing event.
- `isDiscreteEvent(block)`: Indicates if the block is being called at a discrete event.
- `isMeshPoint(block)`: Indicates if the block is being called at a mesh-point.
- `isDiscreteLeftLimit(block)`: Indicates if the block is being called at a left-limit of a discrete event.
- `isZeroCrossingLeftLimit(block)`: Indicates if the block is being called at a left-limit of a zero-crossing event.
- `isExitInitialization(block)`: Indicates if the block is being called at the end of the initialization.
- `isSimulatorInInitializationPhase(block)`: Indicates if the simulator is still in the initialization stage.
- `isSimulatorInTerminationPhase(block)`: Indicates if the simulator is in the termination stage.
- `Coserror(block, msg, â€œ)`: Emits an error message. This message usually follows an error in the block and is followed by a return from the block.
- `Coswarning(block, msg, â€œ)`: Emits a warning message.
- `Cosmessage(block, msg, â€œ)`: Emits a purely informative message.
- `GetSimulatedModelName(block)`: returns the name of the model being simulated.
- `GetSimulatedModelFilePath(block)`: returns the path of the model being simulated.

- `GetSimulatedModelTempDir (block)`: returns the path of the temporary folder for the current model.
- `isInDebugMode (block)`: Indicates if the simulation is done in the debug mode.
- `Discard (block)`: This The block can indicate the numerical solver that the state derivative cannot be computed and the solver should take smaller step size. This feature is used in `flag=VssFlag_Derivatives`, for ther situation is treated as error.
- `isinRepeatEventBlock (block)`: Indicates if this call to the block is the repetition of a previous event.

## 9.2 Block builder

The construction of simulation functions has been presented in the previous section. A C simulation function can be associated with a **CCustomBlock** and be used in **Activate** models. The **CCustomBlock** however has a generic user interface in the BDE which cannot be customized. The customization can be done by hiding this block inside a masked Super Block, however the level of customization is limited. A new basic block provides a much higher level of customization, as it will be seen in the next section.

Basic blocks and programmable super blocks can be constructed using the Block builder tool. These two types of blocks are similar at the BDE level and so is their construction. The block must provide its graphical and strcutural properties: parameters, numbers of ports of different types, block size, color, icon, etc., in an XML file. It must also provide in the case of the basic block: the list of simulation functions used by the block and three **OML** functions instantiating the block. In the case of the programmable super block, only one **OML** function is required.

By using the Block builder, user needs not worry about creating XML files or even defining **OML** functions. He/she needs simply provide the required information and the block builder creates the XML file. For the **OML** functions, only part of the code must be provided, the rest is auto-generated by the Block builder.

The Block builder interface contains 7 tabs:

1. Label: used to define the block default name, its location and whether or not it is visible by default.
2. Atom: used to define the internal properties of the block. Its definition depends on whether the block is a basic block or a prgorammable super block. Both cases will be discussed in details later.
3. Properties: used for documenting the block.
4. Parameters: used to define the number, type, name and default values of the block parameters. Callback functions can also be associated with parameters.
5. Graphics: used to set the block shape, size and color.
6. Ports: used to define the number of ports, their types and locations.
7. Icon: used to associate an image to the block icon. A second image may be used when the block is mirrored.

The block builder can be invoked from scratch or by selecting, in the current diagram, a basic block or a masked Super Block. In the latter case, the properties of the selected block are reflected in the

Block builder interface so that if no modifications are done, the block builder creates a duplicate of the selected block (in the case the selected block is a masked Super Block, the new block will be an equivalent programmable super block). The ability to initialize the Block builder interface using an existing block is very helpful for defining new blocks: user can start with an existing block similar to the new block and edit only the differences.

Atom is where the internal structure of the block is defined. The content of all the tabs, except Atom, are straightforward to understand and edit, and are identical for both basic blocks and programmable super blocks.

### 9.2.1 Atom: Basic block

For a basic block, Atom provides the list of simulation functions. In general a block has more than one simulation function, in particular if it supports different data types. For each simulation function, the shared library and the corresponding entry point is specified. A basic block definition requires up to 3 **OML** functions. The partial code for these functions are provided in Atom.

The first function, `SetParameters`, is the main function; it creates the block structure based on block parameters and port properties. The other two functions are used to modify this structure if needed. The **OML** code that must be provided to define the `SetParameters` function assumes that the followings are available:

- the name of the block, defined in the variable `_label`,
- the block parameter values under the name of the corresponding parameters,
- the diagram where the block is to be placed, defined in the variable `_diagram`.

For a basic block, the block added to diagram is of type “block”, so the **OML** code invariably contains the following lines

```
_block = vssAddNewObject ('Block', _diagram);  
vssSetBlockName (_block, _label);
```

The structure `_block` is then supplemented using provided APIs. For example simulation parameters `opar` are defined based on the value of block parameters. The code used here vary considerably depending on the properties of the new block. Instead of providing an exhaustive list of APIs and their usage, we encourage users to examine the codes associated with existing blocks, in particular blocks having similar properties to that of the new block. By doing so, the creation of the new code is reduced to changing a few lines of an existing code.

The `SetParameters` function is automatically created based on the user provided **OML** code mainly by adding systematic tests so that in case of error, an informative error message can be generated. This function is called in the model evaluation phase, which constructs the model structure used for compilation and simulation.

The two other functions are used to set the values not set by the `SetParameters` function in the block structure. The non-set values correspond to undefined size and data types of block input and output ports. The `SetParameters` function in general specifies all the properties of the block structure but it can leave out some properties, to be found by the compiler. In particular the size of an input may be left unknown. This is done by setting the size vector to negative values,  $[-1, -2]$ . This means the input has totally arbitrary size. If another input or an output size is also set to  $[-1, -2]$ , then the latter size is also arbitrary but equal to the former. If an input size is set to  $[-1, -1]$ , then the matrix is a



square matrix of arbitrary size. Negative value means unknown, but the same negative values mean same sizes. This way constraints on block port sizes can be imposed for many common blocks. For example the matrix transpose block has input size  $[-1, -2]$  and output size  $[-2, -1]$ . This captures all the constraints imposed by the block on its input and output.

The compiler figures out the actual port sizes by taking into account these constraints and the way the blocks are interconnected. Similarly block port data types may be set to arbitrary and the compiler deduces the actual types. Once the compiler completes the computation of block input/output data sizes and types, the block **OML** function `Reset` is called with argument the block structure including the actual port data sizes and types. This function can then modify other block properties if needed. In most cases, the block `Reset` function selects the appropriate simulation function based on the port data types. The `Reset` function is defined in `Atom` based on user-supplied code.

The last **OML** function associated with a basic block is `SetIO`. This function is defined when the constraints on port data types and sizes cannot be imposed by using negative numbers. For example a block may be such that the size of its input is arbitrary and the size of its output is one bigger than the size of its input. These “exotic” constraints cannot be imposed using negative size values. In such cases the block can provide a `SetIO` function working on the block structure (constructed previously by `SetParameters`). This function, based on current status of port sizes and types, may replace negative port sizes and types by positive values. This function may be called many times during compilation. For the example where the output is one bigger than the input, the function can fully define the port sizes if either the input or the output size is known, when the function `SetIO` is called.

Most user defined blocks don't require `SetIO` and `Reset` functions and the associated code in `Atom` should be left empty.

## 9.2.2 Atom: Programmable Super Block

For programmable super blocks, there is no simulation function to define and there is only the `SetParameters` function that needs to be defined in `Atom`. In this case the content being defined is a “subsystem” and the **OML** code is used to construct the diagram inside the block. The code contains:

```
_block = vssAddNewObject('SubSystem', _diagram);
_diagr= vssCreateObject('Diagram');
```

and then goes on populating `_diagr` with blocks and links constructing a diagram.

The definition of a block inside the diagram is done by defining the block parameters by an **OML** `struct`, loading the block from the corresponding library and instantiating it. Consider for example the block **DiscreteDelay**, which has parameters `init_cond`, `typ`, and `externalActivation`. This block can be instantiated in the diagram `_diagr` for example as follows

```
_params=struct();
_params.init_cond=z0;
_params.typ='double';
_params.externalActivation=0;
vssLoadBlock('system', 'Dynamical', 'DiscreteDelay');
_vss._palettes.('system').('Dynamical').('DiscreteDelay').setparams(_diagr, 'DiscreteDelay', _params);
```

Here the initial condition is set to `z0`, which must be a parameter of the programmable super block.

Links are created using the **OML** function `vssAdd_Link`. The following instruction for example connects the first output of the **Input** block to the first output of the **DiscreteDelay** block by a regular link:

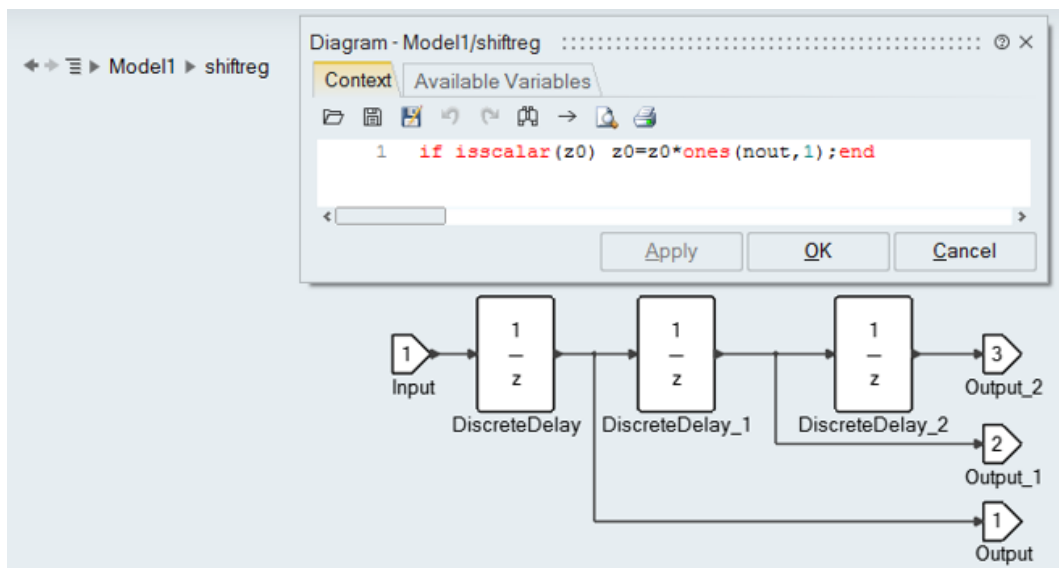
```
vssAdd_Link(_diagram, 'Input', 'DiscreteDelay', 1, 1, 0, 1, 1);
```

When the Block builder is invoked with a masked Super Block, the `SetParameters` function of the block is generated automatically containing the code that constructs the diagram inside the Super Block. In this case the diagram is fixed.

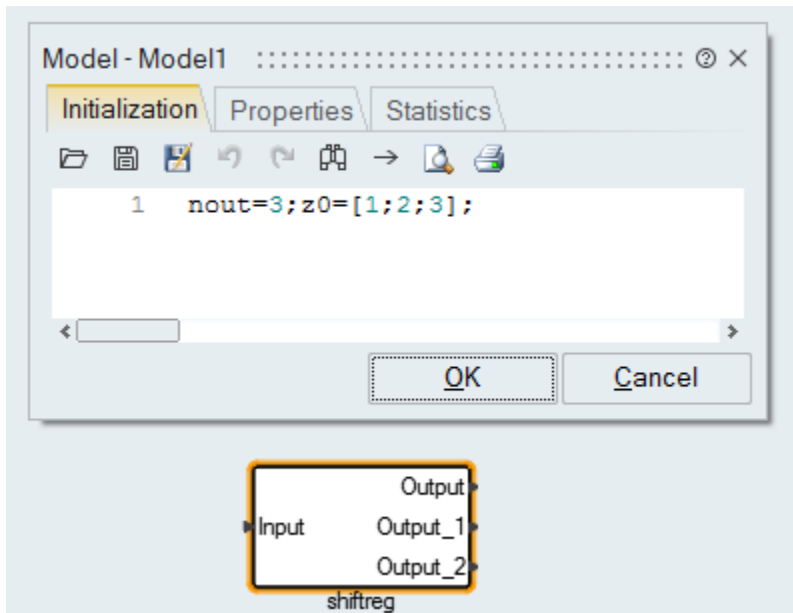
A fixed-diagram programmable super blocks is not particularly useful because it can also be implemented by a standard masked Super Block. It can however be a good starting point for defining the `SetParameters` function of a general programmable super block where the diagram inside and the number of ports can depend on block parameters. This process is illustrated through the following example.

## Example

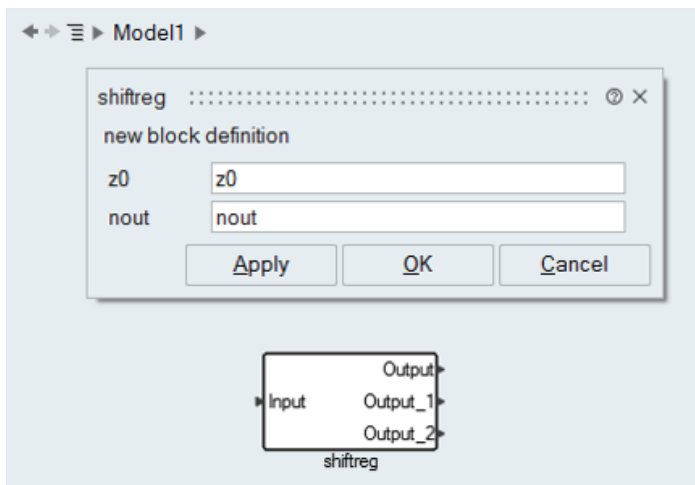
Here the problem is to create a shift register block where each register value is exposed through a separate output. The number of registers (and outputs) is thus variable, equal to `nout`, given as a block parameter. In case of three registers/outputs (`nout=3`), the diagram is simple:



This Super Block will be used as a starting point for the construction of the programmable super block. First, the Super Block is prepared to be masked by defining the undefined variables `z0` and `nout` in the Initialization script of the model. Auto-masking can then be applied:

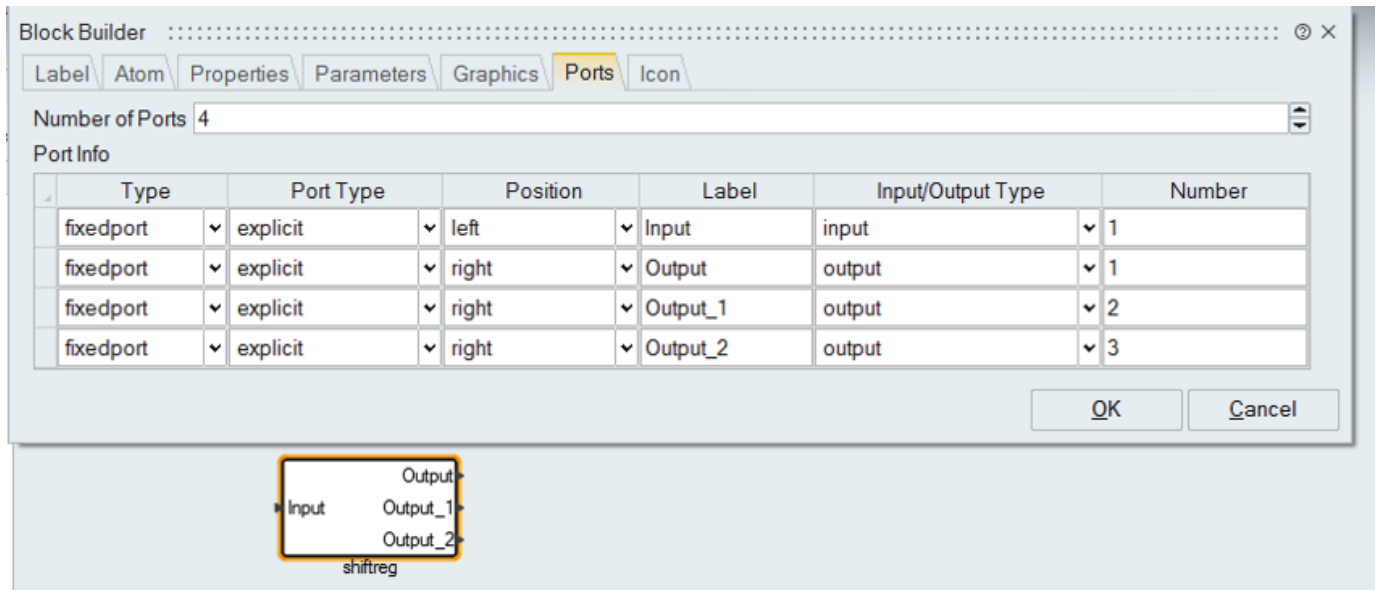


which yields:

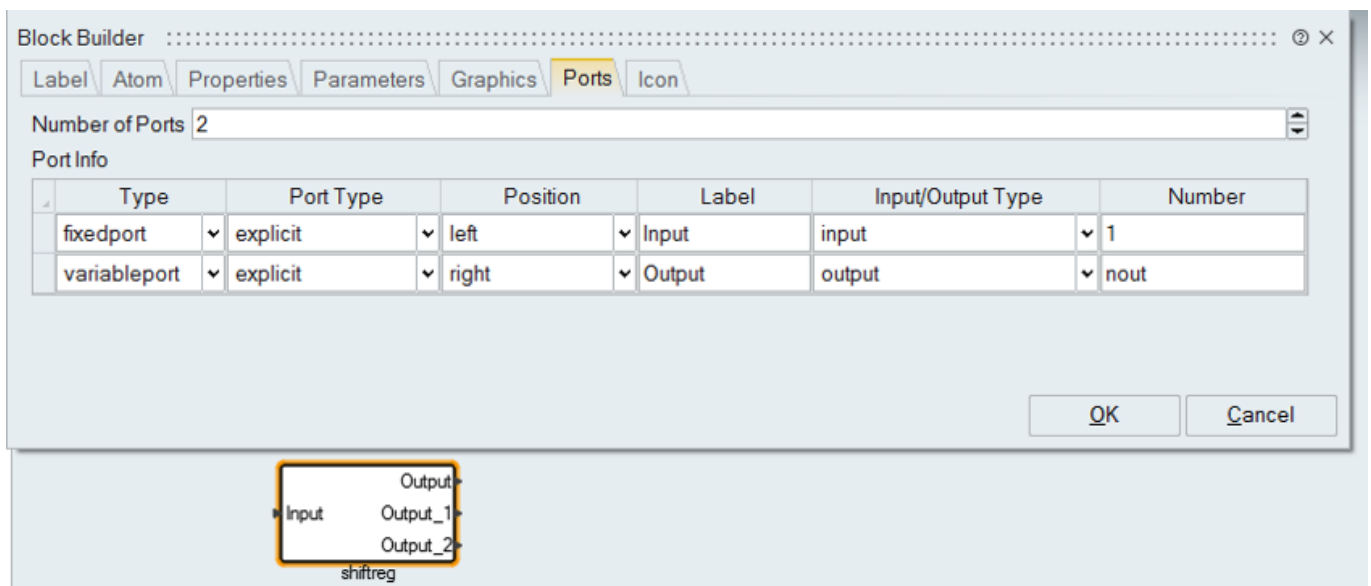


Now that the Super Block is masked, block builder can be applied to create a new block. The Block builder now contains a good starting point for defining the new block.

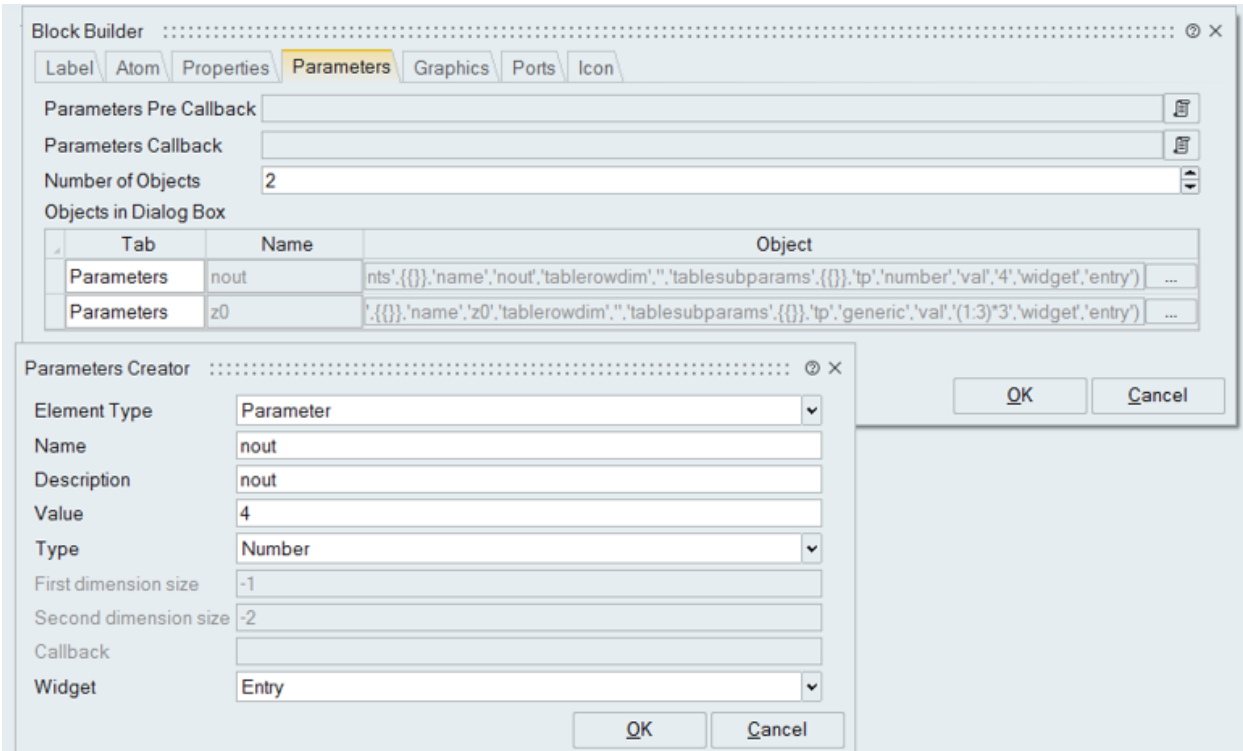
Some of the properties must be modified for the new block. In particular the number of ports in the Ports tab. Originally the ports are fixed:



The Ports tab must be edited to allow the number of outputs to be variable, in particular `nout`:



The Parameters tab must be edited to change the property of the parameter `nout` to make it of type "Number". Structural properties such as the number of ports can only be set by parameters of type "Number".



The main modification to be made is of course in the `SetParameters` code (inside the Atom tab) which defines the diagram inside the Super Block. The current code constructs the diagram by instantiating blocks and connecting them for the case `nout=3`. Note that no graphical properties are used at this point: the diagram does not have a corresponding graphics. The code generated by the Block builder for the fixed diagram is as follows:

```
function _setdiagram_1 (_diagram, _label, __fenv__)
    _curdiagram=_diagram;
    _curlabel=_label;
    importenv(__fenv__);
    _diagram=_curdiagram;
    _label=_curlabel;
    clear _curdiagram;
    clear _curlabel;
    global _vss;
    global __OBJ_ERR__;
    _parent = __OBJ_ERR_.('block');
    __OBJ_ERR_.('type')='context';
    if isscalar(z0) z0=z0*ones(nout,1);end
    __OBJ_ERR_.('block')=vssConstructBlockFullName(_parent,'Output_2');
    __OBJ_ERR_.('type')='parameters';
    _params=struct();
    _params.portNumber=3;
    _params.insize=[-1;-2];
    _params.intyp='inherit';
    vssLoadBlock('system','Ports','Output');
    _vss._palettes.('system').('Ports').('Output').setparams(_diagram,'Output_2',_params);
    __OBJ_ERR_.('block')=vssConstructBlockFullName(_parent,'DiscreteDelay_2');
    __OBJ_ERR_.('type')='parameters';
    _params=struct();
    _params.init_cond=z0(3);
    _params.typ='double';
    _params.externalActivation=0;
    vssLoadBlock('system','Dynamical','DiscreteDelay');
    _vss._palettes.('system').('Dynamical').('DiscreteDelay').setparams(_diagram,'DiscreteDelay_2',_params);
    __OBJ_ERR_.('block')=vssConstructBlockFullName(_parent,'Output_1');
    __OBJ_ERR_.('type')='parameters';
```

```

_params=struct();
_params.portNumber=2;
_params.insize=[-1;-2];
_params.intyp='inherit';
vssLoadBlock('system','Ports','Output');
_vss._palettes.('_system').('Ports').('Output').setparams(_diagram,'Output_1',_params);
__OBJ_ERR_.('block')=vssConstructBlockFullName(_parent,'Output');
__OBJ_ERR_.('type')='parameters';
_params=struct();
_params.portNumber=1;
_params.insize=[-1;-2];
_params.intyp='inherit';
vssLoadBlock('system','Ports','Output');
_vss._palettes.('_system').('Ports').('Output').setparams(_diagram,'Output',_params);
__OBJ_ERR_.('block')=vssConstructBlockFullName(_parent,'Input');
__OBJ_ERR_.('type')='parameters';
_params=struct();
_params.portNumber=1;
_params.outsize=[-1;-2];
_params.outtyp='inherit';
_params.dept=0;
vssLoadBlock('system','Ports','Input');
_vss._palettes.('_system').('Ports').('Input').setparams(_diagram,'Input',_params);
__OBJ_ERR_.('block')=vssConstructBlockFullName(_parent,'Block');
__OBJ_ERR_.('type')='parameters';
_params=struct();
vssLoadBlock('system','Links','Split');
_vss._palettes.('_system').('Links').('Split').setparams(_diagram,'Block',_params);
__OBJ_ERR_.('block')=vssConstructBlockFullName(_parent,'DiscreteDelay_1');
__OBJ_ERR_.('type')='parameters';
_params=struct();
_params.init_cond=z0(2);
_params.typ='double';
_params.externalActivation=0;
vssLoadBlock('system','Dynamical','DiscreteDelay');
_vss._palettes.('_system').('Dynamical').('DiscreteDelay').setparams(_diagram,'DiscreteDelay_1',_params);
__OBJ_ERR_.('block')=vssConstructBlockFullName(_parent,'Split');
__OBJ_ERR_.('type')='parameters';
_params=struct();
vssLoadBlock('system','Links','Split');
_vss._palettes.('_system').('Links').('Split').setparams(_diagram,'Split',_params);
__OBJ_ERR_.('block')=vssConstructBlockFullName(_parent,'DiscreteDelay');
__OBJ_ERR_.('type')='parameters';
_params=struct();
_params.init_cond=z0(1);
_params.typ='double';
_params.externalActivation=0;
vssLoadBlock('system','Dynamical','DiscreteDelay');
_vss._palettes.('_system').('Dynamical').('DiscreteDelay').setparams(_diagram,'DiscreteDelay',_params);
vssAdd_Link(_diagram,'Output_2','DiscreteDelay_2',1,1,1,0,1);
vssAdd_Link(_diagram,'DiscreteDelay_1','Block',1,1,0,1,1);
vssAdd_Link(_diagram,'Block','Output_1',1,1,0,1,1);
vssAdd_Link(_diagram,'DiscreteDelay_2','Block',1,2,1,0,1);
vssAdd_Link(_diagram,'Input','DiscreteDelay',1,1,0,1,1);
vssAdd_Link(_diagram,'DiscreteDelay','Split',1,1,0,1,1);
vssAdd_Link(_diagram,'Split','DiscreteDelay_1',1,1,0,1,1);
vssAdd_Link(_diagram,'Split','Output',2,1,0,1,1);
end
_block = vssAddNewObject('SubSystem',_diagram);
_diagr= vssCreateObject('Diagram');
_setdiagram_1(_diagr,_label,getcurrentenv());
vssSet_SubSystem(_block,1,3,0,0,_label,_diagr);

```

The variable `__OBJ_ERR_` is used to anchor the error locations for generating precise error messages. In the case of programmable super blocks, this is in general not very useful and it is better to reference all error message to the parent block (the programmable super block). User in general does not know about the content of this block.

The above code, which corresponds to the fixed diagram with three delays and outputs, is extended so that it can work for arbitrary `nout` number of delays and outputs. Note that the individual error tags are removed so that all errors are referred to the parent. The script used in the context of the Super Block is also extended to produce better error messages:

```
function shiftreg_setdiagram_1 (_diagram, _label, __fenv__)
    _curdiagram=_diagram;
    _curlabel=_label;
    importenv(__fenv__);
    _diagram=_curdiagram;
    _label=_curlabel;
    clear _curdiagram;
    clear _curlabel;
    global _vss;
    global __OBJ_ERR_;
    _parent = __OBJ_ERR_.('block');
    __OBJ_ERR_.('type')='context';
    if isscalar(z0) z0=z0*ones(nout,1); elseif length(z0)~=nout error('size of z0 must match nb of outputs.');
```

end

```
__OBJ_ERR_.('block')=vssConstructBlockFullName(_parent,'DiscreteDelay');
    _params=struct();
    _params.init_cond=z0(1);
    _params.typ='double';
    _params.externalActivation=0;
    vssLoadBlock('system','Dynamical','DiscreteDelay');
    _vss._palettes.('_system').('Dynamical').('DiscreteDelay').setparams(_diagram,'DiscreteDelay',_params);
    _params=struct();
    vssLoadBlock('system','Links','Split');
    _vss._palettes.('_system').('Links').('Split').setparams(_diagram,'Split_1',_params);
    _params=struct();
    _params.portNumber=1;
    _params.outsize=[-1;-2];
    _params.outtyp='inherit';
    _params.dept=0;
    vssLoadBlock('system','Ports','Input');
    _vss._palettes.('_system').('Ports').('Input').setparams(_diagram,'Input',_params);
    _params=struct();
    _params.portNumber=1;
    _params.insize=[-1;-2];
    _params.intyp='inherit';
    vssLoadBlock('system','Ports','Output');
    _vss._palettes.('_system').('Ports').('Output').setparams(_diagram,'Output',_params);
    vssAdd_Link(_diagram,'Input','DiscreteDelay',1,1,0,1,1);
    vssAdd_Link(_diagram,'DiscreteDelay','Split_1',1,1,0,1,1);
    vssAdd_Link(_diagram,'Split_1','Output',2,1,0,1,1);
    for i=2:nout-1
        _params=struct();
        _params.init_cond=z0(i);
        _params.typ='double';
        _params.externalActivation=0;
        vssLoadBlock('system','Dynamical','DiscreteDelay');
        _vss._palettes.('_system').('Dynamical').('DiscreteDelay').setparams(_diagram,['DiscreteDelay_',...
            num2str(i-1)],_params);
        _params=struct();
        _params.portNumber=i;
        _params.insize=[-1;-2];
        _params.intyp='inherit';
        vssLoadBlock('system','Ports','Output');
        _vss._palettes.('_system').('Ports').('Output').setparams(_diagram,['Output_',num2str(i-1)],_params);
        _params=struct();
        vssLoadBlock('system','Links','Split');
        _vss._palettes.('_system').('Links').('Split').setparams(_diagram,['Split_',num2str(i)],_params);
        vssAdd_Link(_diagram,['Split_',num2str(i-1)],['DiscreteDelay_',num2str(i-1)],1,1,0,1,1);
        vssAdd_Link(_diagram,['DiscreteDelay_',num2str(i-1)],['Split_',num2str(i)],1,1,0,1,1);
        vssAdd_Link(_diagram,['Split_',num2str(i)],['Output_',num2str(i-1)],2,1,0,1,1);
    end
    _params=struct();
    _params.init_cond=z0(nout);
    _params.typ='double';
```

```

_params.externalActivation=0;
vssLoadBlock('system','Dynamical','DiscreteDelay');
_vss._palettes.('_system').('Dynamical').('DiscreteDelay').setparams(_diagram,['DiscreteDelay_',...
    num2str(nout-1)],_params);
_params=struct();
_params.portNumber=nout;
_params.insize=[-1;-2];
_params.intyp='inherit';
vssLoadBlock('system','Ports','Output');
_vss._palettes.('_system').('Ports').('Output').setparams(_diagram,['Output_',num2str(nout-1)],_params);
vssAdd_Link(_diagram,['DiscreteDelay_',num2str(nout-1)],['Output_',num2str(nout-1)],1,1,0,1,1);
vssAdd_Link(_diagram,['Split_',num2str(nout-1)],['DiscreteDelay_',num2str(nout-1)],1,1,0,1,1);
end
_block = vssAddNewObject('SubSystem',_diagram);
_diagr= vssCreateObject('Diagram');
shiftreg_setdiagram_1(_diagr,_label,getcurrentenv());
vssSet_SubSystem(_block,1,nout,0,0,0,_label,_diagr);

```

The new repeated blocks are named by extensions `_2`, `_3`, up to `nout`. This is not needed for links which are not named.

After creating the first layer of blocks, the repeated structure is created within the following for loop:

```

for i=2:nout-1
    ....
end

```

The final layer is created after the for loop. Another change needs to be made in the code to change the number of outputs of the new block:

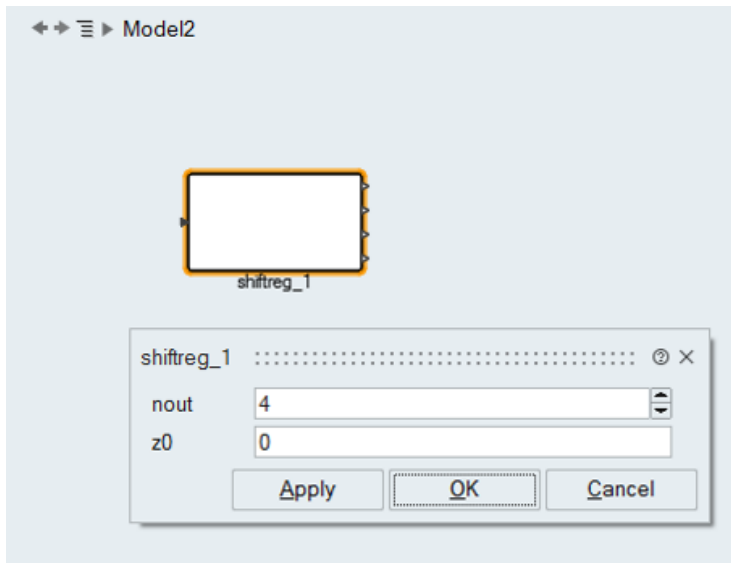
```
vssSet_SubSystem(_block,1,3,0,0,0,_label,_diagr);
```

is replaced by

```
vssSet_SubSystem(_block,1,nout,0,0,0,_label,_diagr);
```

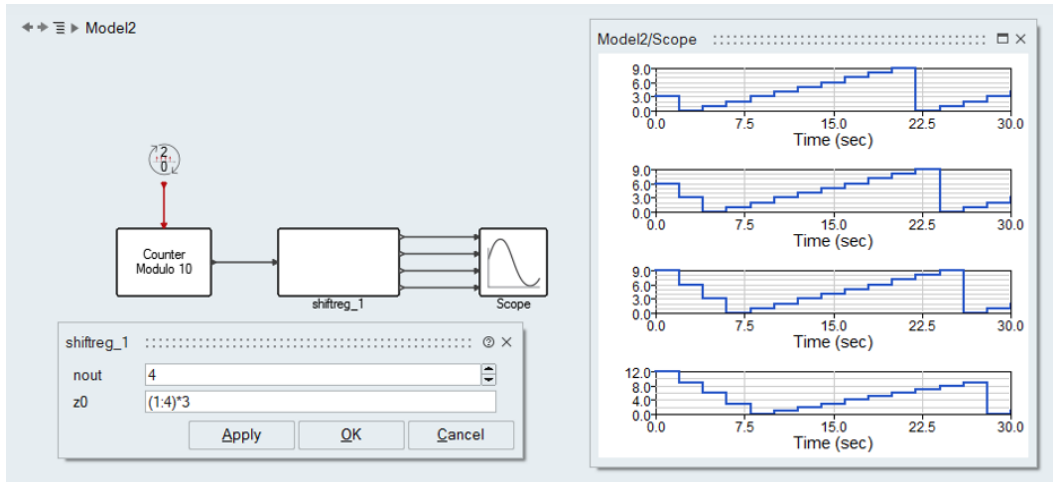
indicating that the block has `nout` outputs and not 3.

The newly created block can be moved to a new model and its parameters can be set to desired values:



and tested:







## Chapter 10

# Activate hybrid simulator and its interface with numerical solvers

**Activate** is a simulator for hybrid systems. A hybrid system is composed of discrete-time and continuous-time subsystems in which the values of continuous-time variables are governed by differential equations and discrete-time variables are updated at time instants called *event times*. In **Activate** several numerical solvers have been interfaced with the simulator to integrate the differential equations. These numerical solvers have many control parameters that should be used to optimize the simulation in different situations. Indeed, in the hybrid context, controlling the solver, which should be done automatically and should remain transparent to the user, is a complex matter due to the interactions between the continuous-time dynamics and the rest of the system. In this document, the simulator of **Activate**, the numerical solvers and the interface between the simulator and the numerical solvers will be discussed.

### 10.1 Introduction

A hybrid system is a dynamical system that involves the interactions of continuous states and discrete states. As an example, consider a bouncing ball which is a hybrid system since the state variables (position and speed) vary continuously according to Newton laws when falling, but they have a discrete change (speed is reversed) when entering in collision with the ground. Jump in state and discontinuities are basic phenomena that cannot be represented and analyzed by methods elaborated either in continuous-time or in discrete-time system theory. In a hybrid system composed of continuous-time and discrete-time subsystems, each subsystem can interact with the other, e.g., if a discrete-time variable used in a differential equation is changed all the continuous-time variables that depend on this value may change. This would imply a discontinuity in these variables.

The discrete-time part of a hybrid system is activated by events and events introduce discontinuities in continuous-time variables or in their time derivatives. Events may come from several sources such as intervention by human operators or from an input control or may be programmed to be generated autonomously.

There are two event types that are more important. They are events that occur at a given time, i.e., a predictable event (or time events), and those which appear when a variable reaches a certain threshold value, i.e., an unpredictable (or a zero-crossing event).

The underlying hybrid formalism in **Activate** allows modeling very general hybrid systems: systems including continuous, discrete-time and event based behaviors. **Activate** includes a graphical editor for constructing models by interconnecting blocks, representing predefined or user-defined functions, a compiler, a simulator, and code generation facilities. The graphical editor allows creating and connecting blocks selected from block palettes. A **Activate** block diagram is composed of blocks and connection links. There are two connection link types; regular and activation links. The regular links represent continuous-time or discrete-time signals or variables in the model, while the activation links transmit activation timings information. A block corresponds to an operation and by interconnecting blocks through links, a model or an algorithm is constructed. These blocks represent elementary systems that can be used as model building blocks. They can have several inputs and outputs, continuous-time states, discrete-time states, zero-crossing functions, etc. Note that it is not possible to treat an activation link as a regular link or to interconnect an activation link with a regular link. In order to get an idea what a **Activate** model looks like, a model of a simple control system implemented in **Activate** is shown in the following figure.

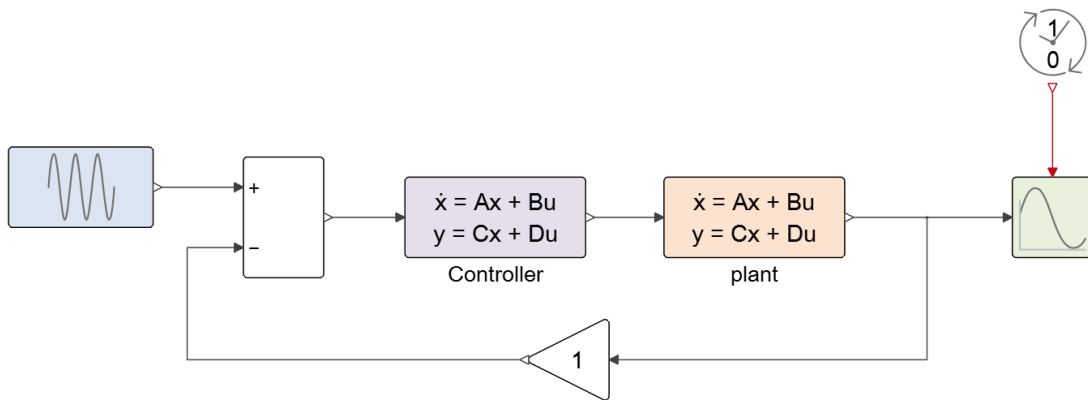


Figure 10.1: Simple control model (simple\_control1.scm)

In this figure (10.1) the clock block generates a periodic activation signal (events) that activates the Scope block. At event times the scope reads its input signal and displays it.

Here is an example of a pure discrete-time system where the state is updated by a periodic clock with period  $T$ .

$$x(k+1) = \begin{cases} x(k) + u(k) & \text{if } u(k) > 0 \\ x(k) & \text{otherwise} \end{cases}$$

$$x(1) = 0$$

The implementation of this model and its simulation result in **Activate** are given in following figures (10.2, 10.3). In this model  $u(k)$  is generated by a random signal generator block with output values between  $[-1, +1]$ .

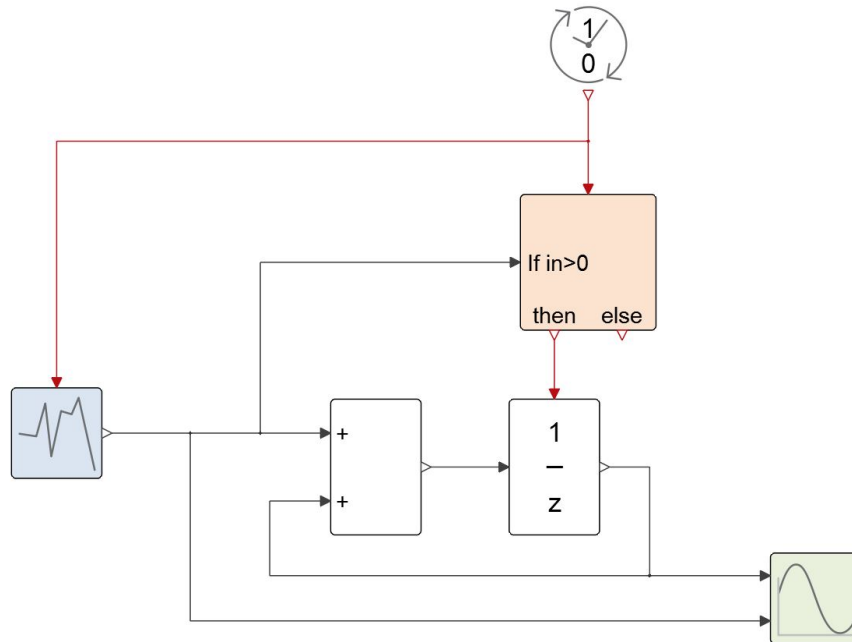


Figure 10.2: Discrete model (discretem.scm)

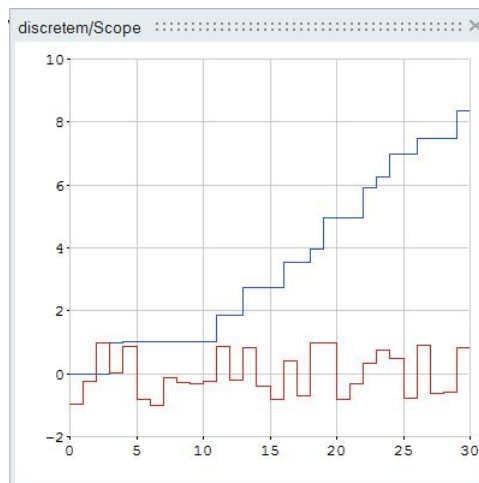


Figure 10.3: Results of Discrete model (discretem.scm)

Now consider the model of the bouncing ball. The model implemented in **Activate** and its simulation result are given in the following figures (10.4, 10.5). In this model when the altitude of the ball ( $x$ ) becomes negative, the zero-crossing block generates an event that resets the Linear Continuous-time block with new state values.

$$\begin{cases} \dot{x} = v \\ \dot{v} = -9.8 \end{cases}$$

when ( $x < 0$ ) then  $v = -0.9 v$

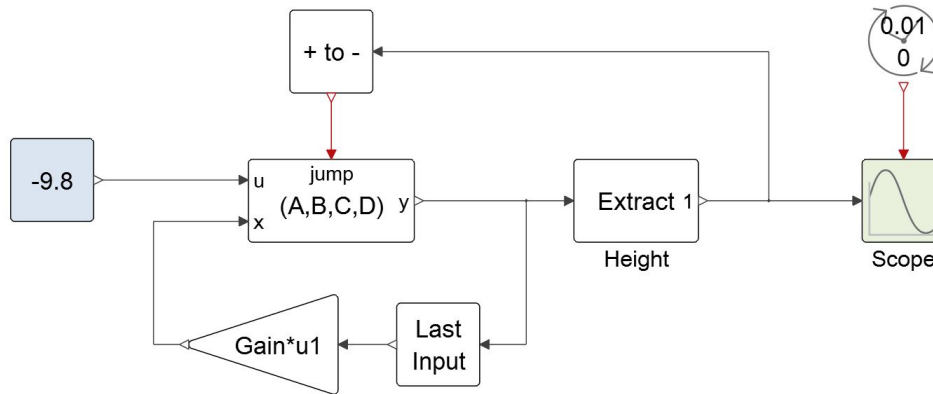


Figure 10.4: Simple bouncing ball model (simple\_bball.scm)

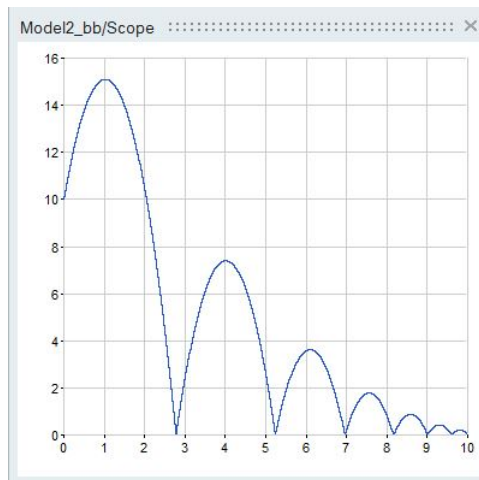


Figure 10.5: Results of Simple bouncing ball model

## 10.2 Simulation of an hybrid model in Activate

The simulator of **Activate** is designed to generate the solution of the **Activate** model, which is very often a hybrid system. A hybrid system simulator should be able to perform, discrete-time scheduling, continuous-time integration, and interactions between discrete-time and continuous-time part (zero-crossing events and event handling).

Simulation of a hybrid system is composed of two phases: the discrete-phase and the continuous-phase. In the discrete phase, event scheduling, event activating and state updating (discrete-time and continuous-time) is done, and the simulation time does not advance. In the continuous-phase, the numerical solver integrates the differential equation and the simulation time advances. Integration of the continuous-time system is done from one event time to the next event time. The exact time instant of an event may not be known in advance, hence during the continuous-time phase an event detection process is being performed to stop the continuous-phase and enter into the discrete-phase.

The continuous-time parts of a hybrid system can be modeled either by Ordinary Differential Equations

(ODE), i.e.,

$$\begin{aligned}\dot{x} &= f(x, u, t) \\ y &= h(x, u, t)\end{aligned}$$

where  $x$  represents the state variable,  $\dot{x}$  denotes the derivative of  $x$  with respect to the time variable  $t$ ,  $u$  is the input vector variable, and  $y$  defines the output vector, or by Differential Algebraic Equations (DAE)

$$\begin{aligned}\dot{x}_1 &= f(x_1, x_2, u, t) \\ 0 &= g(x_1, x_2, u, t) \\ y &= h(\dot{x}_1, x_1, x_2, u, t)\end{aligned}$$

The first equation is the differential part and the second is the algebraic part of the DAE. This equation is a semi-explicit DAE where differential and algebraic parts can be decomposed. In semi-explicit index-1 DAE, the variable whose derivative is present in the equation is referred to as differential equation and others are called algebraic variables. This equation can be cast in the form

$$\begin{aligned}0 &= F(\dot{x}, x, u, t) \\ y &= h(\dot{x}, x, u, t)\end{aligned}$$

where  $x = (x_1, x_2)$ . This system is called a fully implicit DAE. Note that if partial derivative of  $F$  with respect to  $\dot{x}$  is non-singular then it is possible to solve for  $\dot{x}$  in order to obtain an ODE. However, if it is singular, this is no longer possible and the solution must satisfy certain algebraic constraints.

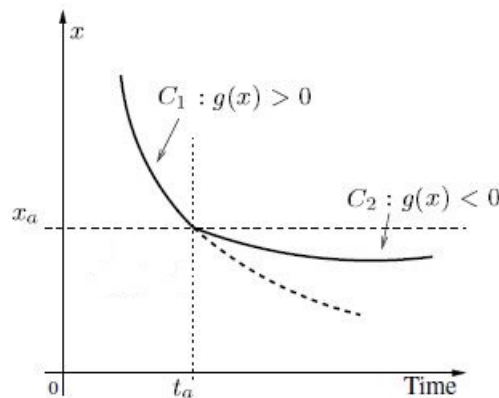
DAEs are characterized by their index. The index of a DAE is the smallest number of differentiations required to obtain an ODE by algebraic manipulations. In general, the higher the index, the greater the numerical difficulty one encounters when trying to solve the system numerically. In a semi-explicit index-1 DAE, variables whose derivatives appear in DAE are called differential variables and the others are called algebraic.

Modeling a hybrid system very often leads to a differential equation with discontinuities or a multi-mode system such as an automaton (see Documentation of the `Automaton` block). In simulation of a hybrid system, state variables or their time derivative may be discontinuous at some time instants. In such a situation special care has to be taken to solve the model by a numerical solver. Some numerical solvers such as `Radau` or `Lsoda` use variable order and variable step-size methods to integrate. In simulation of a discontinuous model, such solvers may repeat several times the same step with smaller step-size to obtain a coherent derivative and meet the accuracy requirements at the discontinuity point. For some problems, the solver may give up with an error message when the step gets too small, and in other cases the solver may continue with an erroneous result. In general, such kind of solvers require that the system be sufficiently smooth over an integration period (at least be  $C_0$ ). This means that simulator must make sure to stop and reinitialize the solver at each potential point of non-smoothness (discontinuity, discontinuity in the derivative, etc.). The discontinuity handling problem is solved in **Activate** by introducing the `mode` variable in **Activate**.

A multi-mode system is a way of describing non-smooth systems in terms of a finite number of smooth systems. The idea is to divide the state space of the system into different regions and associate a `mode` to each region. It is assumed that the system is described in terms of a single smooth model within each region. A simple example of a multi-mode system, where the model in each mode is described as DAE, is:

$$0 = \begin{cases} f_1(\dot{x}, x) & \text{if } g(x) > 0 \\ f_2(\dot{x}, x) & \text{otherwise} \end{cases}$$

This system has two modes: the first one is active when  $g(x) > 0$  and the second is when the condition is not true. The switching between two modes of the system ( $f_1$  and  $f_2$ ) may cause discontinuity in the signals. As a result the system may not be integrated by numerical solvers. Numerical solvers do not like discontinuities in the differential equation and they assume that the state variables and their first derivatives are continuous. If the signal is not continuous, special precautions must be taken at the discontinuity points. To overcome this problem, the discontinuity should be detected and the numerical solver be reinitialized after the discontinuity point. In order to detect and localize the discontinuity time, the simulator uses zero-crossing functions that cross zero at the discontinuity point. For example, for the above model, the zero-crossing function to be used by the simulator is  $g(x)$ . However, to localize this point, the numerical solver should simulator say  $f_1$  and step over the discontinuity point and use the other model ( $f_2$ ). Using the model beyond the discontinuity point ( $f_2$ ) normally results in repeated step failure and a reduction in integration step-size to meet accuracy requirements. For some problems especially models with hard discontinuity the solver gives up with an error message when the step gets too small, and in some other cases the solver may continue with erroneous results. In order to solve this problem we have associated a `mode` variable with each zero-crossing function. We use the `mode` variable to choose a single ODE/DAE when the numerical solver is called. The assigned ODE/DAE does not change till the next discontinuity detection or event. For example, for the above model the general solution has been illustrated in the following figure where  $x_a$  indicates the discontinuity point or the instant where  $g(x_a)$  becomes zero.



Before reaching  $x_a$ , the solver is in (`mode=1`), i.e. , it uses  $0 = f_1(\dot{x}, x)$  , without considering the  $f_2(\dot{x}, x)$  equation even when the solver needs to compute  $f_1(\dot{x}, x)$  for  $g(x) > 0$  (the thick dashed line). The solver employs the  $f_1(\dot{x}, x)$  equation to integrate until it detects a zero-crossing. Then the solver localizes the crossing point and stops just at the discontinuity point (i.e., at  $x = x_a$ ). After detecting the zero-crossing and stopping the integration, the `mode` variable should be updated in order to use another model, i.e.,  $f_2(\dot{x}, x)$ .

In general, when the numerical solver is called, the system of equations should not be changed or show discontinuity. During integration, the zero-crossing functions are monitored by the simulator. In case of any sign change in the zero-crossing functions, the `mode` variables should be updated to provide another ODE/DAE to the solver. In **Activate**, a `mode` variable is assigned systematically to each zero-crossing function, characterized by `If-then-Else` conditions, e.g., to simulate the above system, **Activate** considers the model in the form:



$$0 = \begin{cases} f_1(\dot{x}, x) & \text{if mode} = 1 \\ f_2(\dot{x}, x) & \text{if mode} = 2 \end{cases}$$

where `mode` variables are updated at event time instants, with the following rules.

$$\begin{array}{ll} \text{when } (g(x) > 0) \text{ is true} & \text{then mode} = 1 \\ \text{otherwise} & \text{mode} = 2 \end{array}$$

It should be mentioned that any If-then-Else condition, not resulting in a discontinuity, can be used without `mode` variables. For instance, in this system the `mode` variable is not necessary.

$$0 = \begin{cases} x^2 + x + 1 & \text{if } x \geq 1 \\ 2x^2 - x + 2 & \text{otherwise} \end{cases}$$

Using `mode` variables eliminates solver difficulties over discontinuities but there is still the problem of initialization the `modes` for DAE.

Most of the problems in simulation of hybrid systems are common to both ODE and DAE solvers. However, there is an additional difficulty with the DAE case: the problem of (re)initialization and finding consistent initial conditions. Simulation of an ODE can be started from any initial state, but simulation of a DAE should be started from a consistent initial state. A DAE should be initialized with consistent initial values to be solved by a numerical solver, i.e., initial values of states and their derivatives needs to be consistent with the DAE and some other hidden equations resulted from differentiation of the DAE.

Computation of consistent initial conditions is not in general an easy problem and it becomes particularly difficult for multi-mode DAE (defined with several `modes`). The search for the initial condition in this case is not just the classical problem of finding zeros of smooth functions but it is interlaced with searching for the correct `mode` value. Indeed, each `mode` value implies a smooth function and the solution of this function may imply another `mode` value.

The classification in terms of differential and algebraic is also a key element in the study of multi-model DAEs, and in particular, the problem of finding consistent initial conditions since the classification specifies which variables are solved for. Consider, for example, the following multi-model DAE

$$\begin{cases} \dot{x} = \sin(t) \sin(x) \\ \text{if } (x < 1) \text{ then } y = 2 \text{ else } y = 3 \end{cases}$$

where in **Activate**, it is rewritten as:

$$\text{DAE : } \begin{cases} \dot{x} = \sin(t) \sin(x) \\ \text{if } (\text{Mode}_1 = 1) \text{ then } y = 2 \text{ else } y = 3 \end{cases}$$

$$\text{Mode changing : } \begin{cases} \text{when } (x < 1) \text{ then } \text{Mode}_1 = 1 \\ \text{when } (x \geq 1) \text{ then } \text{Mode}_1 = 2 \end{cases}$$

In this example,  $x$  is a differential variable and its initial value is given. On the other hand,  $y$  is an algebraic variable. This multi-model DAE is well-posed and it is easy to find consistent initial conditions for it. In this case, the condition that needs to be tested involve the variable  $x$  which is given. Now consider the following multi-model DAE

$$\begin{cases} \dot{x} = x + y + z \\ \text{if } (y < 1) \text{ then } z = 0, \text{ else } z = 3 \\ \text{if } (z < 1) \text{ then } y = 0, \text{ else } y = 3 \end{cases}$$

where in **Activate**, we use:

$$\text{ODE : } \begin{cases} \dot{x} = x + y + z \\ \text{if ( Mode}_1 = 1 \text{ ) then } z = 0, \text{ else } z = 3 \\ \text{if ( Mode}_2 = 1 \text{ ) then } y = 0, \text{ else } y = 3 \end{cases}$$

$$\text{Zero-crossing : } \begin{cases} \text{when } (y < 1) \text{ then Mode}_1 = 1 \\ \text{when } (y \geq 1) \text{ then Mode}_1 = 2 \\ \text{when } (z < 1) \text{ then Mode}_2 = 1 \\ \text{when } (z \geq 1) \text{ then Mode}_2 = 2 \end{cases}$$

This problem is more complicated because the conditions are based on the values of  $y$  and  $z$  (both algebraic variables), and the solution cannot be obtained without considering multiple scenarios. In fact, this system does not even have a unique solution for the algebraic variables in terms of the differential variables. If  $x$  is given a value, then  $(y, z)$  can be either  $(0, 0)$  or  $(3, 3)$ .

In **Activate** it is assumed that differential variables are known and the other variables should be computed. As a result, based on the dependence of discontinuity functions on these variables and derivatives, `mode` initialization can be complicated. Consider the the initialization of the following DAE

$$0 = F(\dot{x}_d, x_d, x_a)$$

where  $x_d$  and  $x_a$  are the differential and algebraic states. In this case, if the zero-crossing functions depend on  $x_a$  and  $\dot{x}_d$ , the initialization becomes complicated, because the initial values of these variables are not known and their value may change during the initialization, that causes several `mode` changing during the initialization.

A dynamical system which contains  $N$  Modes, say characterized by  $N$  `If-then-else` conditions, can start in any of the  $2^N$  possible Modes. For the systems with large  $N$  and especially when the conditions are functions of algebraic variables, it is practically impossible to find the starting `mode` via testing all possible Modes. As a result, we have elaborated a search method to find the consistent Modes and initial conditions. The following algorithm has been tested and being used in **Activate**:

1. Assigning the differential states ( $x_d$ ) and take the guess values of  $x_a$  and  $\dot{x}_d$
2. Based on  $x_d$  values and current  $x_a$  and  $\dot{x}_d$  guess values, fix the `modes`.
3. If number of iterations exceeds `nmod`, return by error.
4. Store the current `modes`.
5. Invoke the algebraic solver to find the consistent initial conditions, *i.e.*,  $x_a, \dot{x}_d$
6. Re-evaluate the zero-crossing functions and fix the `modes`
7. Compare new `modes` with stored ones, If there is any change, go to step 3
8. The `modes` and states are consistent. Initialization is finished and the simulation can be started.

### 10.3 Numerical solver classifications and characteristics

Once a model is built in the **Activate** block diagram editor, the solution of the model in a time domain must be computed numerically. There are several solvers available in **Activate**. Each solver implements a specific numerical algorithm for solving the model. As a result, the choice of the solver

becomes important: which solver to use and with what parameters. This section tries to explain general characteristics of solvers that help the user to choose the right solver for right model.

### 10.3.1 Stiff vs. non-stiff ODE

An important characteristics of an ODE solver is to know if it is appropriate for solving a stiff ODE. Based on this information the appropriate solver can be chosen for a particular problem. In order to simulate an ODE, the numerical solver divides the simulation time into small intervals of length step-size and finds the solution at each interval. Smaller step-size generally results in higher accuracy of the solution, but it also means higher computation time. Even if the model to be simulated is stable, the stability of the solution provided by the numerical method depends directly on the step-size. In many cases smaller step-sizes should be used to ensure the stability of the numerical method. Whatever the numerical method, for large step sizes the solution tends to be unstable and stability may be achieved by using sufficiently small step sizes. Generally, the critical step-size depends on the eigenvalues of the dynamics of the model. In problems that have both large and small eigenvalues, the solution may have a short transient solution as the high frequencies settle, followed by a longer period where it is the result of the low frequencies that dominate. In this latter phase, the high frequencies are still present in the system even if they are not visible in the solution. Nevertheless, the integration method continues to use a small step-size to ensure the numerical stability. A system with such behavior is called stiff. Solution of stiff ODEs show sharp changes in the solution with respect to time scale of the overall solution.

### 10.3.2 Explicit vs. Implicit Solvers

Numerical solvers for differential equations are divided into two categories: Explicit and implicit methods. explicit methods calculate the new state of a system as a function of current state and current state derivatives. This is in contrast with implicit methods where the new state is found by solving a (very often) nonlinear equation involving both the new state and the new derivatives as a function of the current state. In other words, if  $x(t)$  is the current system state and  $x(t+h)$  is the unknown new state at the new time  $(t+h)$ , where  $h$  is the time step-size, then, for an explicit method we have

$$x(t+h) = F(x(t))$$

while for an implicit method a nonlinear equation like the following should be solved.

$$G(x(t), x(t+h)) = 0$$

Implicit solvers should solve a nonlinear equation at each time step which makes these solvers slower compared to explicit solvers. If these solvers are slower so why should they be used? They are actually used because many problems in real life are stiff. In simulation of a stiff model, explicit methods need to take small step sizes for a slowly varying solution. For such models, implicit methods, even though more costly, can integrate much faster by taking much larger step sizes. For stiff problem an explicit method requires very small time steps to keep the error small. The need for very small step-size makes the simulation very slow and costly. For such problems, to achieve given accuracy, implicit methods can take big steps and much less computational time event taking into account that they need to solve a nonlinear equation at each step. In summary the choice of the solver depends directly on the model, whether the model is stiff or not and what is the requirement for the simulation time.

### 10.3.3 Fixed-step vs. variable-step solvers

Another aspect of numerical solvers is whether they are fixed-step or variable-step. The numerical solver can use the same step-size all over the simulation or adjust it during the simulation. Based on this property, the solver is fixed-step or variable step. In variable-step solvers the step-size is adjusted as a function of the estimated error in the solution. There are two kinds of error: absolute and relative error. Absolute error is the amount of physical error in a measurement. While relative error gives an indication of how good a measurement is relative to the size of the signal being measured. The acceptable error on a state value is defined as follows:

$$|x| * Rtol$$

where  $x$  is a state in the model. Since the  $x$  may be zero or near zero, the acceptable error is generally computed as follows:

$$|x| * Rtol + Atol$$

where Rtol and Atol are relative and absolute error tolerances respectively. The estimated error can be used to adjust the step-size to maintain the error under the acceptable error tolerances.

Variable step-size ODE solvers take small time steps when the variables change rapidly and take larger time steps when the variables vary slowly. This allows maintaining the estimated error under the error threshold. This is an important feature that increases the efficiency of the solvers. A variable-time step method is usually faster than its constant time step counterpart, because the computational effort is concentrated only on those time intervals that need it most, taking large steps over intervals that do not need small time steps. This feature usually compensates for the extra work required for error estimation and step-size adjustment. Variable step solvers are more reliable to handle fast changes in the solution by reducing the time step, where constant time step methods may give wrong results. On the other hand in some situations where the computational effort should be limited in each step, such as real-time applications fixed step-size solvers are privileged. The user will often want the solution at specified points in time, for example at equally spaced intervals, to produce tables or plots. For these cases, we do not need using fixed-step solvers. Fortunately, most variable solvers can provide the solution values at requested times by interpolating the computed solution points. This approach is more efficient than using fixed-step solvers.

### 10.3.4 Analytical vs. numerical Jacobian matrix

In solving the ODE,

$$\dot{x} = f(x)$$

implicit solvers need the matrix

$$J = \frac{\partial f}{\partial x}$$

and for solving the DAE

$$0 = F(\dot{x}, x)$$

implicit solvers need the matrix

$$J = \alpha \frac{\partial F}{\partial x} + \beta \frac{\partial F}{\partial \dot{x}}$$

where  $J$  is called the Jacobian matrix. The Jacobian matrix can either be provided as an analytical matrix by the user or can be computed numerically by the solver via the finite difference method. The numerical method works fine in most cases, but if the model is stiff or complex, using a numerically computed Jacobian matrix may cause the solver to fail. In **Activate**, if a block provides its local Jacobian matrix analytically, the simulator can take advantage of it and build the overall Jacobian matrix and deliver it to the solver.

An **Activate** block can provide analytical Jacobian of its internal model, i.e., the matrix  $J(x)$ . If a block does not provide this matrix analytically, the simulator computes it numerically via finite difference method. The mixture of analytical and numerical Jacobian matrices of blocks are used to compute the global Jacobian of the whole model. Currently only few blocks provide the analytical Jacobian information, such as FMU and linear ABCD block.

### 10.3.5 Single-step vs. multi-step solvers

Single-step solvers approximate the solution of the model at time  $t + h$  as a function of the solution of the model between times  $t$  and  $t + h$  (current step). On the other hand, multi-step solvers approximate the solution at the end of the time step as a function of the model solution at previous steps. In order to achieve high order accuracy, single-step solvers may need to evaluate the differential equation more often per time step than multi-step solvers. As a result, single step solvers may have higher computational cost per step than multi-step solvers. Due to this cost, multi-step solvers may be more efficient than single-step solvers. The downside of multi-step solvers is the requirement for the initialization phase. These solvers should start with lower order methods or with other single-step methods for very first steps. This may affect the accuracy. Furthermore, multi-stage methods are less efficient if the solution contains frequent discontinuities with respect to single-stage solvers. Indeed, at discontinuities, all solution history of multi-step solvers is obsolete and should be thrown away and the solver needs another costly initialization. A single-step solver may be more efficient for applications with high number of discontinuities. In **Activate** Euler, Radau and Runge-Kutta are examples of single-step solver, and Lsoda, Dopri, Ida are examples of multi-step solver.

### 10.3.6 ODE vs. DAE solver

A **Activate** model is either represented as ODE or DAE. Simulation of a DAE model needs special solver. **Activate** provides IDA, Daskr, and Radau for simulation of DAE models. Although these solvers can be used for simulation of ODEs, they are slower than ODE solvers such as Lsoda for the simulation of an ODE. DAE solvers are however good for simulation of stiff ODEs.

Some kind of DAEs can be represented as ODEs with algebraic constraints, i.e.,

$$\begin{cases} \dot{x} = f(x) \\ 0 = g(z) \end{cases}$$

where  $x$  is the state vector and  $z$  is a subset of vector  $x$ . This model can be simulated using an ordinary ODE solver without taking into account the constraint  $g(z)$ . The solution of course does not satisfy the constraint and that may lead to results in a solution which does not respect physical laws such as energy or mass conservation laws. In order to simulate this kind of models, **Activate** provides the CPODE solver. In this case the user should provide the constraints or a projection that brings back the solution on the constraints.

When an ODE is simulated, any initial value can be used as start values of states. DAEs, on the other hand, do not have this freedom. In a DAE, initial value of states and their derivatives should be consistent, i.e., they should satisfy a set of initial algebraic equations. In order to simulate a DAE these algebraic equations should be solved at the beginning of the simulation as well as at any discontinuity instants. In order to solve these initial equations an algebraic solver should be used. `IDA` and `DASKR` has their own internal algebraic solvers, but `Radau` needs an external one. **Activate** provides `IDACALC`, `DASKRCALC` and `FSOLVE` algebraic solvers that can be used with any of DAEs solvers.

## 10.4 Numerical Solver interface with the simulator

Any numerical solver has several parameters and a correct and fast simulation requires the fine tuning of these parameters and a good interface with the simulator. In this section we explain some important common parameters of solvers and the way they are handled by the simulator of **Activate**.

### 10.4.1 Forward looking in time (stopping time)

Another important feature of most solvers is the possibility of providing a dense output result. Consider a situation where the user needs to integrate a model from  $T_{\text{initial}}$  to  $T_{\text{final}}$  and many intermediate output points are required. It is not efficient to run the solver from one output point to the next and then return to the main program to print the results at the output points, even if the solver is called with a hot-start option. To handle this situation efficiently, the solver may integrate past the output points and interpolate back to obtain the results at requested output points. This increases the efficiency, but sometimes it is not possible to integrate beyond some point  $t_{\text{stop}}$  because the equation changes there or simply because the solution or its derivative is not defined past  $t_{\text{stop}}$ . To overcome this obstacle, most solvers have another important property to set a time limit on the solver. Time limit can be imposed to forbid the solver from advancing the time beyond a given time called the stopping time. In **Activate** during the simulation, the nearest event that requires a cold-restart is considered as the stop time and is given to the solver.

### 10.4.2 Zero-crossing and event-detection

An important feature of a hybrid simulator is the ability to detect and handle zero-crossings or state-events. Zero-crossings are introduced in models to correctly handle discontinuities or generate events. The ability to detect the time when a discontinuity occurs, or more precisely, the time when a function crosses some given value (by default considered zero) is of capital importance in the hybrid environment. When a zero-crossing occurs, the simulator pinpoints the exact crossing time and stops the simulation at the crossing point. Zero-crossings are not just used to generate events in order to communicate with the discrete-time part of the model. In most cases zero-crossings are used to properly control the solver to simulate non-smooth continuous-time dynamics. Consider a continuous-time model in which the absolute value function is used. For example, the system

$$\begin{cases} \dot{x} = |u| \\ u = \sin(t) \end{cases}$$

can produce a non-smooth signal even if the input  $u = \sin(t)$  is smooth. It is important to halt the solver and do a cold restart at the point where the non-smoothness occurs.

The simulator triggers a zero-crossing event if the sign of a zero-crossing function changes or the function becomes zero at the end of a solver's step. This works fine to some extent. In fact in some particular cases, where for example there is a model change, or having loose error tolerances, the simulation may show chattering and detects numerous unphysical zero-crossings. In **Activate** simulator the user can set a threshold value to the zero-crossing function. If this threshold is defined, after detection of a zero-crossing event, that zero-crossing is disabled (masked) until its absolute value becomes larger than the threshold value and leaves the threshold region (unmasked). This mechanism avoid unwanted chattering. The default value of the threshold is zero.

### 10.4.3 Cold-start versus hot-start of the solver

There are two strategies for starting the integration with the multi-step solvers like `IDA` or `Lsoda` solvers: cold-start and hot-start. A cold-start is the first call to the solver for a given problem. In this case, many cumbersome memory setting tasks are done and the integration is started with a very small step-size and a first order method. A hot-start, on the other hand, is a call to the solver that is not the first call for the given problem. In this case, assuming no change in the problem or discontinuity, the solver does not perform the memory setting tasks and can use all past information about the solution. This makes a hot-start more efficient than a cold start. Cold restart is costlier for multi-step solvers with respect to single step ones. Since in multi-step solvers memory about solution in previous step is lost and the solver needs a new reinitialization and restart from lower order methods. In **Activate** the simulator has been designed to avoid unnecessary costly cold-restarts.

In order to demonstrate the effect of a bad design and excessive cold restarts, consider the model illustrated in Fig.10.6. In this model, instead of using a clock to generate an event sequence, a Sine block is connected to a zero-crossing block. At each zero-crossing detection, the numerical solver is cold restart-ed. The model contains continuous-time states, and detection of zero-crossings forces the multi-step solver (LSODA in this example) to do a cold-restart. The simulation time becomes much shorter, if a Clock block is used to activate the Counter block instead of the Zero-crossing block.

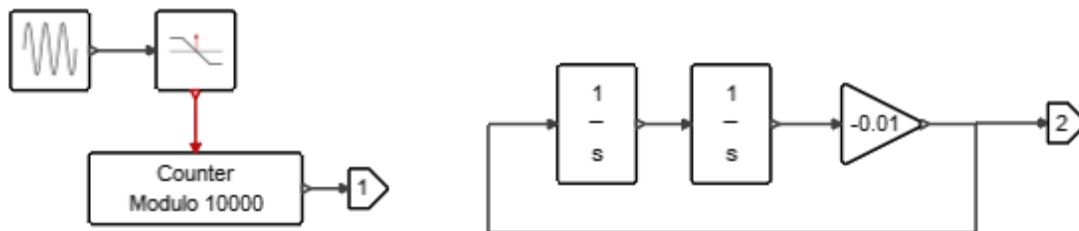


Figure 10.6: Effect of excessive cold-restart (coldrestart.scm)

### 10.4.4 Absolute and relative error tolerances

The absolute and relative error tolerances specified for the integration of an ODE affects the speed and accuracy of the computation for variable step solvers. Small error tolerances may result in higher accuracy, but the simulation time will become large. On the other hand looser error tolerance values, speeds up the simulation but the error introduced in the solution will be higher. As a consequence, it is important to find a compromise between speed and accuracy. It is not recommended to choose very tight (small) or very loose (large) `Atol` and `Rtol` values. If Very small error tolerances are chosen the

integration may become too slow or even fail to achieve the desired accuracy. In general a relative error tolerance around 1% or 0.1% is acceptable. The value of absolute error tolerance is only important if the value of signal being observed is close to zero.

#### 10.4.5 Maximum, minimum, and initial step-size

Variable step solvers adjust the step-size during the integration, if the model becomes stiff or the error becomes large, the solver reduces the step-size to reduce the error. If the error estimation remains constant or reduces, the solver tries a larger step-size to advance faster in time. Thus, the step-size value can be very small or very large. **Activate** provides the possibility of setting high and low limits on the step-size chosen by the solver. As an example to demonstrate how setting a high limit on the step-size may be useful, consider a simple non-stiff model containing a zero-crossing. The model is not stiff, thus the solver tries to advance the time by taking large step sizes. If it happens that the zero-crossing function changes the sign several times during a large step, the solver will miss the sign change in the zero-crossing function and it may introduce error in the result. In order to avoid such kind of errors, the user can limit the maximum step-size. If the maximum step-size set to *auto* the solver chooses according to the following formula.

$$H_{max} = \frac{T_{final} - T_{initial}}{100}$$

Where  $T_{final}$  is the final time and  $T_{initial}$  is the initial simulation time.

The size of the very first steps after a cold-restart in variable step solvers does not usually have much importance, since the solver can adjust it and find an appropriate value. In some situations, the solver is unable to find a good value for the step-size and fails. In order to help the solver, the user can set the initial value of the step-size.

#### 10.4.6 Fixed-step solver interface with the simulator

Fixed-step solvers are usually used for non-stiff models where convergence is ensured by choosing small enough step-sizes. The way fixed step solvers have been interfaced with the simulator is a bit different from that of variable step solvers. First important issue is handling events when a fixed-step solver is chosen. With fixed-step solvers, the simulator considers the step-size as the basic clock of the model. All other events are synchronized with solver steps. No event is allowed to be triggered during a step. If an event is programmed to happen during a solver step, it is postponed to the end of the step. It is also important to indicate that when a fixed-step solver is chosen, no zero-cross iteration is performed. The sign of zero-crossing functions are checked only at the beginning of the step. An if a sign-change happens, an event is triggered instantly. `mode` variables are also completely ignored.

#### 10.4.7 Numerical solvers available in Activate

**Activate** provides several solvers for simulation of ODE and DAE models. Here is the list of available solvers. If a model is represented by an ODE, an ODE solver can be used. On the other hand if the model is represented by a DAE, a DAE solver should be used. A model using Modelica blocks, implicit blocks or `Automaton` block usually results in DAE. Note that for simulation of ODE models, DAE solvers can also be used but they are less efficient than ODE solvers.



## ODE solvers

1. **Lsoda** is a variable step and variable order solver [1, 2, 3, 5]. It is a multi-step solver based on BDF method. This solver is the best choice when the user does not know if the model is stiff or not. The solver switches automatically between stiff and non-stiff methods during the integration. It automatically selects between nonstiff (Adams) and stiff (BDF) methods. It uses the stiff method initially, and dynamically monitors data in order to decide which method to use. The maximum step-size for this solver is unlimited. The maximum integration order for stiff and non-stiff methods is set to 5 and 12 respectively. `Lsoda` is a part of `ODEPACK`<sup>1</sup>. `ODEPACK`[4] is a collection of Fortran solvers for the initial value problem for ordinary differential equation systems. It consists of nine solvers, namely a basic solver called `LSODE` and eight variants of it `LSODES`, `LSODA`, `LSODAR`, `LSODPK`, `LSODKR`, `LSODI`, `LSOIBT`, and `LSODIS`. The collection is suitable for both stiff and nonstiff systems. It includes solvers for systems given in explicit form,  $\frac{dy}{dt} = f(t, y)$ , and also solvers for systems given in linearly implicit form,

$$A(t, y) \frac{dy}{dt} = g(t, y)$$

In **Activate** only `Lsoda` has been used.

2. **CVODE** which is a part of the SUNDIALS package [6]<sup>2</sup> is a solver for stiff and nonstiff ordinary differential equation (ODE) systems (initial value problem) given in explicit form  $y' = f(t, y)$ . The methods used in `CVODE` [8] are variable-order, variable-step multistep methods. For nonstiff problems, `CVODE` includes the Adams-Moulton formulas, with the order varying between 1 and 12. For stiff problems, `CVODE` includes the Backward Differentiation Formulas (BDFs) in so-called fixed-leading coefficient form, with order varying between 1 and 5. For either choice of formula, the resulting nonlinear system is solved (approximately) at each integration step. For this, `CVODE` offers the choice of either functional iteration, suitable only for nonstiff systems, and various versions of Newton iteration. In the cases of a direct linear solver (dense or banded), the Newton iteration is a Modified Newton iteration, in that the Jacobian is fixed (and usually out of date). The direct solvers can only be used with the provided serial vector structure. These are written in C, but assume that the user calling program and all user-supplied routines are in Fortran. In **Activate** the `CVODE` solver is provided in four variants:
  - **CVODE-BDF-NEWTON** is a variable step and variable order solver. The multi-step method is based on BDF with maximum order of 5. The Newton iterations are used to solve the nonlinear equation resulting from discretization. This solver is used when we know the model stays always in the stiff region.
  - **CVODE-BDF-Functional** is a variable step and variable order solver. The multi-step method is based on BDF with maximum order of 5. A functional iteration (some sort of fixed-point iteration) is used to solve the nonlinear equation resulting from discretization.
  - **CVODE-ADAMS-NEWTON** is a variable step and variable order solver. The multi-step method is based on Adams method with maximum order of 12. The Newton iterations are used to solve the nonlinear equation resulting from discretization.
  - **CVODE-ADAMS-Functional** is a variable step and variable order solver. The multi-step method is based on Adams method with maximum order of 12. A functional iteration is used to solve the nonlinear equation resulting from discretization. This solver which is usu-

<sup>1</sup>Available at <https://www.netlib.org>

<sup>2</sup>Available at <https://computation.llnl.gov/casc/sundials/main.html>

ally used when the model is known to be non-stiff, allows to get a high accuracy and fast integration.

3. **RADAU-IIA for ODE** [9, 10, 11] is a single-step method based on implicit Runge-Kutta methods (Radau IIa) of order 5, 9, 13. These methods are L-stable. These methods are suitable for  $Mf' = f(t, y)$  with a possibly singular matrix  $M$ . It is therefore suitable for stiff ODEs and DAEs. A Newton iteration is used to solve the nonlinear equation resulting from discretization. The Radau solver<sup>3</sup> is a variable step and variable order solver. The higher order methods perform better than low order methods as soon as the convergence of the Newton iteration is sufficiently fast. This solver is used when the model is stiff and since it is a single-step solver it is more efficient in presence of frequent discontinuities.
4. **Backward Euler** is an order one method with fixed step size. The backward Euler method (or implicit Euler method) is one of the most basic numerical methods for the solution of ordinary differential equations. This solver is widely used in industry as the simplest fixed-step solver for stiff models. It is similar to the forward Euler method, but differs in that it is an implicit method. The backward Euler method has order one and is A-stable.
5. **Implicit Trapezoidal** is second order, fixed step and single-step solver. It is an implicit method from family of Runge-Kutta method.
6. **DOPRI (Dormand-prince)** is variable step, 5th order and single-step solver. It is based on a family of Runge-Kutta order 5 method. The Dopri solver<sup>4</sup> provides a fast integration for non-stiff ODE. the Dormand-Prince method, is an explicit method for solving ordinary differential equations [12, 9, 10]. This solver uses six function evaluations to calculate fourth- and fifth-order accurate solutions. The difference between these solutions is then taken to be the error of the (fourth-order) solution. This error estimate is very convenient for adaptive stepsize integration algorithms.
7. **Runge-Kutta** is a fixed-step, 5th order explicit Runge-Kutta method solver. It is based on a family of Runge-Kutta order 5 method. This solver is used for non-stiff models.
8. **Classical Runge-Kutta** is a fixed-step, 4th order Runge-Kutta method solver.
9. **Forward Euler** is a first order, fixed step, single-step solver. This solver is widely used in industry as the simplest fixed-step solver. the Euler method is the simplest numerical method for ODEs with a given initial value. The Euler method is a first-order method, which means that the local error (error per step) is proportional to the square of the step size, and the global error (error at a given time) is proportional to the step size. The Euler method often serves as the basis to construct more complex methods, e.g. Predictor-corrector method. This method is widely used in industry when the computation time is critical such as in embedded systems.
10. **Explicit Trapezoidal** is a second order, fixed step, single-step solver. It is an explicit method from family of Runge-Kutta method.
11. **CPode(ODE with constraint)** is a variable step and variable order solver. The multi-step method is based on BDF with maximum order of 5. The Newton iterations are used to solve the nonlinear equation resulting from discretization. This solver is especially used when the model is an ODE with several additional constraints. The constraints need to be respected during the simulation. This solver handles algebraic constraints between states and also accepts the projection to bring back the states on the constraints.

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<sup>3</sup>Available at <http://www.unige.ch/~haire/>

<sup>4</sup>Available at <http://www.unige.ch/~haire/>

## DAE solvers

1. **IDA** which is a part of the SUNDIALS package [6]<sup>5</sup> is a numerical solver for the solution of differential-algebraic equation (DAE) systems in the form  $F(t, y, y') = 0$ . It is written in C, but derived from the package DASPK which is written in Fortran. The integration method in IDA [7] is variable-order, variable-coefficient BDF, in fixed-leading-coefficient form. The method order varying between 1 and 5. The solution of the resulting nonlinear system is accomplished with some form of Newton iteration. In the cases of a direct linear solver (dense or banded), the nonlinear iteration is a Modified Newton iteration, in that the Jacobian is fixed. When the model is DAE this solver can be used. The minimum step-size is zero for this solver.
2. **RADAUV-IIA for DAE** [9, 10, 11] is a variable step, variable order solver. The Radau solver<sup>6</sup> is based on implicit Runge-Kutta methods (Radau IIA) of order 5, 9, 13. These methods are L-stable. This is a single-step method based of Radau method of order 5, 9, 13. These methods are suitable for  $Mf' = f(t, y)$  with a possibly singular matrix  $M$ . It is suitable for stiff ODEs and index-1 and index-2 DAEs. A Newton iteration is used to solve the nonlinear equation resulting from discretization. The higher order methods perform better than low order methods as soon as the convergence of the Newton iteration is sufficiently fast. This solver is used when the model is stiff and since it is a single-step solver it is more efficient in presence of frequent discontinuities.
3. **DASKR** is a variable step, variable order method solver for systems of differential-algebraic equations (DAEs). **DASKR**<sup>7</sup> [13] is a multi-step solver based on BDF method. In contrast to the DASSL solver, DASKR includes a procedure for calculating consistent initial conditions for a large class of problems (which includes semi-explicit index-1 systems). The package also includes an option to omit the algebraic components from the local error control. If the model is DAE this solver can be used. The minimum step-size is zero for this solver.

The following table classifies the solver to clarify which solver is appropriate for stiff or non-stiff ODE or for DAE models.

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<sup>5</sup>Available at <https://computation.llnl.gov/casc/sundials/main.html>

<sup>6</sup>Available at <http://www.unige.ch/~hairer/>

<sup>7</sup>Available at <http://www.netlib.org/ode/>

<b>Solver type</b>	<b>Stiffness</b>	<b>Solver Name</b>
Fixed step-size	non stiff ODE	Forward Euler Explicit Trapezoidal Classical Runge-Kutta Runge-Kutta
	stiff ODE	Backward Euler Implicit Trapezoidal
variable step-size	non stiff ODE	CVODE-BDF-Functional CVODE-ADAMS-Functional DOPRI (Dormand-prince)
	stiff ODE	Lsoda CVODE-BDF-NEWTON CVODE-ADAMS-NEWTON RADAU-IIA for ODE CPODE
	DAE	IDA RADAUV-IIA for DAE DASKR

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# Chapter 11

## Physical component modeling in Altair Activate

### 11.1 Introduction

**Activate** blocks communicate data with other blocks through ports that are specified as inputs and outputs. Produced data is placed on a block output port and retrieved through the input port of one or more other blocks. Even though it is possible to model many physical systems containing components in **Activate**, this generally requires translating the implicit equations representing the behavior of the physical components into explicit form and as a result, the resulting **Activate** diagrams may differ significantly from the original component based diagrams.

In **Activate**, the behavior of physical components is specified through symbolic equations. Unlike regular blocks that may be regarded as black boxes, physical components expose the implicit equations of the corresponding component. **Activate** then can, through symbolic manipulation, use these equations to produce efficient code for models obtained from the interconnection of physical components. The symbolic specification of the behavior of physical components in **Activate** is done using the [Modelica language](#).

### 11.2 Causal vs. acausal modeling

**Activate** allows using components for construction of models. The ports of a component are not labeled, a priori, as inputs and outputs, and the block imposes a dynamical constraint on the values on its ports, contrary to regular blocks which compute explicitly their outputs as functions of their inputs. Components are essential for constructing models which include physical components such as resistors, capacitors, etc., in electricity, or pipes, nozzles, etc., in hydraulics. They also arise in many other areas such as mechanics and thermodynamics.

Modular model construction can be classified in two major categories: causal and acausal. To make an analogy with computer programming languages, causal modeling corresponds to the use of assignment statements where the right-hand sides of the equations are evaluated and the result of the evaluation is assigned to the variables on the left-hand side of the equations. This corresponds closely to the evaluation of the outputs based on the inputs in a causal module. Acausal modeling on the other hand

is closer to *equations* in the mathematical sense of the term. For an equation in a set of mathematical equations, it is not possible to classify (at least a-priori) its variables as inputs and outputs. In this case, for constructing the solution to the set of equations, the choice of the variable which should be computed in terms of the rest of the variables, in one particular equation, depends on the complete set of equations. In other words, a mathematical equation contains potentially a number of assignment statements; the choice of the statement which should be used for constructing the solution of the complete problem depends on all the equations.

An acausal module is like a mathematical equation. It has ports of communication with other modules but these ports are not labeled a-priori as inputs and outputs.

Most physical components are more naturally modeled as acausal modules simply because physical laws are expressed in terms of mathematical equations. Consider a resistor in an electrical circuit. If the resistor is to be used as a module, it can only be an acausal module because depending on the way the resistor is used, the input can be a current and the output a voltage, or the input a voltage and the output a current.

Often in modeling and simulation softwares, only causal modules (blocks) are considered. This situation is not only easier to implement but it provides a stronger notion of modularity which corresponds closely to the notion *subsystem* used by systems and control engineers. This is the reason why such way of modeling is referred to as *system level modeling*, as opposed to *component level modeling* where acausal modules (blocks) are used.

It is in general possible to convert a component level model into a system level model by rewriting the equations and finding the appropriate causality structure in each module, but this task is time consuming, and error prone. Furthermore the resulting (system level) model has very little resemblance with the original model making subsequent modifications difficult to realize. Indeed, a small change in the original model may require a complete redesign of the system level model.

To illustrate the difference between component level and system level modeling, consider the simple electrical circuit shown in Figure 11.1.

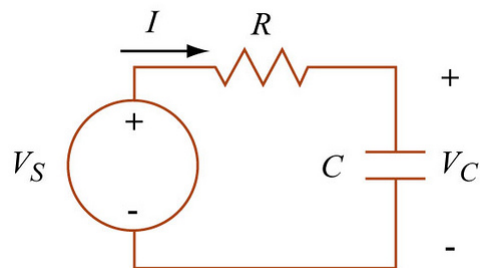


Figure 11.1: The original electrical system to be modeled.

The circuit in Figure 11.1 clearly corresponds to a component level model. This is what an electrical engineer would like to work with. Using elementary models for each electrical component, we obtain a corresponding equivalent system level model depicted in Figure 11.2. We see clearly an important drawback in using such a model: there is no one to one correspondence between original circuit elements and the blocks in the diagram. It is not difficult to see that for example replacing the resistor with a diode, would require starting over the construction of the system level model. This is an important point because even for this very simple example, the analysis to convert the intuitive physical model to a system level model is non-trivial and cumbersome.



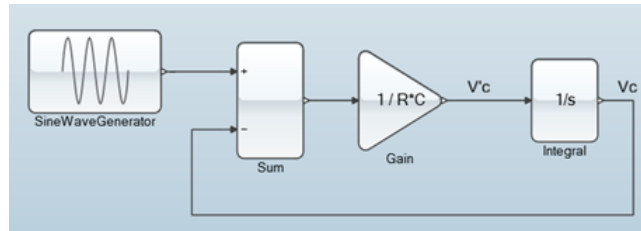


Figure 11.2: The system-level (causal) implementation of model depicted in figure 11.1

It turns out however that in many situations, in particular in systems and control applications, the use of causal modules is very useful. As an example in figure 11.3, a control system has been illustrated. This system is composed of several parts such as the plant, feedback, and the controller. In such control systems it is much easier for the control engineer to have a system level model to use for the design of the controller and to optimize the control parameters.

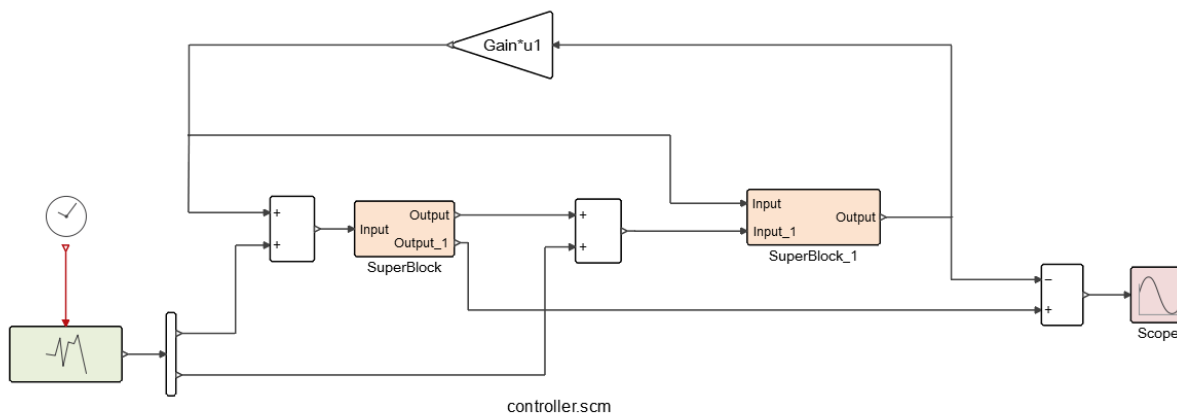


Figure 11.3: A typical system level modeling and design model

Even though causal models can in some sense be considered as special cases of acausal models, the restrictions imposed by causality implies that the behavior of the module can be implemented using an almost block box program which evaluates its outputs as a function of its inputs. The implementation of the behavior of the module in the acausal case is more complicated and requires either the construction of multiple causal modules or a formal description. In the latter case, the construction of the complete model requires formal manipulation of equations obtained from the description of acausal modules and their connection topology. It is for this reason that in **Activate** we allow the co-existence of both type of modules: causal and acausal.

### 11.3 Modelica: a standard in component level modeling

In order to extend the capacity of **Activate** to allow component level modeling, we needed a language for describing the algebraic-differential constraints imposed on the input/outputs of the acausal blocks. We found **Modelica** [1] to be an excellent choice.

Modelica is primarily a modeling language, sometimes called hardware description language, for specifying mathematical models of physical systems, in particular for the purpose of computer simulation.

Modelica is a modern object-oriented programming language based on equations instead of assignment statements (it can thus be classified as a declarative language)[3]. This makes it particularly useful for the purpose of acausal modeling, and this is not a surprise because Modelica is developed exactly for such applications. As an example the electrical circuit in figure 11.1 can be expressed in modelica as follows:

```
class RC_circuit
  Resistor    Res(R=1);
  Capacitor   Cap(C=1, v(start=0));
  Ground      Gnd;
  VsourceAC   VS(VA=1, f=50);
equation
  connect (VS.n, GND.p);
  connect (VS.p, Res.n);
  connect (Res.p, Cap.p);
  connect (cap.n, Gnd.p);
end RC_circuit;
```

where each component has been defined in the Modelica's library. For example here is the model of a simple Inductor.

```
class Capacitor "Ideal capacitor"
  Pin p, n;
  Real v;
  Real i;
  parameter Real C "Capacitance";
equation
  i = C*der(v);
  v = p.v - n.v;
  0 = p.i + n.i;
  i = p.i;
end Capacitor;
```

We are not going to present Modelica here in this document. More information can be found on [Modelica website](#) and in [1].

Modelica language is very rich and has the capacity also to handle discrete time systems. Our use of Modelica however, at least for the moment, is limited to the continuous-time part. We think that, except for very special cases, modeling discrete events can be done more naturally at the system level, where **Activate** provided already the necessary functionalities for modeling and simulation. We thus have found that the best compromise is to allow the usage of acausal blocks in the existing **Activate** environment while keeping the traditional causal blocks the way they were. Both types of blocks of course should be usable in the construction of diagrams.

Since the information available on the links connecting acausal models corresponds in general to physical quantities such as (voltage, current), (flow, pressure), etc, and are very different from information on links connecting causal models which transport signals, even a simple derivation placed on a link has a very different significance in two cases: a link that splits in two in the causal setting simply means that the information is duplicated whereas in the acausal setting, say in an electrical circuit, such a split must be implemented using the Kirchof's voltage and current laws. Clearly it would be meaningless to allow the direct connection of a port of an acausal model to that of a causal model. That is why,

special blocks were introduced in **Activate** to be used as interface between the causal and the acausal world. Such blocks which have very clear physical significance, may have input ports, output ports *and* "acausal ports". An example of such a block is a voltmeter. This block has two acausal ports; physically they correspond to the place where the two wires are to be connected. But it also has an output where the measured value of the voltage is sent out of the block. In general, sensors and actuators make the interface between the component level part of the model and system level part.

### 11.3.1 Example: modeling a DC motor

Consider a DC motor as shown in figure.11.4. The DC motor can be modeled using some basic electrical and mechanical components, such as resistor, inductor, and rotational inertia, as shown in Figure.11.5.

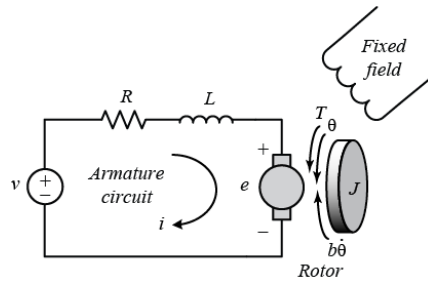


Figure 11.4: Representation of a DC motor.

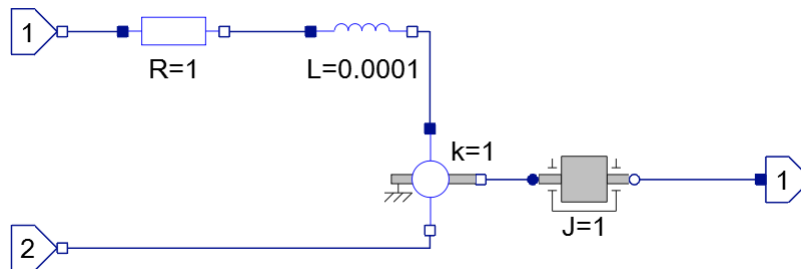


Figure 11.5: The model of a DC motor in **Activate**.



Figure 11.6: Representation of the DC motor component in **Activate**.

From electrical engineering courses you may remember that a DC motor can be either used as ordinary motor or as DC voltage generator. Since the ports of the components are not input or outputs, the components in Figure.11.6 can be used as motor (see Fig.11.7) or as generator (see Fig.11.8).

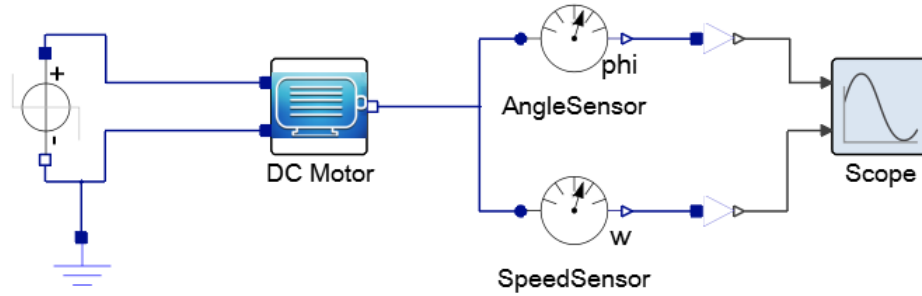


Figure 11.7: Representation of the DC motor component (motor.scm)

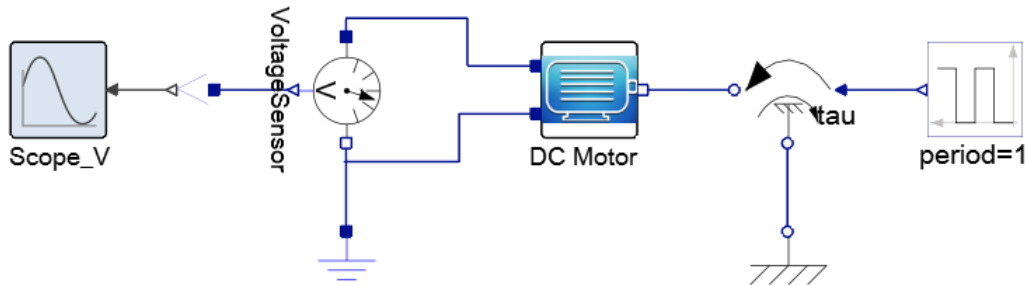


Figure 11.8: Representation of the DC motor component (motor.scm)

## 11.4 Implementation in Activate

The way acausal blocks are handled in **Activate** is as follows. First, all the acausal blocks (including the interfacing blocks) are grouped and replaced with a single causal block. This operation is of course totally transparent to the user. For example, the model in Fig.11.7 is seen by the **Activate** compiler as shown in Fig.11.9.

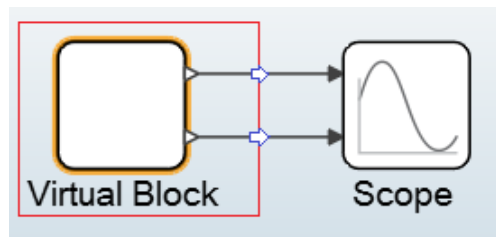


Figure 11.9: Aggregation of acausal blocks (physical components) into a causal block

Once all acausal blocks (physical components) are aggregated, only the explicit ports of the interfacing functions which are at the boundaries of the two worlds are left visible and thus the result is a causal block.

The behavior of this new causal block is defined by the Modelica program resulting from the Modelica programs of each of the acausal blocks and the way they are interconnected. This program is automatically generated, compiled and incrementally linked to the model.

This new block is almost a standard **Activate** block and can be treated like any other **Activate** block so that the way compilation and simulation were performed can be carried out as before. There is however

one small difficulty. So far in **Activate**, not only we had assumed that the blocks were causal but also that if a block had an internal state, the dynamics of this system were explicit. If we note the inputs, outputs and the state of the block respectively by  $u$ ,  $y$  and  $x$ , this condition amounts to imposing the following structure on the internal dynamics of the block

$$\begin{aligned}\dot{x} &= f(t, x, u) \\ y &= g(t, x, u)\end{aligned}$$

But the equations governing the new block can very well not be explicit. In many applications in particular in mechanics and electronics, we end up with differential-algebraic systems of equations which cannot always be converted into explicit form.

$$\begin{aligned}0 &= f(t, \dot{x}, x, u) \\ y &= g(t, x, u)\end{aligned}$$

Allowing internal implicit dynamics did not change the overall design of the **Activate** compiler but did require the implementation of DAE solvers; Consequently `Ida`, `Daskr`, and `Radau` solvers were added to **Activate** [4, 5, 6].

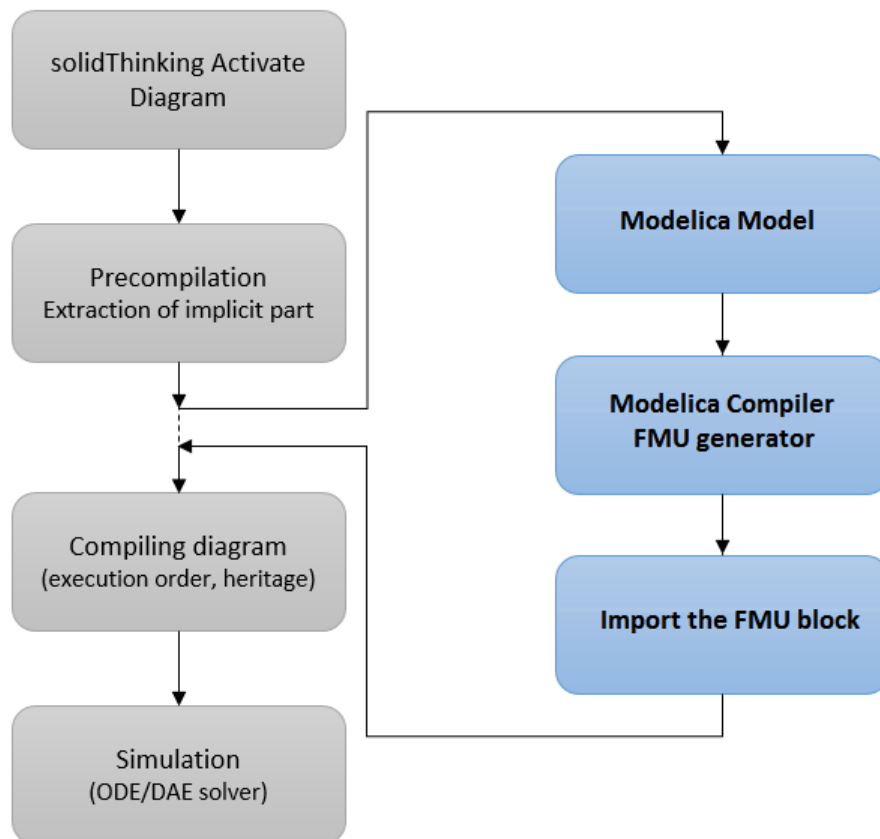


Figure 11.10: **Activate** Compiler flowchart

**Important Note** - The ability to use Modelica-based blocks in **Activate** and in particular the use of a

Modelica compiler to convert the selected Modelica-based **Activate** blocks into one Activate block, in particular an FMU block, as described below, is provided by an additional external library.

Once an **Activate** diagram edited, to simulate this system several operation are performed in an automatic and transparent way. First, the precompiler removes the super blocks to obtain a flat diagram. Then if model contains physical blocks representing components, precompiler extracts them and their connecting links to generate a Modelica model. The Modelica model is then saved in a temporary Modelica file to be treated by the Modelica compiler. A Modelica compiler is used to translate the Modelica file(s) into an FMU block. The FMU block is connected to the rest of original **Activate** diagram in place of removed parts. After these operations, **Activate** diagram is a standard one and is compiled and simulated as usual.

## 11.5 Modelica custom block

Beside the Modelica components available inside the Modelica toolbox, **Activate** provides a simple way to write Modelica code and test it. **Activate** provides **Modelica Custom block** available in `CustomBlocks` palette. This block allows developing Modelica models from scratch.

In order to illustrate the way this block can be utilized, an example will be given in this section. Consider the model of a simple differential equation as follows:

$$\begin{cases} \dot{x} = 2 - 15y + u \\ \dot{y} = x + 6 \end{cases} \quad (11.1)$$

where the initial value of  $x$  and  $y$  is 5.0 and 2.0 respectively. The Modelica code for this ODE follows.

```
model simpleODE
  Real x(start =5);
  Real y(start=2);
  Real u;
equation
  der(x)=2-15*y+u;
  der(y)=x+6;
end simpleODE;
```

This simple ODE can be easily modeled and simulated with the Modelica Custom block. As shown in the Fig. 11.11, in the GUI of the block, the user can provide the number of explicit inputs and outputs of the block, as well as their names used in the Modelica code.

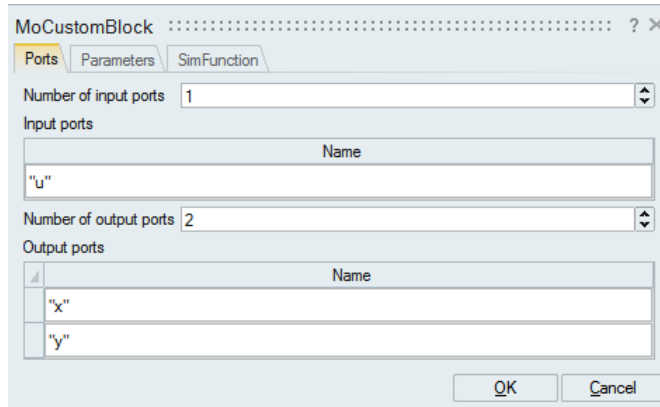


Figure 11.11: the GUI of the Modelica custom block for simulating 11.1.

Then in the Simfunction tab of the GUI, the function name and the Modelica code can be given, as shown in Fig.11.12 and Fig.11.13 .

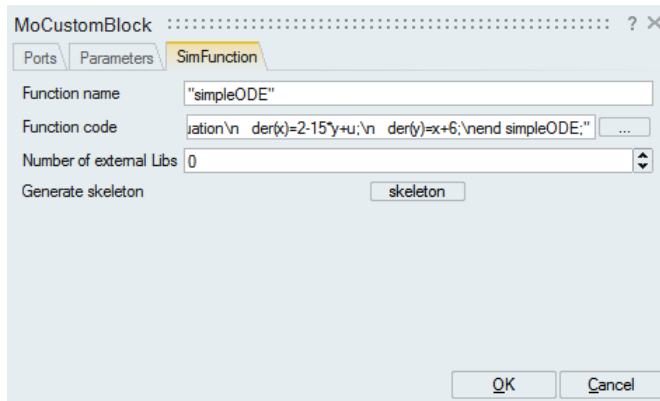


Figure 11.12: the simFunction tab of the GUI of the Modelica custom block for simulating 11.1.

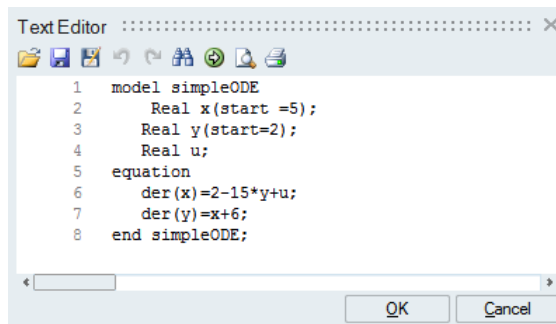
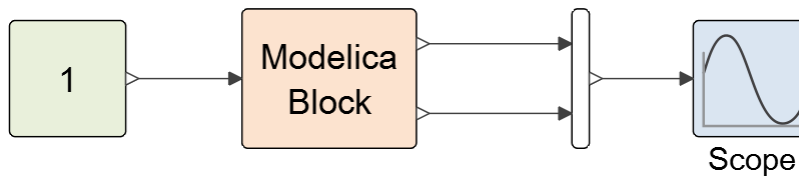


Figure 11.13: The Modelica code for simulating 11.1.

After building the model, as shown in Fig.11.14, it can be simulated. The simulation result is given in Fig.11.15.



simpleODE.scm

Figure 11.14: Complete model for simulation of 11.1.

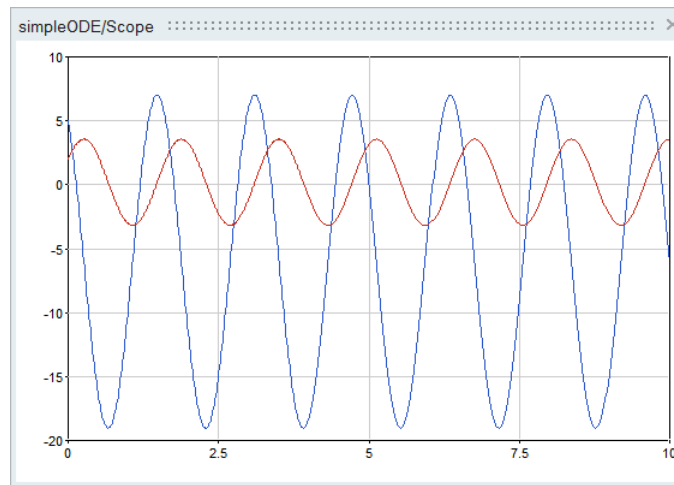


Figure 11.15: Simulation result for model 11.14.

Note that the input and output ports of this block is explicit and they can be connected to any other standard **Activate** block. In this example, the input  $u$  is just a constant block.



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# Chapter 12

## FMI (Functional Mock-up Interface)

### 12.1 Introduction

Automotive OEMs, who may have several hundreds of suppliers, have difficulties when they need to integrate many sub-systems from those suppliers.

Since they are specialized sub-systems, such as hydraulics, mechanical, electrical, ... the suppliers use different modeling and simulation programs for developing their sub-systems or components. In order to make sure that all of these subsystems work actually all together, the OEM should simulate them all together and in combination. For that, the system integrator should be able to work with all programs and also programs should be able to work with others. This is not of course only a problem in automotive industry but it is prominent in this industry.

A solution would be to share the source code of models and compile them in the host simulator, but very often these models have proprietary rights and can only be delivered in binary form or as libraries, which means they need special tools.

Co-simulation between simulators is a popular solution to make simulators work together in parallel. This solution works into some extent. In order to implement a co-simulation interface between simulators, a specialized interface should be developed. That means an interface per every two simulators. There are many tools and it is usually very hard to replace one by another or do all simulations in a single tool. There are some programs specialized in Co-Simulation that can interface several simulation tools with several versions, but it is not practically easy to interface all simulation tools.

Developing export and import facilities to exchange models between tools seems also to be a good solution, but that needs developing special export and import between tools. Also imported models do not usually work as well as in the original simulator due to difference in the simulator characteristics and lack of a standard format. There are several well-known interface formats, such as Modelica language, `s-function` from MathWorks, user routines in ADAMS, etc. But most of them are proprietary and cannot be used as standard for communicating between tools.

Even though Modelica is a tool independent language, it cannot be used as a standardized interface for model exchange because it needs a compiler to convert the language into the executable code.

In 2008, a large number of software and automotive companies as well as several European research centers worked in a cooperation project named MODELISAR to create the **FMI**<sup>1</sup> (Functional Mock-up

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<sup>1</sup>More information, Detailed interface specification and images used here can be found at [www.fmi-standard.org](http://www.fmi-standard.org).

Interface) standard. The development of the FMI specifications was initiated and coordinated by the *Daimler* AG company. The FMI standard defines a general interface to other simulators. A component (model) which implements this interface is called **FMU** (Functional Mock-up Unit). The goal of FMI is that the calling of an FMU in a simulation environment becomes standardized and remains reasonably simple. The FMI functions can be imported and called by any external simulation environment such as **Activate**. The host environment can create one or more instances of the FMU and simulate them. This is very similar to the way a model composed of several blocks can be created and simulated in **Activate**. An FMU may either need a simulation environment to import it and perform numerical integration (FMI for Model Exchange) or have its own solvers/simulator (FMI for Co-Simulation).

In 2010 the first version of FMI-1.0 for Model Exchange and for Co-Simulation was released[1, 2].

In 2014, FMI-2.0 standard including both Model Exchange and Co-Simulation, was released[3]. FMI-2.0 fixed few issues in FMI-1.0 and is not backward compatible with FMI-1.0. In this document we concentrate on the FMI-2.0 version. Currently, the development of FMI is conducted by the Modelica Association.

### 12.1.1 FMI interface types

FMI standard which was introduced to cope with tool interface problems, provides two major solutions for interfacing tools:

1. **Interface as Model Exchange (in this document often abbreviated as ME):** FMI standardizes the model export as binary or as source code. In this interface type the model can be a system of algebraic-differential equations (at most index-1 DAE) with discrete equations. With this approach a large range of transient dynamical systems can actually be modeled. In this approach, the numerical solver which is external to the model should be compatible with the model. This interface type will be discussed more in section 12.3.



Figure 12.1: FMI for Model exchange

2. **Interface as Co-Simulation (in this document often abbreviated as CS):** FMI standardizes the co-simulation between simulation tools. In this interface type, the FMU can contain the model, the numerical solver, and the simulator. It is also possible that FMU only provide the link between two tools without containing the solver or simulator. In this approach, the tool that imports the FMU, provides the input to the FMU and receives the output of the FMU. This approach is more general and any kind of simulator and any format such as physical models, 1D-3D and 0D for Controls can be interfaced. This interface type will be discussed more in section 12.4.

### 12.1.2 Advantages of using FMI in industry

FMI is a free and open standard published under a BSD license and currently supported by most important actors in the simulation software domain. Up to now more than 70 tools support this standard

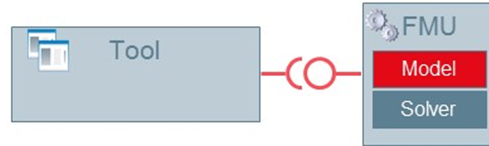


Figure 12.2: FMI for Co-simulation

according to the FMI-standard web-page. Both of FMI interface types allow preserving the intellectual property of the supplier over the exported model. FMI compliant tools often allow export of models freely licensed for distribution in the organization or to partners. As a result, exported FMUs most often do not require extra license from the export tool to be accessed or executed<sup>2</sup>. In industry, there are always only handful of specialists on modeling who develop detailed high fidelity models using modeling tools. They develop the model and deliver it to a large number of users who exploit the model for other applications such as

- Model parameter identification,
- Model sizing,
- Controller design,
- Model specific analysis such as frequency analysis,
- Pre and post-processing,

None of these applications needs the modeling tool which very often requires costly license and a good knowledge to use. In this situation, it is extremely useful to embed the model in an FMU and share it with other users. The FMU or model can be deployed from simulation specialists to designers, domain specialists, control engineers in other departments. That is the advantage of FMI, less costly modeling tool and more less expensive execution tools. As an example consider the model of the engine developed by the mechanical engineering department. This FMU can be deployed to Electrical department for ECU design, to Thermal department for thermal requirements verifications, or to control department for design appropriate controllers. In each of these departments, several engineers can work on the same FMU with different FMI-compliant tools and each engineer can run several variants of FMUs at the same time.

An FMU can be used by different groups in different domains, i.e., multi-domain collaboration. Engineers in different domains such as mechanical, Electrical, system, control, fluid, and thermal can work together with FMUs. They can share FMU models but staying with their tool of choice. Now, with advent of a new standard, every tool supporting FMI can virtually be connected without developing new interface.

It is, however, important to highlight that the FMI is basically developed for system level modeling where signals are scalar. As a result, FMI is not really suitable for interfacing finite-element or finite-volume problems where huge number of variables should be interfaced. If the number of connecting variables in finite-element or finite volume problems such as CFD is kept low, they can be interfaced by this standard. An example of such connection would be the placement of flow or temperature sensors in a fuel system modeled by a CFD tool.

<sup>2</sup>Except for FMI for Co-Simulation with external solver where the remote simulator is still required.

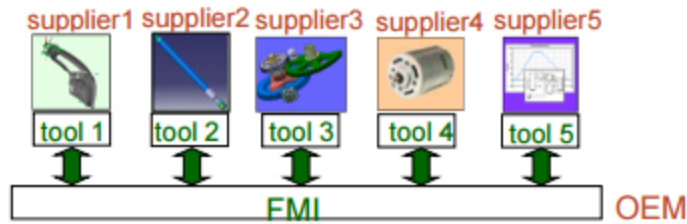


Figure 12.3: Using FMUs to share models

## 12.2 Internal structure of an FMU

An FMU implementing the FMI interface is a model distributed in one zip-file containing several files with a structure similar to the diagram shown in Fig.12.4:

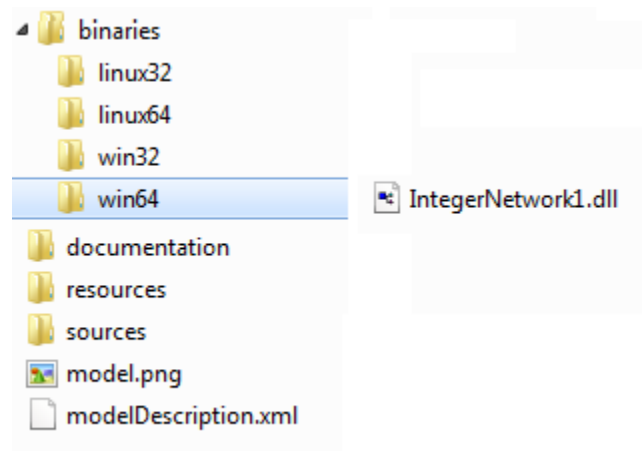


Figure 12.4: Internal structure of an FMU.

An FMU is composed of several files that will be discussed as follows:

- **An XML file (modelDescription.xml):** This XML file contains information on the model and the definition of all variables of the FMU in a standardized way. XML file indicates the FMI version, whether the FMU is for Model Exchange and/or for co-simulation, and the features of FMI standard supported by the FMU. The XML file also lists the exposed model variables including inputs, outputs, states, state derivatives, and model parameters. Model parameters can be changed at the beginning of the simulation or even during the simulation for tunable parameters. An important information included in the XML is the model structural information, i.e., the variables who are output, derivatives, or unknowns at initial time. The dependency of output, derivative and initial unknowns on other variables are also provided. This information is crucial for detecting algebraic loops and correctly handle them in a model aggregated by several FMUs. The XML can also contain vendor information. It is important to indicate that information stored in the XML is not needed for execution of the model. That allows having an executable code with a minimum overhead. Below is an example of an FMI model description issued from Modelica. For more details on the XML file, interested readers are referred to the section `FMI description Schema` in

[3].

```
<?xml version="1.0" encoding="UTF8"?>
<fmiModelDescription
  fmiVersion="1.0"
  modelName="ModelicaExample"
  modelIdentifier="ModelicaExample_Friction"
  ...
  <UnitDefinitions>
    <BaseUnit unit="rad">
      <DisplayUnitDefinition displayUnit="deg" gain="23.26"/>
    </BaseUnit>
  </UnitDefinitions>
  <TypeDefinitions>
    <Type name="Modelica.SIunits.AngularVelocity">
      <RealType quantity="AngularVelocity" unit="rad/s"/>
    </Type>
  </TypeDefinitions>
  <ModelVariables>
    <ScalarVariable
      name="inertial.J"
      valueReference="16777217"
      description="Moment of inertia"
      variability="parameter">
      <Real declaredType="Modelica.SIunits.Torque" start="1"/>
    </ScalarVariable>
    ...
  </ModelVariables>
</fmiModelDescription>
```

- **C source file or compiled C dynamic linked libraries:** These files define the model equations converted into causal form. If the source code is provided, the source code is put in the [fmu://sources](#) folder. If the model is provided in binary form, it is placed inside the folder [fmu://binaries/win64](#). Binaries for several platforms, such as `win32`, `linux32`, and `linux64`, can be provided by an FMU.
- **Documentation (optional):** Additional optional documentation can be included in the FMU inside the [fmu://documentation](#) folder.
- **resources (optional):** Additional optional data can be included in the FMU inside the [fmu://resources](#) folder. This folder can be used for storing tables, maps or any additional data.

## 12.3 FMI for Model Exchange (ME)

The FMI for Model Exchange (ME) interface defines an interface to the model of a hybrid dynamic system described by differential, algebraic and discrete-time equations. In this kind of the interface, ordinary differential equations with events and zero-crossings are handled. Algebraic equation systems might be contained inside the FMU. Also, the FMU might consist of discrete-time equations activated by events, for example a sampled-data controller. Since the overhead is small, this kind of FMU can be used for very large models as well as for small embedded systems. An FMU for Model Exchange operates in three main modes, *Initialization mode*, *Continuous-time mode*, and *event-mode*. The state-machine of the FMU/ME has been illustrated in Fig. 12.5.

In *initialization mode* the initial values of continuous-time states, the previous (internal) discrete-time states, are computed at the start time. In this mode some extra equations that may not be present in other modes may be used. As an example, suppose that an FMU should start at equilibrium point. The equations defining the start value of state at equilibrium are defined only in this mode.

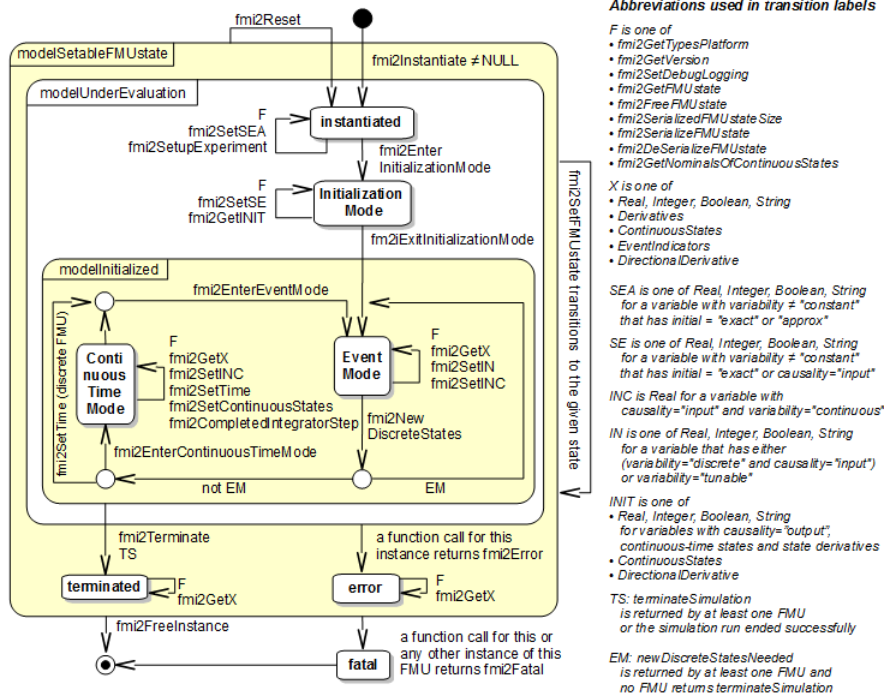


Figure 12.5: State Machine for FMI-2.0-Model Exchange.

The *continuous-time mode* is used to compute the values of all continuous-time variables between events by numerically solving ordinary differential and algebraic equations. All discrete-time variables are fixed during this phase and the corresponding discrete-time equations are not evaluated.

The *Event Mode* is used to compute new values for all continuous-time variables, as well as for all discrete-time variables that are activated at the current event instant, given the values of the variables from the previous instant. This is performed by solving algebraic equations consisting of all continuous-time and all active discrete-time equations.

The operating mode jumps from continuous-time into event-mode, if an event happens. There are four event kinds in FMI:

- **State-event (zero-crossing event):** Event which is triggered at the time instant where the event indicator function changes from  $z > 0$  to  $z \leq 0$ , or vice versa. Discontinuities in a model are usually defined as zero-crossing functions (event-indicators).
- **Time events:** Event that is triggered at a predefined time instant. Since the time instant is known in advance, the integrator can select its step size so that the event instant is directly reached. Therefore, this event can be handled efficiently.
- **Step-event:** Event that might occur when the simulator finishes a completed integrator step. This event is not usually defined by the FMU. The simulator may use it, for example, to dynamically switch between different states, or storing data into a file.
- **External event:** This kind of event is triggered by the environment. This event type is triggered if at the current time instant, at least one discrete-time input changes its value, a continuous-time input has a discontinuous change, or a tunable parameter changes its value.

Upon happening any of these events, the continuous-time integration is stopped and the FMU goes into



the event-mode.

## 12.4 FMI for Co-Simulation (CS)

Modeling complex heterogeneous systems in engineering usually leads to hybrid systems of differential and algebraic system of equations with discrete-time equations. Such complex multi-disciplinary systems cannot often be modeled and simulated in one simulation tool alone. Sub-system models are often available only for a specific simulation tool. For example special tools for CFD (Computational Fluid Dynamics) models or integrated circuit systems. In many situations, the sub-systems shall be simulated with the simulator which suits best for the specific domain. Thus for the simulation of multi-disciplinary models it is often reasonable or even necessary to couple different simulation tools with each other or with real world system components. Co-simulation is a general approach for the joint simulation of models developed with different tools where each tool treats one part of a modular coupled problem. The data exchange (input and output variables, status information) between sub-systems is restricted to discrete communication points. In the time between two communication points, the sub-systems are solved independently from each other by their individual solver.

Master-slave is a common method in Co-simulation. In a master-slave approach, the slaves simulate sub-models whereas the master is responsible for both coordinating the overall simulation as well as transferring data. In the master-slave approach, instead of coupling the simulation tools directly, it is assumed that all communication (input/output data exchange) is handled via a master. Master algorithms control the data exchange between sub-systems and the synchronization of all slave simulation solvers. Besides distribution of communication data, the master checks the connection graph, chooses a suitable simulation algorithm and controls the simulation according to that algorithm. The slaves are the simulation tools, which are prepared to simulate their subtask. The slaves are able to communicate data, execute control commands and return status information.

FMI for Co-Simulation unifies the master-slave interface and supports from simple co-simulation algorithms to more sophisticated ones. FMI provides a small set of C-functions to implement the interface. It is important to note that the master algorithm itself is not part of the standard FMI for CoSimulation. The master is free to choose the appropriate co-simulation algorithm.

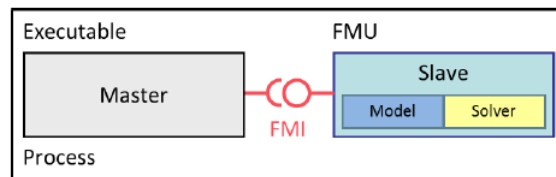


Figure 12.6: Co-simulation with generated code on a single computer.

FMI for Co-Simulation supports two possible ways to provide slave sub-systems for co-simulation: sub-systems (models) packaged along with their respective solver, which can be simulated as stand-alone components, see Fig.12.6, or a wrapper FMU which calls its respective simulation tool to simulate the sub-system (model), which is installed externally to the FMU, see Fig.12.7. The FMI import in **Activate** supports both implementations of FMI for Cosimulation. In FMI export in **Activate**, the solver is embedded in the FMU and no **Activate** installation is required to run the FMU in other environments. In the first case, the model and its solver are exported together as runnable code, whereas in the second, the FMU plays as a wrapper to communicate with the exporter simulator.

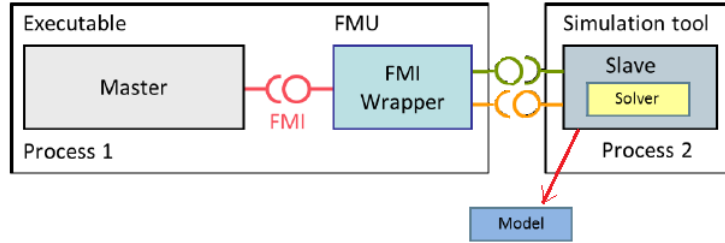


Figure 12.7: Co-simulation with tool coupling on a single computer.

In FMI for co-simulation, all information relevant for the communication in the co-simulation environment is provided in the XML-file. The XML file includes a set of capability flags to characterize the ability of the slave to support advanced master algorithms, e.g. the usage of variable communication step sizes, higher order signal extrapolation, or others. In **Activate**, the simulator is considered as the master and each FMU block as a slave.

In FMI for co-simulation, an slave operates in two main modes, *initialization mode*, and *step mode*. The *initialization mode* is used to compute the initial values for internal variables of the co-simulation slave at the start time.

This mode supports a few special cases:

1. some extra equation may be used that are not active during the rest of the simulation.
2. algebraic equations can be solved.
3. If the slave is connected in loops with other FMUs, iterations over the FMU equations are possible.

The *step mode* is used to compute the values of all variables at communication points. In this mode, the master provides the input of slaves and requests their outputs.

### 12.4.1 Co-simulation algorithms

Consider a model composed of two parts (submodels) defined in two tools as shown in Fig.12.8.

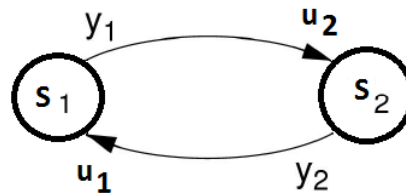


Figure 12.8: Co-simulation with tool coupling on a single computer.

The outputs of submodels are computed as following equations.

$$\begin{cases} y_1 = F_1(y_2) \\ y_2 = F_2(y_1) \end{cases}$$

In order to co-simulate this system, this equation should be solved at each communication instant.  $F_i$  functions are usually complex, non-linear, and time-dependent relationships, as a result, iterative methods are the only choice for solving such equations. Three common methods used for solving

them are Gauss-Seidel and Gauss-Jacobi and Newton methods. The Gauss-Seidel iteration method is represented as follows.

$$\begin{aligned} y_1^{i+1} &= F_1(y_2^i) \\ y_2^{i+1} &= F_2(y_1^{i+1}) \\ i &= 0, 1, 2, \dots \end{aligned}$$

The Gauss-Jacobi iteration method is defined as

$$\begin{aligned} y_1^{i+1} &= F_1(y_2^i) \\ y_2^{i+1} &= F_2(y_1^i) \\ i &= 0, 1, 2, \dots \end{aligned}$$

These iterative methods are simple fixed-point methods and their convergence (at most linear) depends on  $F_i$  functions. If the Fixed Point converges very slowly or diverges the Newton-type methods can be used. These methods are, however, more complicated, because in each step the Jacobian must be computed which results in higher number of simulator calls. The method which is used actually in **Activate** is GS1, i.e., the Gauss-Seidel method with only one iteration. This method has been illustrated in Fig.12.9;

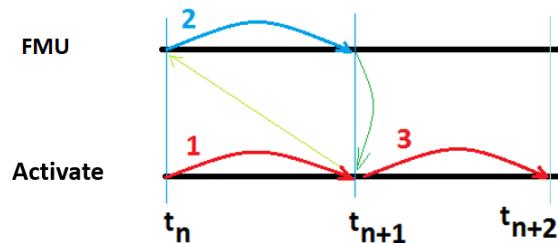


Figure 12.9: Co-simulation method used in **Activate**.

The simulator of **Activate** which acts as the master performs the following method for the co-simulation and synchronization between tools, as shown in 12.9.

1. The simulator advances the time from  $t_n$  to  $t_{n+1}$ , and updates the output of all blocks, except those of external tools (slaves). Optionally, the slave (FMU) outputs are extrapolated at a desired order if available from the FMU.
2. External tools (slaves) are called one by one and asked to advance their time from  $t_n$  to  $t_{n+1}$  and their output is requested at  $t_{n+1}$ . Optionally, the slave (FMU) inputs are linearly interpolated.
3. The **Activate** simulator takes a new step from  $t_{n+1}$  to  $t_{n+2}$  using the new output value of slaves computed at  $t_{n+1}$ .
4. goto 1

Usually in order to get a correct result, the communication step-size (time instant between two communication instants) should not be larger than two times the highest frequency content (one-half the smallest time constant; Nyquist theorem) of the slaves, and in practice should usually be smaller than this. Smaller communication step-size results in better convergence but longer simulation time. In **Activate** the user can control the communication step-size, see 12.5.5.

## 12.4.2 Advanced: FMI import preserving full output/input dependency property

<sup>3</sup> A challenge in importing an FMI generated is the treatment of output/input dependencies. In an Activate basic block output/dependencies are expressed as a vector of dependencies specifying the inputs that affect any of the outputs. So the dependency is solely a property of an input port. A basic block computes all its outputs in the same call, so all its dependent inputs must be up to date when the call is made. An FMU, on the other hand, specifies output/input dependencies as matrix specifying which output depends on each input. This input/output dependency matrix may also be different during the initialization. It provides routines that allow the computation of output ports separately and take advantage of variable caching.

A way to deal with this situation, which was the way Activate imported Modelica parts, is to simply project the matrix of dependencies into a vector. This conservative approach properly assigns dependencies in Activate but "loses" information along the line. This may lead in particular to detection of algebraic loops by the Activate compiler that are not true algebraic loops (artificial algebraic loops). Even though there are ways to break algebraic loops in an Activate model, we decided to detect artificial algebraic loops and treat them properly. A very simple example that illustrates the problem is shown in Figure 12.10.

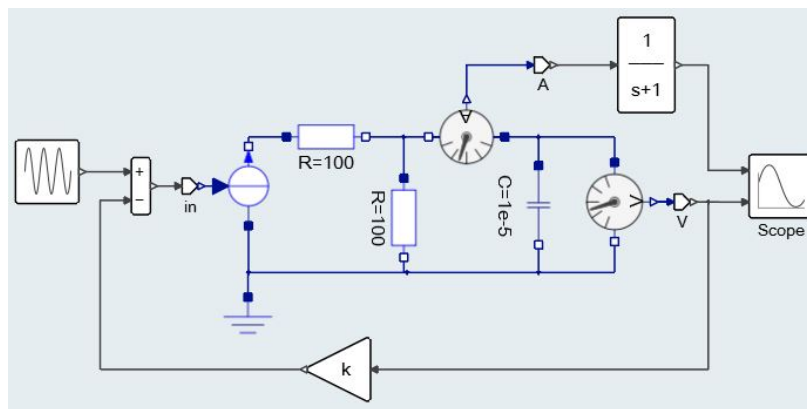


Figure 12.10: A simple model mixing Modelica and Activate blocks

After compiling the Modelica part, a virtual model similar to what is shown in Figure 12.11 is obtained. In the generated FMU, there is a direct dependency between the `SignalCurrent` input port (`in`) and the `CurrentSensor` output signal (`A`). The dependency is depicted by a red dashed line in Figure 12.11. If the dependency matrix is converted into a vector, output ports `A` and `V` depend on input port `in`. This direct dependency results in an artificial algebraic loop that does not really exist.

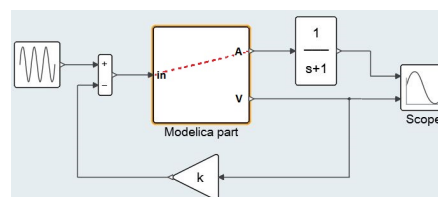


Figure 12.11: The model in Figure 12.10, after converting the Modelica part into an FMU block.

In order to solve this problem, the output/input dependency property of the basic blocks can only be set

<sup>3</sup>This section can be skipped without losing the continuity.

for ports separately so the imported FMU cannot be represented by a single FMU; but it can be properly represented as a network of basic blocks. The topology of this network depends on the number of inputs and outputs of the FMU and the FMUs output/input dependency matrix. A programmable super block is thus used for importing the FMU. The block main parameter is the FMU filename. By reading and parsing the XML inside the FMU, the block generates a network of basic blocks, as shown in Figure 12.12, during the model compilation stage. The network contains a central basic block, always present, and other blocks associated with each output port. Figure 12.12 illustrates the resulting network for an imported FMU with two inputs and four output ports.

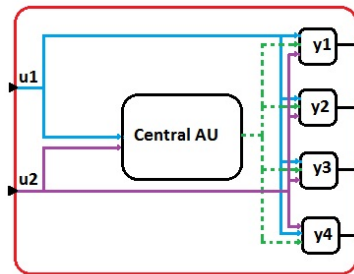


Figure 12.12: Automatically generated network of basic blocks from FMU import for an FMU with one input and four output ports.

In resulting network of basic blocks, each output unit keeps the direct-feedthrough property of the corresponding FMU output port. The central unit on the other hand does not have the direct feed-through. In order to communicate within this network, the central block outputs the internal structure which contains the FMU context and shares it with other blocks of the network. Note that this process is completely transparent for the user, who never sees the network in Figure 12.12. Hence generating a programmable network of blocks instead of single block for FMU import, avoids creation of artificial algebraic loops.

## 12.5 FMI Import in Activate

**Activate** can simulate FMI-1.0 and FMI-2.0 block of both Model-exchange and co-simulation kinds. The **FMU Import** block (see Fig. 12.13) available in the **CoSimulation** palette is used to import and simulate FMU blocks. This block automatically detects the FMI version and FMI kind and chooses appropriate methods to simulate them. By double clicking on the block, a block dialog (GUI) is opened as shown in Fig. 12.14



Figure 12.13: **FMU Import** block from **CoSimulation** palette

This dialog allows for selecting and loading an FMU. Once selected, the user can either fill the FMU information fields such as number of input and outputs, or click on the **Reload** button. The button temporarily reads the FMU and fills the information fields of the dialog automatically, as shown in Fig. 12.15.

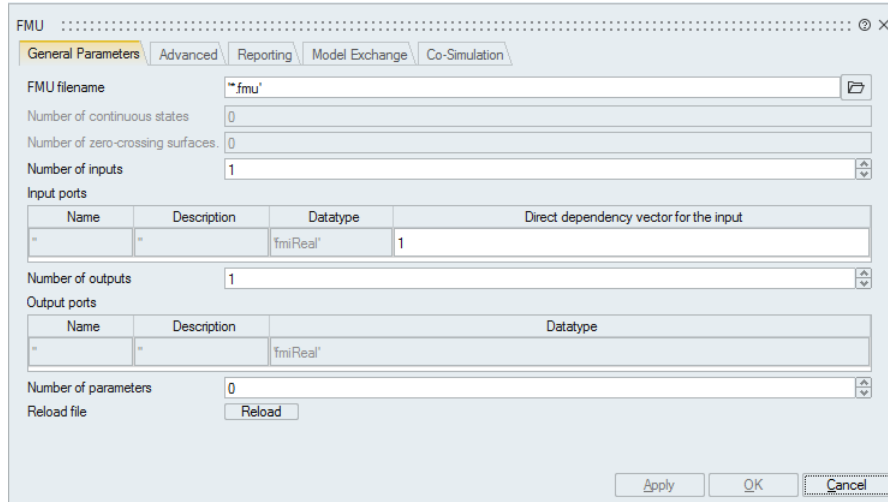


Figure 12.14: Block Dialog for **FMU Import** (empty)

In the Parameter field, the FMU variables whose start value can be changed are displayed. The user can provide new values for these variables. The new values are taken into account before the start of the simulation.

There are, however, few features of FMI that are not supported by the FMI Import block in **Activate**.

- Tunable parameters are not supported.
- The `FmuSetState` and `FmuGetState` features of FMU are not fully exploited for repeating the steps and solving algebraic loops.

### 12.5.1 Direct dependency vector for inputs (Feedthrough)

An FMU block usually provides the dependency of output variables on its variables including its input variables. This information is transformed into feedthrough information of the FMU block in **Activate** (see 9.1.1). This information is displayed in the FMU GUI, as shown in Fig.12.15.. For each input port, a dependency vector is provided which indicates the output ports that depend on the this input port. In this vector zero(0) means no dependency and one(1) means direct dependency of output on input.

### 12.5.2 Advanced tab

**Run as "Model Exchange"**: In FMI-2.0, the FMU can provide both ME and CS implementations for a model. In this case, the user can choose the implementation that should be simulated. The "Run as Model Exchange" check-box, if checked, forces the simulator using the Model Exchange implementation.

**Tolerance controlled**: This check-box allows choosing if the FMU should be tolerance-controlled or not. If the checkbox is On, then the FMU model is called with a numerical integration scheme where the step size is controlled by using the relative error tolerance for error estimation. In such a case, all numerical algorithms used inside the model operate with an error estimation of an appropriate smaller relative tolerance. If the fixed-step solver is used, this checkbox may be Off.

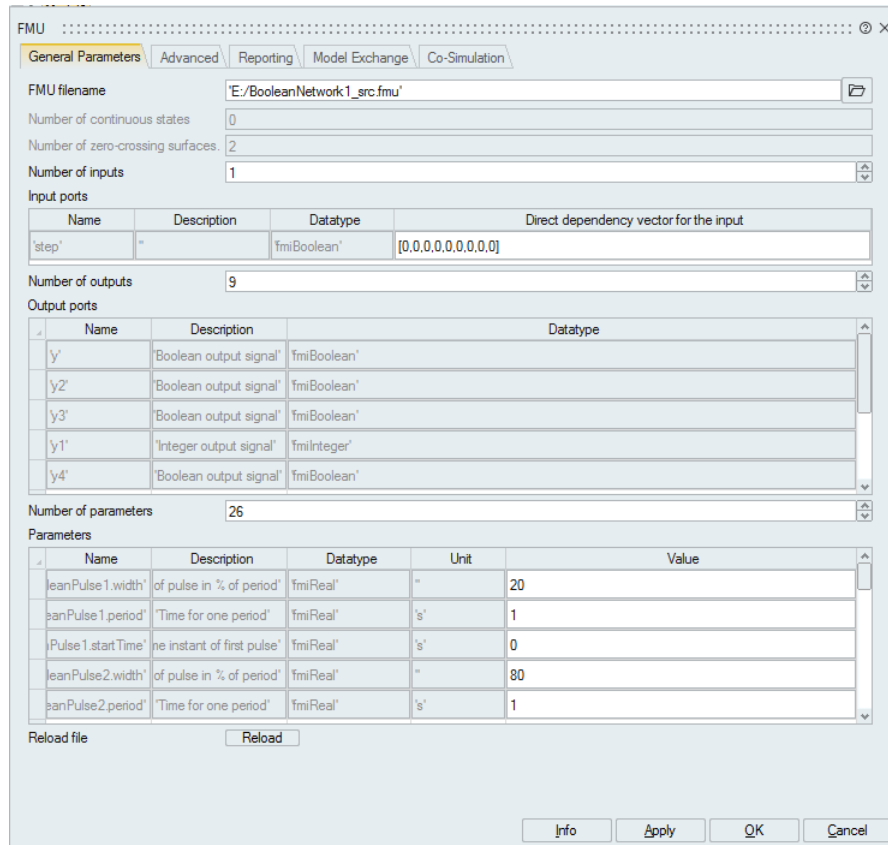


Figure 12.15: Graphical user interface of the block **FMU-Import** (filled)

Delete the unpacked FMU before exit: The simulator unzips the FMU in a folder on each run and deleted the folder once the simulation finished. This check-box allows keeping the folder after finishing the simulation.

Show FMU input event ports: This check-box allows displaying/hiding the input event ports of the block. The FMU import block has got two input event ports, one for external events and another for End-event. The external event which can be activated by discrete events of **Activate** pushes the FMU (of type ME) into the event mode. This is important if, for example, the input of the block is discontinuous. The End-event pushes the block into the terminate mode, where the output of the block remains unchanged until the end of the simulation.

Show FMU output event ports: This check-box allows displaying/hiding the output event ports of the block. The FMU import block has got two output event ports, one for discrete event that happen inside the FMU and another for End-event. The discrete event is triggered for example when a time event is programmed inside the FMU block. The output End-event is triggered when the FMU requests a termination. This event can be used to terminate the simulation.

FMU resource location: This field is used to provide the location of the resource folder to be given to the FMU. If this field is left empty, the FMU is the default location which is [fmu://resources](#).

Directory for unpacking the FMU: This field allows unzipping the FMU in a specified folder. If this field is left empty, the FMU is unzipped in the temporary folder of **Activate**.

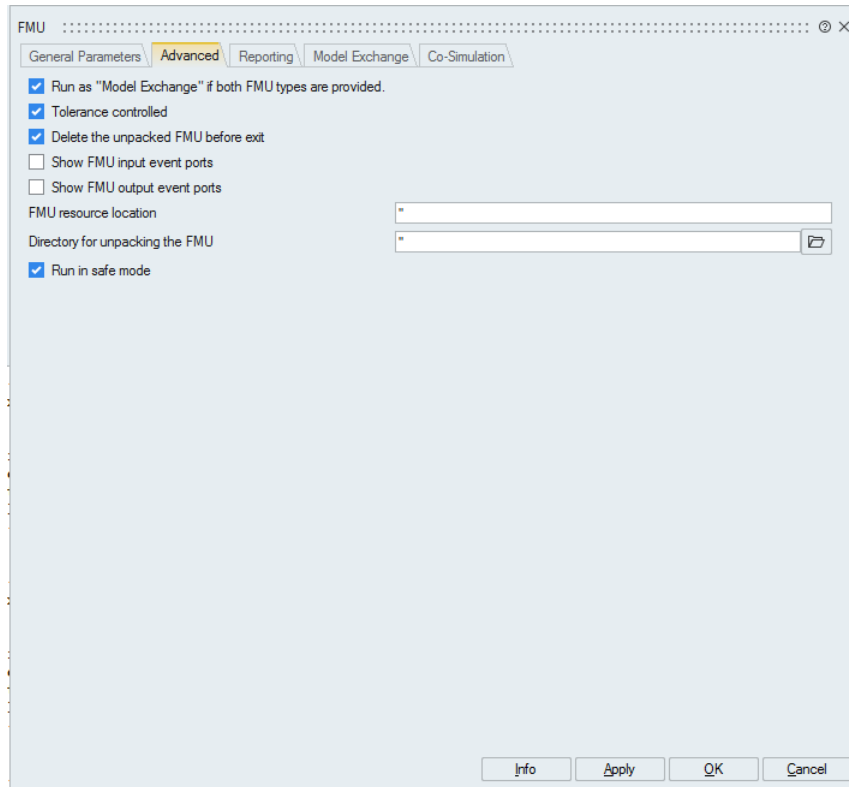


Figure 12.16: Graphical user interface of the block (Advanced tab)

`Run in safe mode`: If enabled, the FMU is loaded inside another process. This protects **Activate** from possible crashes in the FMU. The simulation is slower in this mode. Both FMI-1.0 and FMI-2.0 of ModelExchange and CoSimulation kinds can be simulated in this mode.

### 12.5.3 Reporting tab

`FMU interface logging level`: The list-box allows choosing the logging level for the interface to the FMU block. For example, if the user chooses the *Warning* level, all logger messages emitted by FMU with status *fmi2Warning*, *fmi2Error* and *fmi2Fatal* will be stored in the logfile. Possible values *Nothing*, *Fatal*, *Error*, *Warning*, *Info*, *Verbose*.

`Enable FMU logging`: This check-box enables or disables the whole logger.

`Add Activate messages`: If this check-box enabled, the messages emitted by Activate interface to the FMU will be added to the logfile. If you need to catch only the messages emitted by the FMU, you may turn this checkbox off.

`FMU logging categories (for FMI-2.0)`: In FMI 2.0, the FMU is given the possibility to filter logger messages. The user can enter the required log categories. Log categories should be protected by double quote and separated by comma, for example "fmiCall;events;logger2". Only messages with given categories will be stored in the logfile. If left empty, all messages are stored, regardless of their category.

`Logfile name (*.log)`: The log filename to be created. If this field is left empty, no logfile is created. `stderr` as a filename is supported.



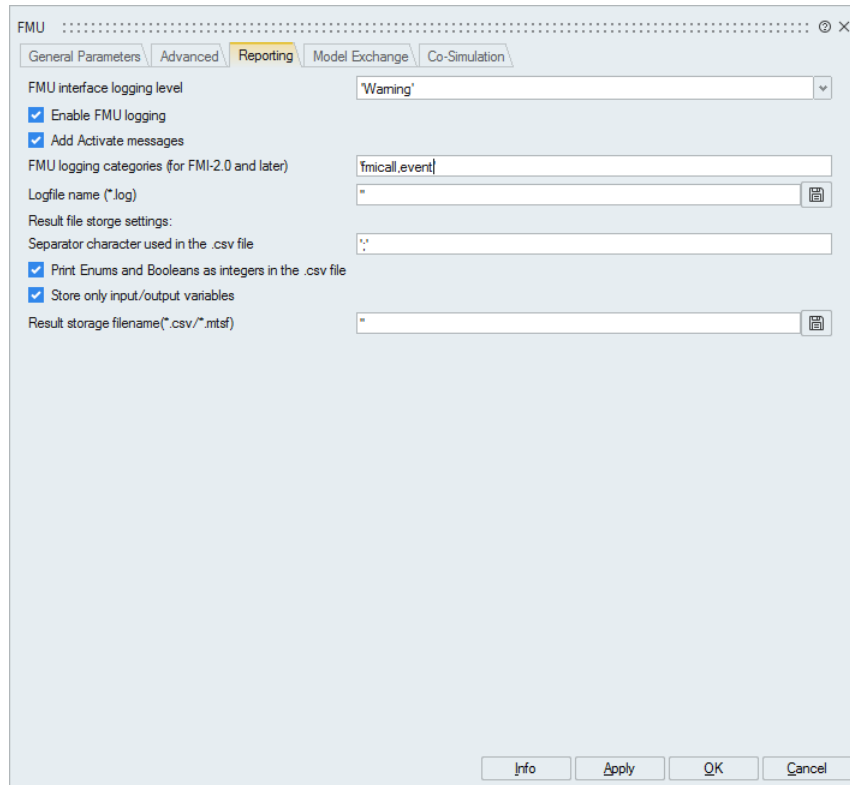


Figure 12.17: Graphical user interface of the block (Reporting tab)

`Separator character used in CSV file`: The separator character in the output CSV file.

`Print Enums and Booleans as integers in CSV file`: If this check-box is activated, the Enum values are stored as integer, otherwise their text values are stored.

`Store only input/output variables`: If this check-box is activated, the CSV or HDF file will store only input and output variables. This allows reducing the result file volume.

`Result storage filename (*.csv/*.mtsf)`: The full path of the output result file. The output file can either be a CSV file or an HDF file (with extension \*.mtsf)[\[4\]](#).

### 12.5.4 ModelExchange tab

`Solver mesh-points events (step events)`: If checked, whenever a step event happens, the value of variables at that time instant is stored in the CSV file.

`Zero-crossing events (state events)`: If activated, whenever a state event (zero-crossing event) happens, the value of variables at that time instant is stored.

`Time events`: If activated, whenever a time event happens, the value of variables at that time instant is stored.

`Every output update`: If activated, whenever the block outputs are updated, the value of variables at that time instant is stored.

`Meshpoints points`: If activated, the value of variables at that solver mesh-points are stored.

Super dense time instants: If activated, the value of variables at super dense time in discrete event iterations are stored.

## 12.5.5 CoSimulation tab

Preferred fixed communication step size: An FMU of kind co-simulation is simulated by its internal solver. The FMU and Activate exchange (update) their respective output and input values at intervals called communication points. The interval between communication points is called communication step-size which may vary during the co-simulation. The communication step size can be set by the "Preferred fixed communication stepsize" parameter in the FMU block under the Co-Simulation tab as shown in Fig.12.18.

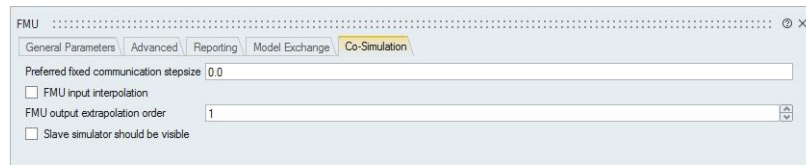


Figure 12.18: Setting co-simulation parameters in the FMI import block

The default value is set to zero (0), which means that the communication instants are governed by the time-step of the Activate solver. Therefore, in order to control the communication interval you can adjust the "maximum step size" parameter for the Activate solver. Note that, if the Activate solver is a fixed-step size solver, the "maximum step size" parameter will be the co-simulation communication step size. In **Activate**, "maximum step size" parameter is found under the Solvers tab of the Simulation Parameters dialog.

Alternatively, you can change the "Preferred fixed communication step size" to a non-zero value and this will set the rate of communication between the FMU and Activate, regardless of the step-size chosen by the Activate solver. Note that, in this case the communication step size is fixed. If two sub-systems (FMU and Activate part) tightly interact, fixed communication points methods does not work and the co-simulation may diverge. In this case, Newton methods should be employed, although this is not yet supported in Activate.

**FMU Input interpolation:** In order to enable the slave to interpolate the continuous real inputs between communication steps, **Activate** can provide the first derivatives of the inputs with respect to time using the numerical differentiation method. If this checkbox is activated, the first derivative of inputs is computed and is given to the FMU. This may be useful if the FMU can exploit the derivative of its input. Note that since the derivative is computed numerically, if the signal is not smooth enough, the computed derivative is poor. In this case the checkbox is better to be off.

**FMU Output interpolation:** FMU can provide higher order derivatives of their real outputs to allow extrapolation of the output variables between communication steps. In order to exploit the derivative and read smoother signal from the FMU, the user can choose the extrapolation order to be used. Based on the chosen extrapolation order, the derivative of outputs of FMU/CS block is used for extrapolation of outputs. Note that no extrapolation order bigger what is provided by FMU can be used, otherwise a warning message will be raised. The FMU provides the highest order available and you can set the extrapolation to this order or any lower order.

**Slave simulator should be visible:** If activated, the FMU/CS is co-simulated in interactive manner.

## 12.5.6 Example

In order to demonstrate the way an FMU can be imported, export a model as FMU with another simulator, e.g., MapleSim from MapleSoft. You can also choose an FMU from the FMI public SVN server<sup>4</sup>. Make sure that the FMU you choose is compatible with the **Activate** binaries. For example if the **Activate** is compiled for a win64 machine, your FMU should contain win64 DLLs.

Consider for example the bridge rectifier modeled in MapleSim as shown in 12.19. With input signal  $u = 10 \sin(20\pi t)$ , the simulation result is given in Fig.12.20.

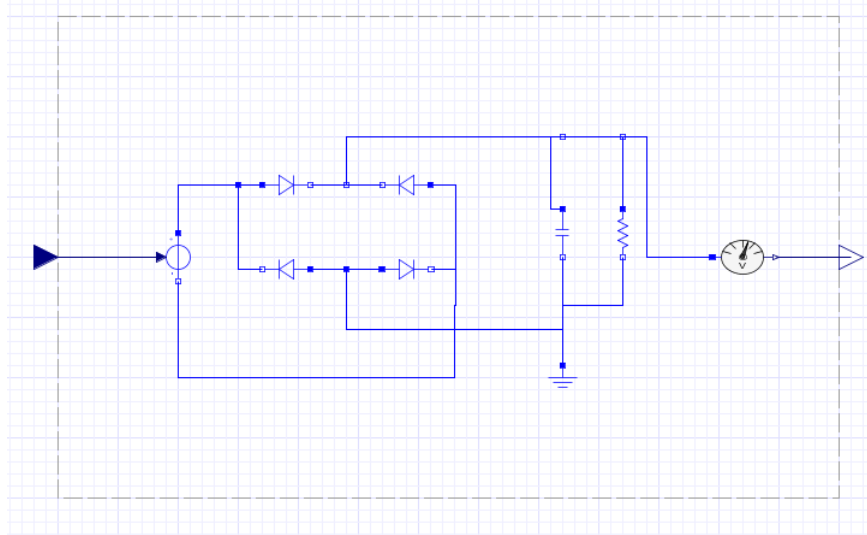


Figure 12.19: A bridge rectifier modeled in MapleSim.

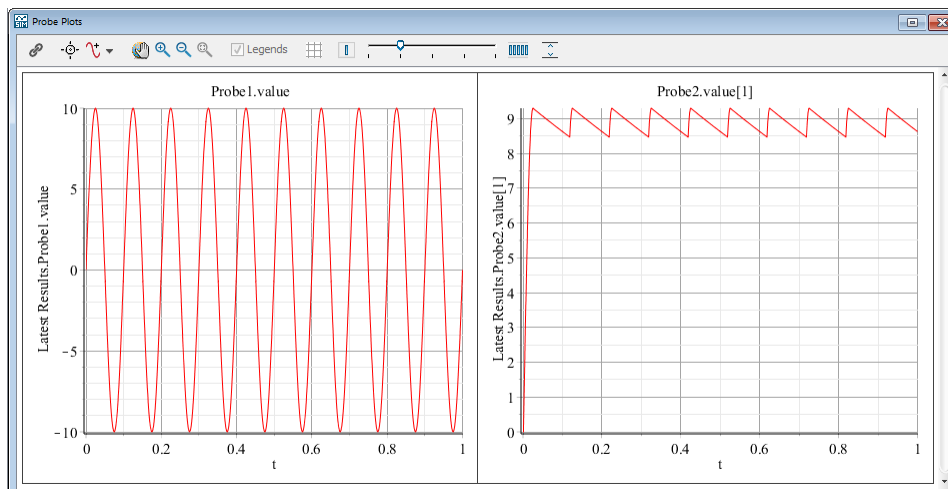


Figure 12.20: Simulation result of model in Fig.12.19 in MapleSim.

In MapleSim, export this model as FMI-2.0 for ModelExchange or Co-Simulation and load it in **Activate** as shown in Fig.12.21 and Fig.12.22.

<sup>4</sup><https://svn.fmi-standard.org/fmi/branches/public>

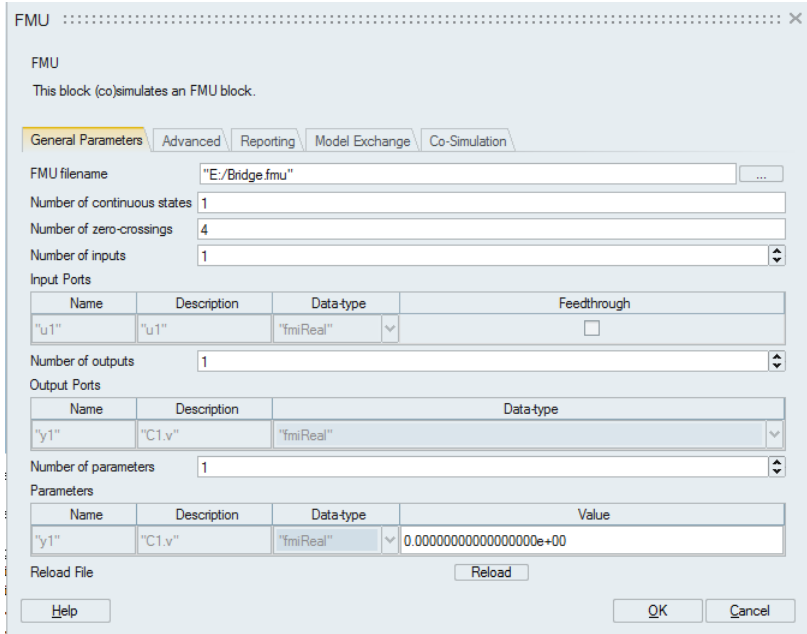


Figure 12.21: FMI Block dialog in **Activate**

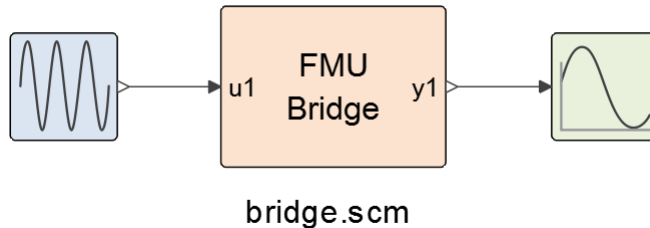


Figure 12.22: Bridge model (bridge.scm)

Choose appropriate simulation parameters in **Activate**, i.e., Final time=1.0, Maximum stepsize=0.0001. and run the model. You should get the same result as in MapleSim, as shown in Fig.12.23.

## 12.6 FMI export in **Activate**

**Activate** can export super blocks as FMI-2.0. The FMU exported in **Activate** is standalone, i.e., in the form depicted in Fig.12.6. In the FMU, the numerical solver used in the model setting is embedded in the FMU. So no **Activate** installation is required to run the FMU in other environments.

In order to export a model or a part of a model, the interested region should first be placed into a super block. Once converted into a super block, in the Tools/FMU 2.0 drop down menu, select the export method: either Model Exchange, Co-Simulation, or both can be selected as an option to export an FMU. The name and identifier of the exported FMU will be the name of the super block. Hence, if the name of the super block contains unsupported characters, they are converted to "\_". For example if the name of the super blocks is "SuperB !%lock", "sb\_SuperB\_\_lock" will be used instead. After the

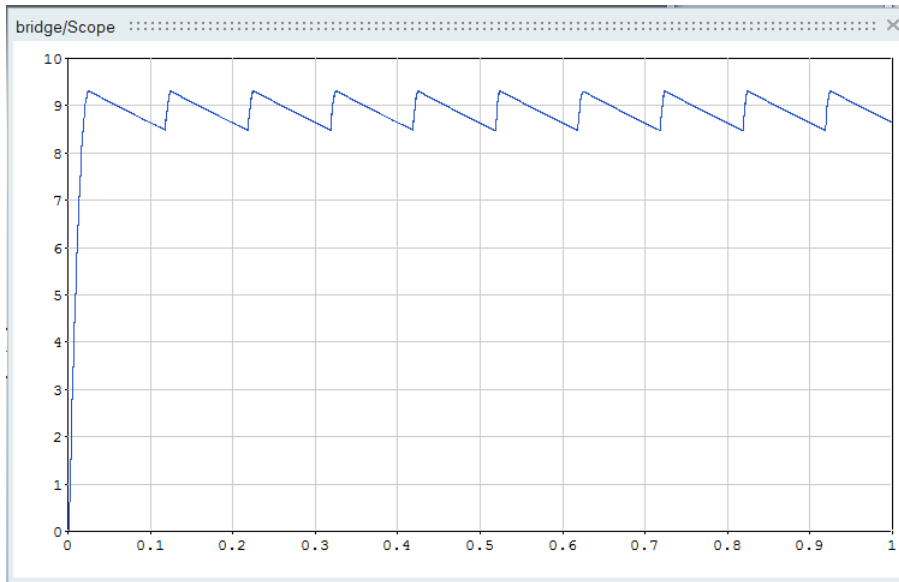


Figure 12.23: Result in **Activate**.

FMU generation, the user is asked to select a folder to save the generated FMU. The default location is next to the model. If the super block is exported as FMI for co-simulation, the **Activate** simulator as well as the simulation parameter values in the "setup" menu will be used in the exported model. If a model is implicit, i.e., contains implicit blocks, such as ImplicitDerivative, Constraint, or Automaton, it can only be exported as FMU for CoSimulation. In this case, an implicit solver such as, Ida, Daskr, or Radau(DAE) is used inside the FMU.

### 12.6.1 Nested FMU

A superblock containing other FMUs (FMI-1.0 or FMI-2.0 of kind ModelExchange or cosimulation) can be exported as a new FMU.

### 12.6.2 Exporting Modelica

A superblock containing Modelica components can be exported as FMU. Note that the superblock cannot have Modelica ports (implicit Activate ports). All input and output ports should be of explicit signal type to be exported as FMU.

### 12.6.3 Shortcomings

Current version of the FMI export of **Activate** does not support following features:

- Signal import blocks FromBase and SignalIn are not supported.
- Signal export blocks ToBase and SignalOut are not supported.
- Signal Viewer blocks such as Scope, Display, etc. are ignored.
- Co-simulation blocks such as MotionSolve or Flux blocks are not supported.

- Memory block is not supported.

## 12.6.4 Requirements

The FMI export facility needs a MicroSoft Visual Studio C++ compiler installed on the system. Otherwise the generated **C** code cannot be compiled and no binary will be available for the FMU and FMI export fails. Note that **Activate** automatically detects the **C** compiler on the system. In order to detect the **C** compiler, you may use the command: `vssGetCompilerName()`

Note that the `tcc` compiler cannot be used in FMI export.

## 12.6.5 Example

In order to demonstrate the way a model in **Activate** can be exported, here is an example. Open the model `Controller_Me_CS.scm` available in the product installation directory, under `tutorial_models/Extended_Book` folder. Suppose that we want to export the indicated region in the model, as shown in Fig.12.24.

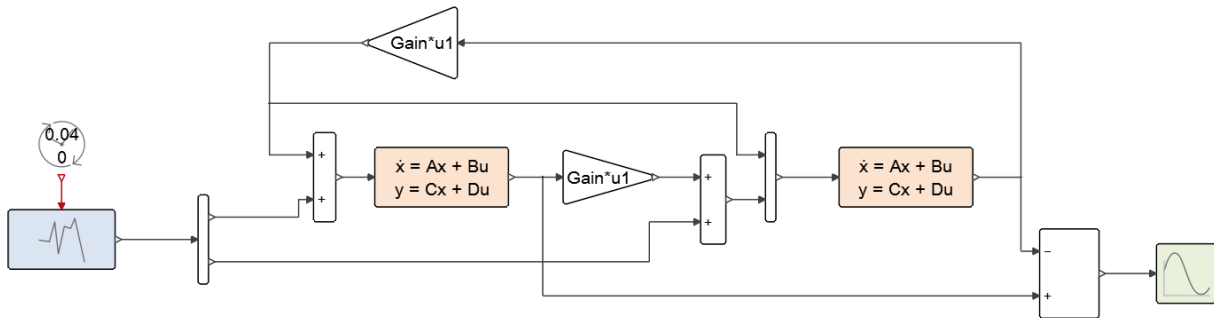


Figure 12.24: Controller model (AFM\_controller.scm)

- Select the region and convert it into a super block as shown in Fig.12.25. Rename the super block as needed to the name of the FMU to be generated.

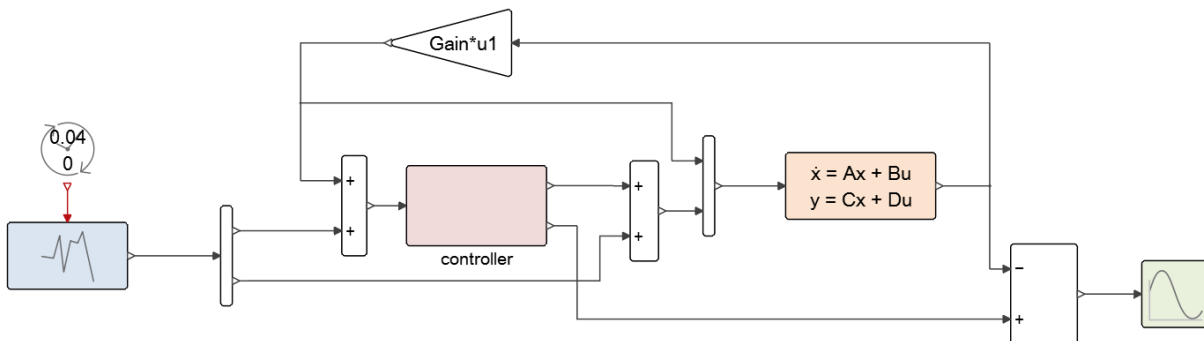


Figure 12.25: Controller model after superblock creation (SFM\_controller.scm)

- If the superblock contains Input ports, inside the super block, click on the Input blocks and activate the "Time-Dependency" checkbox and fill the input dimensions with correct values (as shown in

Fig.12.26). Otherwise an FMU with only discrete-time variables will be generated and the size of input ports will be one.

- Get outside of the super block and after selecting the super block, select the menu Tools/FMU 2.0 and choose the export type: Model Export, Co-Simulation, or both.
- The C code and the binaries are generated in the **Activate** model temporary folder.
- Once the FMU is generated, a dialog window opens and asks the user to choose a folder to copy the FMU into.
- The generated FMU can be imported as an FMU with the "FMI import" block and be tested to verify if it gives the same result.

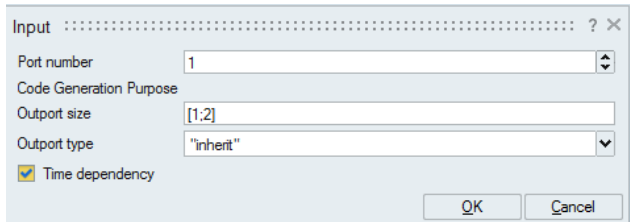


Figure 12.26: Input-port block dialog.

## 12.7 Advanced user topics:

The co-simulation algorithm is not a part of the FMI standard and tools are free to choose the best method to perform the co-simulation with an FMU. The FMI import block is basically an Activate block that interfaces an FMU (ModelExchange or CoSimulation) with Activate. Similar to any other Activate block, the FMI import block is called with several flags during the simulation. The model is updated through the propagation of block outputs throughout the whole model. In the output propagation, the input/output dependency of the blocks plays an important role. For example, if at least one output of a block depends directly on the  $i^{th}$  input of the block, the block is not called (i.e., its outputs are not updated) unless  $i^{th}$  input is updated. This is guaranteed by the Activate compiler to ensure that at any time instant and for a given continuous-time state, the outputs are correctly evaluated. Once all input and outputs are evaluated, other jobs, like reading derivative of continuous-time states, evaluating the zero-crossing surfaces, programming a discrete event, or in the case of co-simulation with an FMU calling `fmiXdoStep` can be done. For more details about the simulation flags and calling blocks the reader is referred to the chapter 9.

For the sake of simplicity, consider an Activate model containing only one FMU, as shown in Figure 12.27. The FMU has only one input and one output port. The whole Activate part is abstracted into a single block. In co-simulation, the standard part of the Activate model and the FMU exchange their input and output signals at some time instants called communication points (CP). Between two subsequent CPs Activate simulator engine and FMU simulate separately with their respective solver. At the communication step  $t_i$ , the Activate simulator engine takes the lead and performs the simulation until the next communication step, i.e., simulate the interval  $[t_i, t_{i+1}]$ . During the integration between  $t_i$  and  $t_{i+1}$ , the outputs of the FMU can be extrapolated. Mathematically speaking, the Activate simulator engine performs the simulation of this model for the interval  $t_i \leq t \leq t_{i+1}$ ,

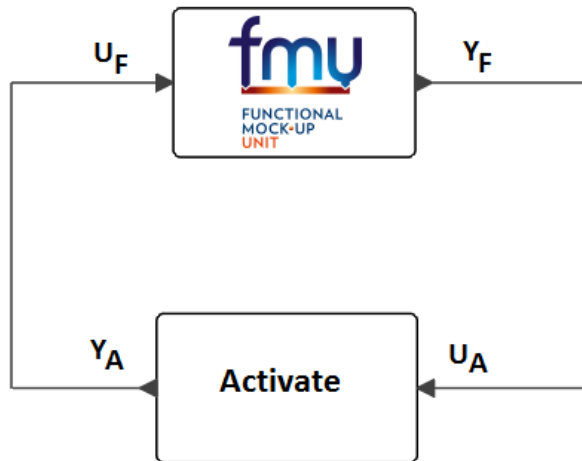


Figure 12.27: Simple co-simulation model

$$\begin{cases} \dot{x}_A &= f(x_A, u_A(t)) \\ u_A(t) &= y_F(t_i) + (t - t_i)y'_F(t_i) + \frac{(t-t_i)^2}{2}y''_F(t_i) + \dots \end{cases} \quad (12.1)$$

where  $x_A$  is Activate continuous-time states and  $u_A$  is the input of the Activate part.  $y_F$ ,  $y'_F$  and  $y''_F$  are output of the FMU block, and their first and second time derivatives, respectively. If extrapolation is disabled (i.e., zero order extrapolation method is selected) then  $u_A(t) = y_F(t_i)$ .

During the integration of the Activate part, the FMU may be invoked several times to retrieve  $y_F(t_i)$ ,  $y'_F(t_i)$ ,  $y''_F(t_i)$ , etc.

Once Activate engine simulated the interval  $[t_i, t_{i+1}]$ , and  $y_A(t_{i+1})$  is evaluated, it is the FMU turn to advance and simulate the same interval. Right before invoking `fmiXDoStep`, the input values of the FMU at  $t_i$  (i.e.,  $u_F(t_i)$  and possibly its first order derivative ( i.e.,  $der(U_F(t_i))$ )) are given to FMU (using `fmi2SetXXX` and `fmi2SetRealInputDerivatives` commands). Also, the value of  $u_F(t_{i+1})$  is read and saved for subsequent `fmiXDoStep` calls.

The derivative of input is computed as follows:

$$der(U_F(t_i)) = \frac{u_F(t_{i+1}) - u_F(t_i)}{t_{i+1} - t_i}$$

Once FMU inputs are set, the FMU is called to perform `fmiXDoStep(cp=t_i, h=t_{i+1} - t_i)`.

### Controlling the co-simulation communication step

In the co-simulation with an FMU, the size of communication steps is governed by the numerical solver of Activate (see the model settings panel). If a variable step-solver is chosen, the communication step will be variable and with a fixed-step solver, the communication steps will be fixed. In some cases, this behavior is not desired. Consider, for example, a complex Activate model where a variable step-solver



is needed but calling `fmiXDoStep` in an FMU is costly. In this case, the user has the possibility of choosing a fixed-communication step size in the FMU parameter settings (section 12.5.5) .

Unlike the variable communication step size where every Activate numerical solver step is followed by a `fmiXDoStep` in the FMU, if a fixed-Communication step is chosen by the user, `fmiXDoStep` is called periodically, regardless of Activate Solver steps. Upon dependence of the output of the FMI block on the rest of the model, Activate solver meshpoint may or may not coincide with the communication points of FMI. For example, if the output of the FMI block does not go to any block with continuous-time state, the solver mesh-point and communication points do not coincide.

### Calling `fmiXDoStep`

As described in section 12.4.1, Activate and the FMU for co-simulation alternate between calling their respective solvers to solve and advance time between communication points and coupling shared data (inputs/outputs) at these communication points. At any particular time ( $t_i$ ), Activate will take a time step from  $t_i$  to  $t_{i+1}$  first and then call to the FMU to advance the time with `fmiXDoStep`. Activate calls `fmiXDoStep` for either the flag `VssFlag_OutputUpdate` or `VssFlag_EventScheduling` depending on the model and FMU parameter settings.

Blocks in Activate are called in a predefined order with flag `VssFlag_OutputUpdate`, in order to read the output ports. The simulator propagates the updated output port values all over the model. Once all input and output ports of blocks are updated, the blocks can be called (maybe in an arbitrary order) to do the update job with flag `VssFlag_EventScheduling` and `VssFlag_StateUpdate` (see the section 9.1.1 for more details).

In one case for calling `DoStep` in FMU for co-simulation blocks, after the input/output update phase at  $t_{i+1}$ , the input value of the FMU block at  $t_i$  (and possibly their derivatives) are given to the FMU. Then `fmiXDoStep` is called in flag `VssFlag_EventScheduling` to advance the FMU time from  $t_i$  to  $t_{i+1}$ <sup>5</sup>.

It is important to note that if `fmiXDoStep` is called in flag `VssFlag_EventScheduling`, the output of the FMU are not immediately read because this is the purpose of the flag `VssFlag_OutputUpdate` which was called prior to `VssFlag_EventScheduling`. The outputs of the FMU will be updated in the next model input/output update with flag `VssFlag_OutputUpdate` .

This usually introduces a communication step delay in the output signal versus the input signal. In some situations, it is possible to perform `fmiXDoStep` during the input/output update phase, i.e., with flag `VssFlag_OutputUpdate` . In this case, the newly updated output can be propagated through the model and the delay be avoided. The situations where `fmiXDoStep` will be called in flag `VssFlag_OutputUpdate` are:

- If the FMU block does not have any input port, no need to wait all input/output update propagation to call `fmiXDoStep`. At the block invocation to retrieve  $u_A(t_{i+1})$  (which is equal to  $y_F(t_i)$ ), `fmiXDoStep` will be called and immediately  $y_F(t_{i+1})$  can be obtained and propagated into the model. In this way, the delay is avoided.
- If the variability of all inputs is *discrete* or none of input data-types is *Real*,  $u_F(t)$  will be equal to  $u_F(t_i)$  all over the interval  $[t_i, t_{i+1}]$ . As a result, `fmiXDoStep` will be called at input/output update with flag `VssFlag_OutputUpdate` .

---

<sup>5</sup>The flags and the job that the block should do in each flag has been explained in chapter 9

- If the input interpolation is off or the FMU does not support interpolation,  $u_F(t)$  will be equal to  $u_F(t_i)$  all over the interval  $[t_i, t_{i+1})$ . As a result, `fmiXDoStep` will be called at input/output update with flag `VssFlag_OutputUpdate`.
- Once all input values of FMU at  $t_{i+1}$  are read during the input/output update phase with flag `VssFlag_OutputUpdate`. As an example, consider the case where an output of the FMU depends directly on all input ports of the FMU. Once the value of the output port is requested by the simulator, it means that all inputs are already updated by the simulator at  $t_{i+1}$ . As a result, `fmiXDoStep` can be immediately called and the FMU output  $y_F(t_{i+1})$  can be propagated for the FMU outputs whose value are not already propagated.

### Input interpolation and output extrapolation in co-simulation

**Activate** is a hybrid simulation tool which means it can simulate a model with continuous-time and discrete-time parts. The discrete-time part is activated by discrete-time events and the continuous-time part of the model is integrated by the numerical solvers of Activate. Discrete-time events cause discontinuities in signals. As a result, signals in a hybrid model may be piece-wise continuous. Since several discrete-time events may happen at the same time, a signal may have several values at the same time instants, as shown in Fig. 12.28.

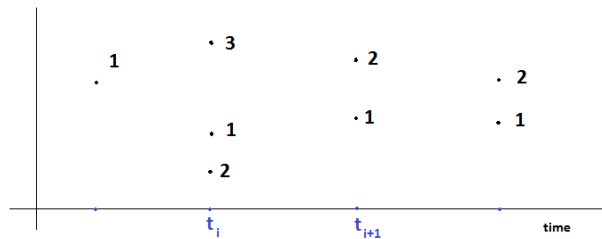


Figure 12.28: evaluation of a model at different time instants

The multiple time instants at an event time is also called super dense time instants. During the integration of the continuous-time part, the numerical solver takes steps to advance the time. At the beginning of each step, the time dependent part of the model are evaluated with the flags `VssFlag_OutputUpdate`, `VssFlag_EventScheduling` and `VssFlag_StateUpdate`. The time instant where the model is evaluated at the beginning of the solver step is also called a solver 'mesh-points'. Discrete-time events are classified in two categories, critical and non-critical events. Critical events are discrete-time events that may cause discontinuities in the continuous-time part of the model. If a critical event happens, the numerical solver needs to be cold-restarted<sup>6</sup>. Following one or several critical events, there is always a solver mesh-point to update the continuous-time parts. In other words, after the evaluation of a model at critical events, the continuous-time part should be evaluated before starting the integration of the continuous-time part. Unlike critical events, after a non-critical events, there is not necessarily a mesh-point. Non-critical events may happen between two mesh-points.

If the "Preferred fixed communication stepsize" in the co-simulation tab of the FMU block, is left zero, then the Activate solver mesh-points are taken as the co-simulation communication points, i.e., in Fig. 12.29  $t_i$  and  $t_{i+1}$  are Activate mesh-points and also communication points. Since the model may

<sup>6</sup>For more details about events, and solver, interested readers are referred to the chapter on solvers.

have critical events, the model may get evaluated several times at communication instants. Before calling `fmixDoStep` at the mesh-point  $t_{i+1}$ , the input of the FMU at the mesh-point (i.e.,  $u_F(t_i)$ ) is given to the FMU. If the derivative of the input is also provided to the FMU, Activate computes the first order derivative of the input by using the following equation.

$$\text{der}(u_F(t_i)) = \frac{u_F(t_{i+1}) - u_F(t_i)}{t_{i+1} - t_i}$$

Note that in order to compute the derivative of the input, the input value at the mesh-point evaluation, as well as the first super dense time at the next communication points is used, see Fig.12.29.

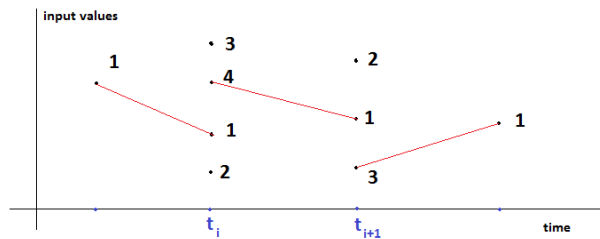


Figure 12.29: First order interpolation of FMU inputs in presence of events at communication steps.

If the "Preferred fixed communication stepsize" in the co-simulation tab of the FMU block is non-zero, then the communication points are periodic. i.e., in Fig.12.30,  $t_i$  and  $t_{i+1}$  are communication points. Between communication points, there may be several Activate mesh-points, see Fig.12.30. The FMU is activated by a periodic events where at each event time, `fmixDoStep` is called. In this case, before calling `fmixDoStep` at the event time of  $t_{i+1}$ , the input of the FMU at the previous communication point (i.e.,  $u_F(t_i)$ ) is given to the FMU.

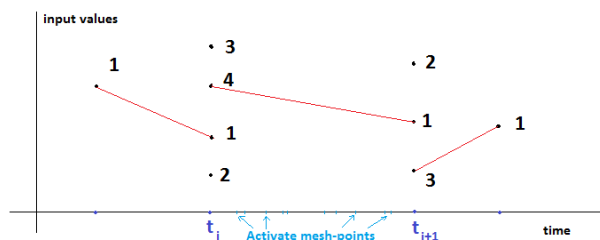


Figure 12.30: First order interpolation of FMU inputs in presence of events at communication steps (fixed communication step).

Based on FMU attributes, Activate chooses to call `fmixDoStep` at  $t_{i+1}$  either during the call with flag `VssFlag_OutputUpdate`, or during the call with flag `VssFlag_EventScheduling`. When the Activate solver integrates the interval  $[t_i, t_{i+1}]$ , the output of the FMU block is computed with the following formula (if a linear output extrapolation is used).

$$y_F(t) = y_F(t_i) + (t - t_i)y'_F(t_i)$$

At  $t_{i+1}$ , the output of FMU is  $y_F(t_{i+1}^1) = y_F(t_i) + (t_{i+1} - t_i)y'_F(t_i)$  until `fmixDoStep` is called at the mesh-point and the output of FMU is retrieved. The output is retried and displayed at the next communication point.

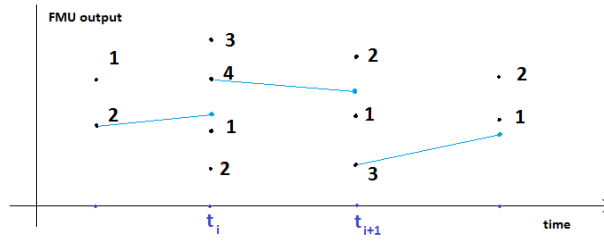


Figure 12.31: First order extrapolation of FMU output in presence of events at communication steps.

# Bibliography

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- [2] Functional Mock-up Interface for Co-Simulation. Version-1.0, October 2010.
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- [4] Proposal for a Standard Time Series File Format in HDF5, Andreas Pfeiffer, Ingrid Bausch-Gall, Martin Otter, Linköping Electronic Conference Proceedings, 2012.
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## Chapter 13

# Co-Simulation with Multi-body Simulation

### 13.1 Introduction

Modeling complex heterogeneous systems in engineering usually leads to hybrid systems of differential and algebraic system of equations with discrete-time equations. Such complex multi-disciplinary systems cannot often be modeled and simulated in one simulation tool alone. Subsystem models are often available only for a specific simulation tool. For example special tools for CFD (Computational Fluid Dynamics) models or integrated circuit systems. In many situations, the sub-systems shall be simulated with the simulator which suits best for the specific domain. Thus for the simulation of multi-disciplinary models it is often reasonable or even necessary to couple different simulation tools with each other or with real world system components. Co-simulation is a general approach for the joint simulation of models developed with different tools where each tool treats one part of a modular coupled problem. The data exchange (input and output variables, status information) between subsystems is restricted to communication points. In the time between two communication points, the subsystems are solved independently from each other by their individual solvers. If intermediate input or output are needed by one of simulators, it is very often computed through interpolation or extrapolation.

Master-slave is a common method in Co-simulation. In a master-slave approach the slave simulates the sub-model whereas the master is responsible for both coordinating the overall simulation as well as transferring data. The slaves are the simulation tools, which are prepared to simulate their subtask. The slaves are able to communicate data, execute control commands and return status information. Several MSplant blocks representing different **Altair MotionSolve** models can be instantiated in an **Altair Activate** model.

### 13.2 Co-Simulation with MBS

The Co-simulation interface lets the user simulate a complex system that includes one or more multi-body systems (defined by **MotionSolve** blocks) and several subsystems modeled with **Activate** blocks. In order to effectively simulate the entire system, the MBS subsystems are simulated with **MotionSolve** models <sup>1</sup> while the control subsystem is simulated with **Activate**.

---

<sup>1</sup>The installation of Altair **MotionSolve**™ software is required for co-simulation. **MotionSolve** is a multi-body modeling, analysis, visualization and optimization solution for performing multi-disciplinary simulations that include kinematics and dynamics, statics and quasi-statics, linear and vibration studies, stress and durability, loads extraction, co-simulation, effort

In the **Activate-MotionSolve** interface, **Activate** is the master and **MotionSolve** (MS) is slave. The co-simulation interface has been implemented in a block called **MS Plant** which is available in the *Advanced/CoSimulation* palette, as shown in Fig.13.1. **MS Signals** is a variant of **MS Plant** block that displays the name of MS signals at the input/output ports of the blocks.

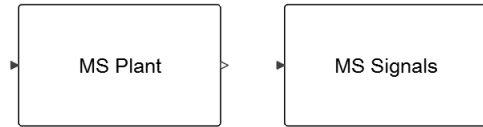


Figure 13.1: Blocks in **Activate** to simulate a **MotionSolve** model.

The block provides a small set of C++ functions to implement the **Activate-MotionSolve** interface. The path of the **MotionSolve** model is given to MS Plant and **MotionSolve** solver is called to simulate the model. In order to perform the simulation, it is necessary to have **MotionSolve** installed on the machine. This process is shown in fig.13.2.

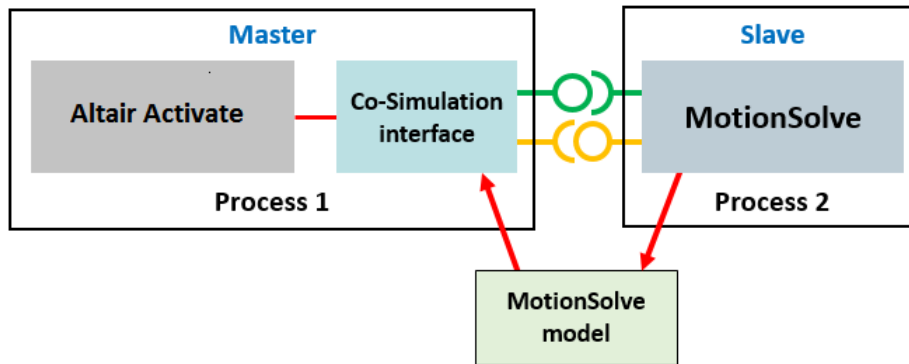


Figure 13.2: Co-simulation between **Activate** and **MotionSolve**.

The co-simulation connection variables to define the inputs/outputs from the **MotionSolve** model are defined within the `Control_PlantInput` and `Control_PlantOutput` **MotionSolve** statements, which is a list of runtime (solver) variables for inputs and outputs to the mechanical plant, respectively. These also have settings for the `hold_order`, `sampling_period`, and `offset_time`.

In co-simulation, after instantiation and initialization of the **MotionSolve** solver, inputs to the **MotionSolve** model is provided and the output of the **MotionSolve** block is requested. In order to explain the way a co-simulation is performed, consider a model composed of two simulators as shown in Fig.13.3.

If the output of each model is computed as following equations:

$$\begin{cases} y_{act} = F_{act}(y_{ms}) \\ y_{ms} = F_{ms}(y_{act}) \end{cases}$$

in order to co-simulate this system, this equation should be solved at each communication instant.  $F_i$  functions are usually complex, non-linear, and time-dependent relationships, as a result, iterative methods are the only choice for solving such equations. Three common methods used for solving them are Gauss-Seidel and Gauss-Jacobi and Newton methods. The Gauss-Seidel iteration method is

estimation and packaging synthesis.



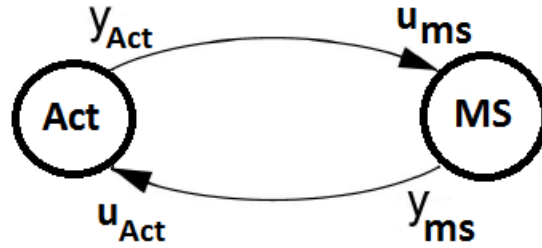


Figure 13.3: Co-simulation between **Activate** and **MotionSolve**

represented as follows.

$$y_{act}^{i+1} = F_{act}(y_{ms}^i)$$

$$y_{ms}^{i+1} = F_{ms}(y_{act}^{i+1})$$

$$i = 0, 1, 2, \dots$$

The Gauss-Jacobi iteration method is defined as

$$y_{act}^{i+1} = F_{act}(y_{ms}^i)$$

$$y_{ms}^{i+1} = F_{ms}(y_{act}^i)$$

$$i = 0, 1, 2, \dots$$

These iterative methods are simple fixed-point methods and their convergence (at most linear) depends on  $F_i$  functions. If the Fixed Point converges very slowly or diverge the Newton-type methods can be used. These methods are, however, more complicated, because in each step the Jacobian must be computed which results in higher number of simulator calls. **Activate** does not support roll back in time, as result, the method which is used actually in **Activate** is GS1, i.e., the Gauss-Seidel method with only one iteration. In this CoSimulation scheme there is no error control.

This method has been illustrated in Fig.13.4;

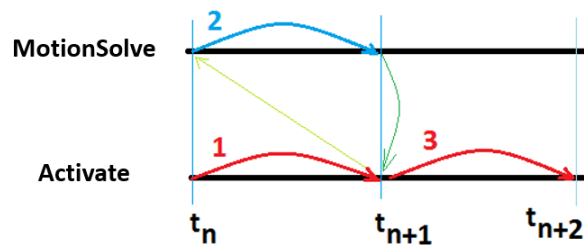


Figure 13.4: CoSimulation method used in **Activate**.

The **Activate** simulator performs the following steps for the co-simulation and synchronization between slaves, as shown in 13.4.

1. The **Activate** simulator advances the time from  $t_n$  to  $t_{n+1}$ , and updates the output of all blocks (except slaves).

2. Slaves are called one by one and asked to advance their time from  $t_n$  to  $t_{n+1}$  and their output is requested at  $t_{n+1}$ .
3. The **Activate** simulator takes a new step from  $t_{n+1}$  to  $t_{n+2}$  using the new output value of slaves computed at  $t_{n+1}$ .
4. Goto 1

Usually in order to get a correct result, the communication step-size (time between two communication instants) should not be larger than the smallest time-constant of slaves. Often smaller communication step-size results in better convergence but longer simulation time.

During the CoSimulation, **Activate** is the lead solver and takes the lead and advances the time, as shown in 13.4. Once **Activate** took its step and reached  $t_{n+1}$ , then **MotionSolve** solver which is the lag solver takes its own internal steps to reach at least  $t_{n+1}$ .

### 13.2.1 Under the hood

The CoSimulation method shown in 13.4 is a rough representation of the CoSimulation method between **Activate** and **MotionSolve**. Actually, on the **MotionSolve** side there are two buffers for keeping input and output values exchanged at communication points,  $t_n, t_{n+1}$ , etc. , as shown in 13.5.

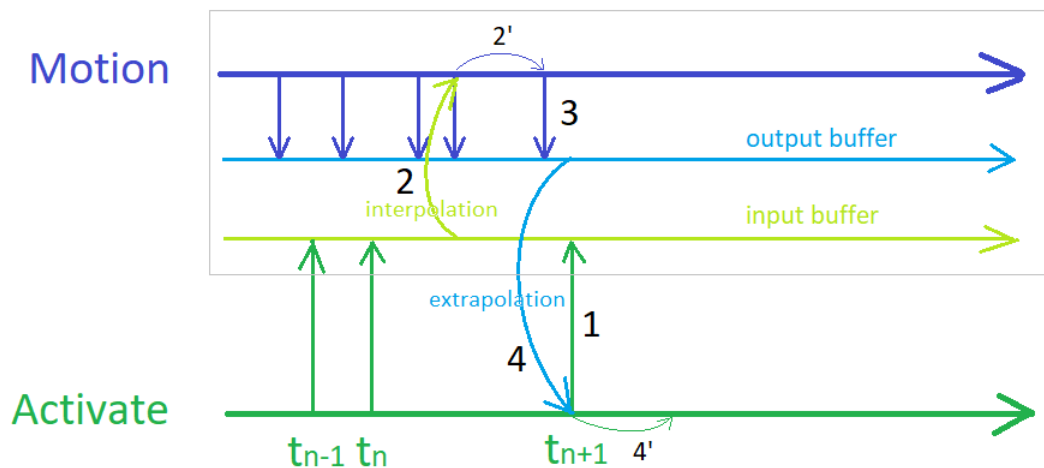


Figure 13.5: Input and output buffers in **MotionSolve**.

1. **Input-buffer:** At communication points  $t_n$ , **Activate** deposits the inputs to **MotionSolve** into this buffer whenever **Activate** calls the **MotionSolve** block. The deposit time instants are completely independent of **MotionSolve** internal steps. **MotionSolve** solver can query this buffer to get the data it needs at a specific time. Since the **MotionSolve** solver is the lag solver, the input buffer will always interpolate the data deposited by **Activate** to provide the input to the **MotionSolve** solver. The interpolate order is 2.
2. **Output-buffer:** **MotionSolve** deposits the outputs from **MotionSolve** into this buffer at the frequency it is supposed to. The deposit time instants are completely independent of **Activate** internal steps or communication points. **Activate** solver can query this buffer to get the data it needs at a specific time. Since **Activate** is the lead solver, the output buffer will extrapolate at communication points  $t_n < t \leq t_{n+1}$  to provide the output to the **Activate** solver. The order of

extrapolation is defined in the **MotionSolve** xml model, as will be explained as follows.

The writing into and reading from these buffers follows three basic rules in **MotionSolve**:

1. At each step for any solver, either **Activate** or **MotionSolve**, makes an attempt to deposit data into its corresponding output buffer, i.e., **output-buffer** for **MotionSolve** and **input-buffer** for **Activate**, and the output buffer immediately returns the control back to the solver. The data is accepted if the step is a successful step, and rejected if it's not. Then, the solver will request data from its input buffer.
2. When input buffer gets request from a solver, the **MotionSolve** solver will need to wait if there is not enough data for interpolation. The **Activate** solver has to wait if the request time is more than one step (the **Activate** solver's step) larger than the last time the **MotionSolve** solver made an attempt to deposit or request data to the same buffer.
3. A buffer-in-use lock should be enforced to prevent buffer from being overwritten (by new deposit) while the same buffer is preparing and returning request for another solver.

In general, one should choose **Activate** solver to be the solver with smaller stepsize and **MotionSolve** to be the solver with larger stepsize. This is because **Activate** solver usually gets extrapolated inputs and **MotionSolve** always gets interpolated inputs, and extrapolating by a large step usually leads to worsen results. Also, the inputs to the **Activate** solver should be something less prone to noise, like displacements. The inputs to the **MotionSolve** solver, on the other hand, can be less restrictive including noisier signals such as forces or torques.

When the **Activate** solver requests the output-buffer the output at  $t_n$ , the **MotionSolve** solver cannot integrate beyond  $t_n$ . Actually, whether **MotionSolve** solver can integrate beyond  $t_n$  or not is not determined by when the **Activate** solver requests the output-buffer for a data. It is determined by when the last time **Activate** solver deposits a data at the input-buffer. Since, this last time is usually  $t_n$ , **MotionSolve** solver cannot integrate beyond this point because the input-buffer will keep the **MotionSolve** solver waiting until there is enough data for interpolation (rule #2).

These two buffers are not visible from **Activate**'s side and filled and queried only by **MotionSolve**. **Activate** communicates with **MotionSolve** only at communication points  $t_n$ . When outputs of the **MotionSolve** block are computed at communication points, **MotionSolve** extrapolates the outputs. The order of extrapolation, is defined by the parameter *HOLD\_ORDER* defined in the **MotionSolve** or MotionView model. This parameter which defines the degree of the polynomial used for extrapolation takes the following values:

1. *HOLD\_ORDER* = 0 means the last value is returned.
2. *HOLD\_ORDER* = 1 means a straight line is fitted through the last 2 points. The line is extended to the new time to return the signal value.
3. *HOLD\_ORDER* = 2 means a parabola is fitted through the last 3 points. The parabola is extended to the new time to return the signal value.
4. *HOLD\_ORDER* > 2 is not supported, because extrapolation is inherently dangerous and higher order extrapolations can lead to large (and unexpected) values.

The decoupling of the solvers through the buffer mechanism allows each solver to pick its own step. In other words, **Activate** and **MotionSolve** solver settings do not directly affect each other's time steps. This leads to a faster simulation. The only way the simulators can affect each other's time step is through the model. If the **MotionSolve** output is smooth enough, that will allow **Activate**'s numerical solver take larger step sizes. On the other hand, if in the **MotionSolve** side, a lower order extrapolation order is

chosen or the **MotionSolve** output shows noisy or high frequency behavior, the **Activate** numerical solver needs to choose smaller steps to reduce the error. When **MotionSolve** output is noisy, data got from 2nd order extrapolation at larger time step will be way off. This is the reason that user needs to choose order 1 or 0. So, in the **MotionSolve** xml, a lower extrapolation order needs to be chosen when the **MotionSolve** output shows noisy or high frequency behavior, otherwise the **Activate** numerical solver needs to choose smaller steps to reduce the error.

Interested users are referred to the **MotionSolve** Users Guide for more information.

### 13.3 MSplant block parameters

The MSplant block implements co-simulation interface between **Activate** and **MotionSolve**. Simulations with this block require that Altair **MotionSolve** solver is installed. The path to the solver and its licensing must be set within **Activate** preferences panel, section Co-Simulation.

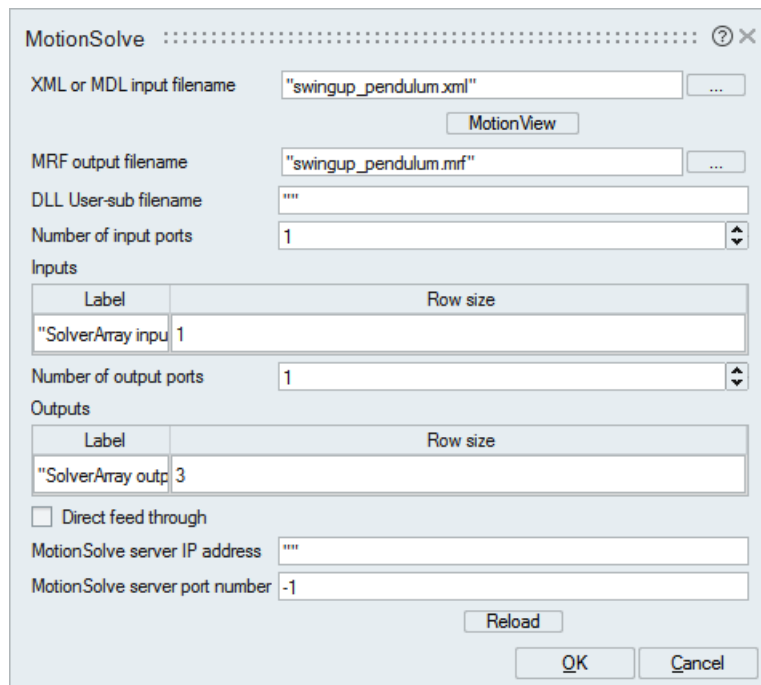


Figure 13.6: Graphical user interface of the MSplant block.

- **XML or MDL input filename:** In this field the name of the **MotionSolve** model (with extension .xml) or MotionView (with extension .mdl) can be given. The filenames can be either given by an absolute path, or a relative path. If **Activate** and **MotionSolve** models are in the same folder, no path is required. In the later case the absolute path is obtained from where the **Activate** model is located. If a MotionView is given the (.mdl) file is automatically converted into a **MotionSolve** (.xml) file.
- **Launch MotionView:** If a MotionView (.mdl) file is provided as the input **MotionSolve** filename, it is possible to visualize and edit the model in MotionView by clicking on this button. Note: When MotionView is open **Activate** remains inactive.

- **MRF output filename:** This field indicates the name of the MotionView result filename. Note that the folder where the file should be generated is not write-protected. If the corresponding feature is active in the **MotionSolve** (.xml) model, the MotionView result file (.mrf) will be converted into an H3D file (.h3d). The .mrf and .h3d files can be visualized in MotionView.
- **DLL User-sub filename (optional):** If the **MotionSolve** model requires extra libraries for execution, the path of these DLLs can be provided in this field. The DLLs provided here will be loaded by **MotionSolve** at the beginning of the co-simulation and will be released at the end.
- **Number of input ports:** The number of input ports of the block. This number must correspond to the number of plant inputs in the **MotionSolve** model. This field is automatically filled if the Reload button is pressed.
- **Inputs:** Each row corresponds to an input port. The label of input as well as the size of the input should be provided here. These fields are automatically filled if the Reload button is pressed.
- **Number of output ports:** The number of output ports of the block. This number must correspond to the number of plant outputs in the **MotionSolve** model. This field is automatically filled if the Reload button is pressed.
- **Outputs:** Each row corresponds to an output port. The label of output as well as the size of the output should be provided here. These fields are automatically filled if the Reload button is pressed.
- **Direct feedthrough:** The **MotionSolve** model does not indicate if an input of the MSplant block directly affects the outputs (feedthrough information). This check box allows setting the feedthrough value. If this check-box is active, it is ensured that the input of the block is always up-to-date when the output of the block is requested. Otherwise a very short delay may be observed in the output of the block.
- **Reload button:** The information about the input and output of the **MotionSolve** model and their names and their sizes are all available in the **MotionSolve** (.xml) file. Instead of filling the above fields manually, the user can fill them automatically by pressing this button. Filling manually the fields is useful when the xml file is not still available and the user needs to build the **Activate** model.
- **MotionSolve server IP address:** This field is not currently used.
- **MotionSolve server port number:** This field is not currently used.

## 13.4 Example: Pendulum Swing-Up

The inverted pendulum is a popular system for illustrating control and multi-body problems. In [1], a control strategy for swinging up a pendulum from downward position to the upright position has been presented. This strategy has been explained and implemented in 16.3.1 where the pendulum and the controller are implemented in **Activate**. Here we will explain the way the pendulum is implemented in **MotionSolve** and the controller in **Activate**. The resulting models are then co-simulated to bring the pendulum in upward position.

The inverted pendulum has been implemented in **MotionSolve** as shown in Fig.13.7. This model is available as a **Activate** model in the `Main_Demos` folder. The model has one input signal `sa_in[1]` of size one and one output vector signal `sa_out[3]` of size three, as shown in Fig.13.8. The force



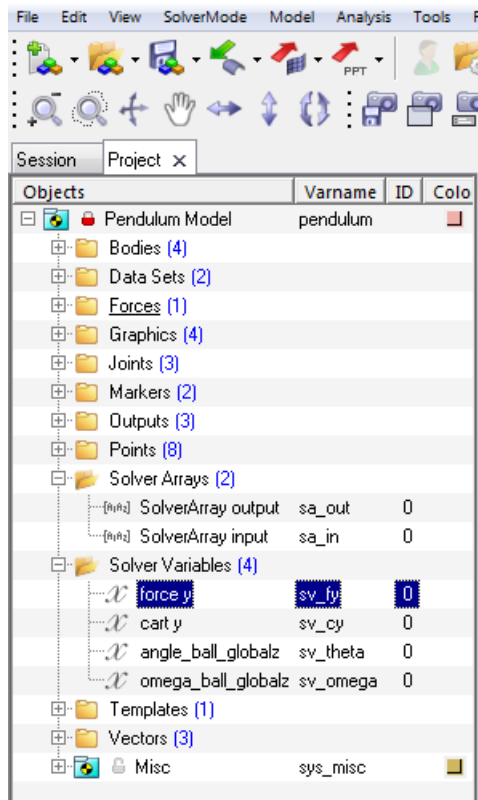


Figure 13.8: Input and output variables of the **MotionSolve** pendulum model.

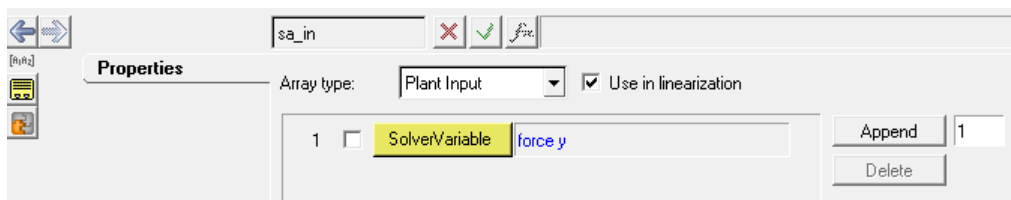


Figure 13.9: Input variable of the **MotionSolve** pendulum model.

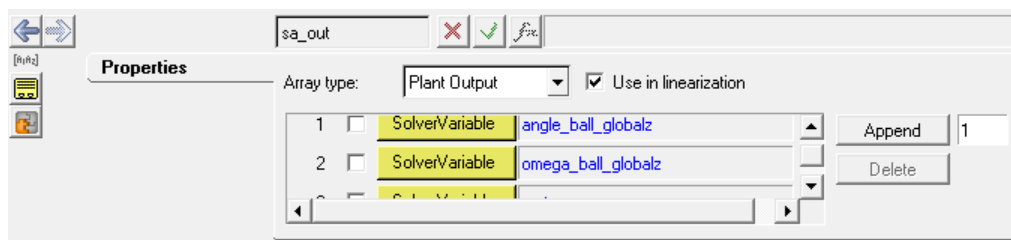


Figure 13.10: Output variables of the **MotionSolve** pendulum model.

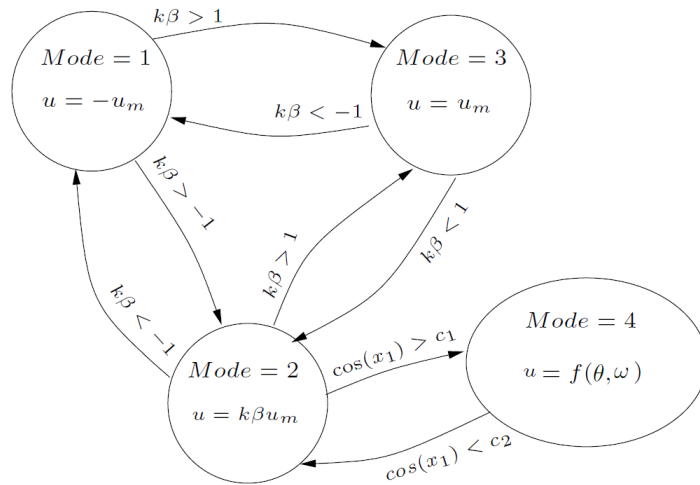


Figure 13.11: Control strategy to swing up and stabilize an inverted pendulum on a cart.

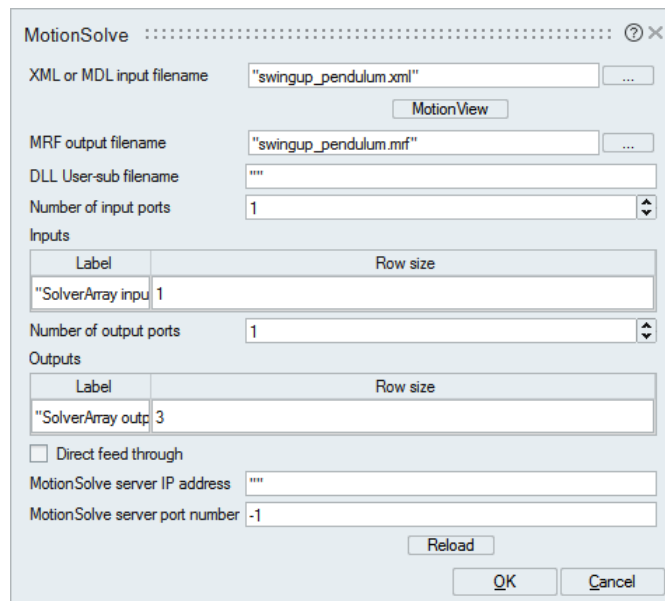


Figure 13.12: Graphical user interface of the MSplant block.



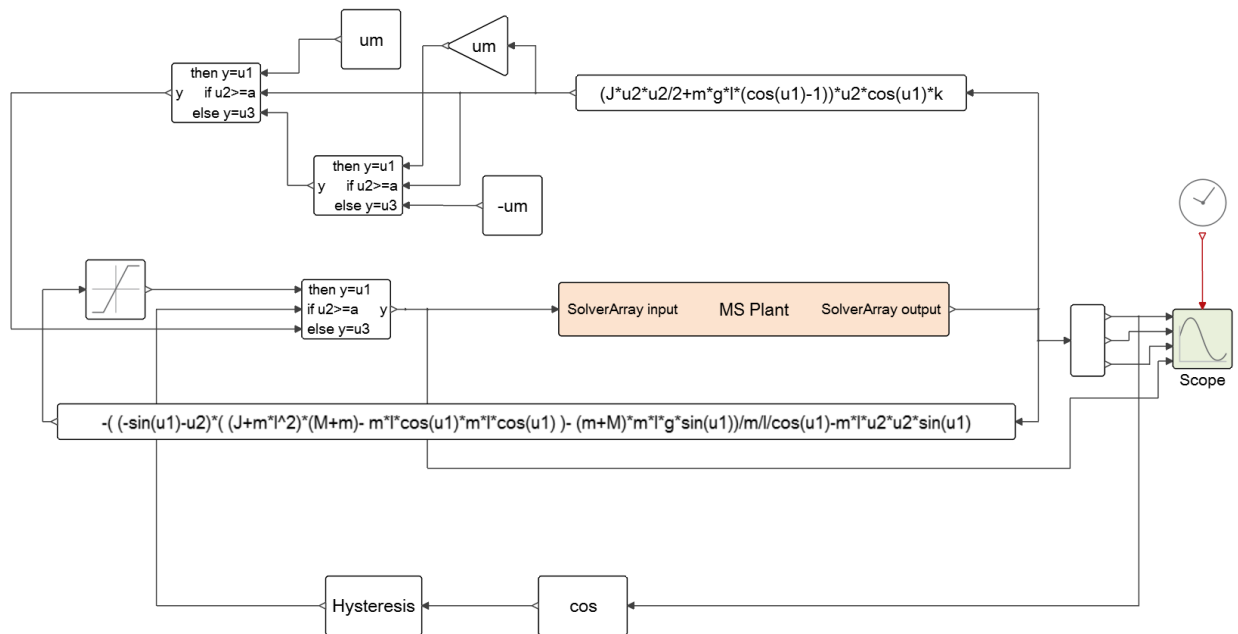


Figure 13.13: The controller built in **Activate**.

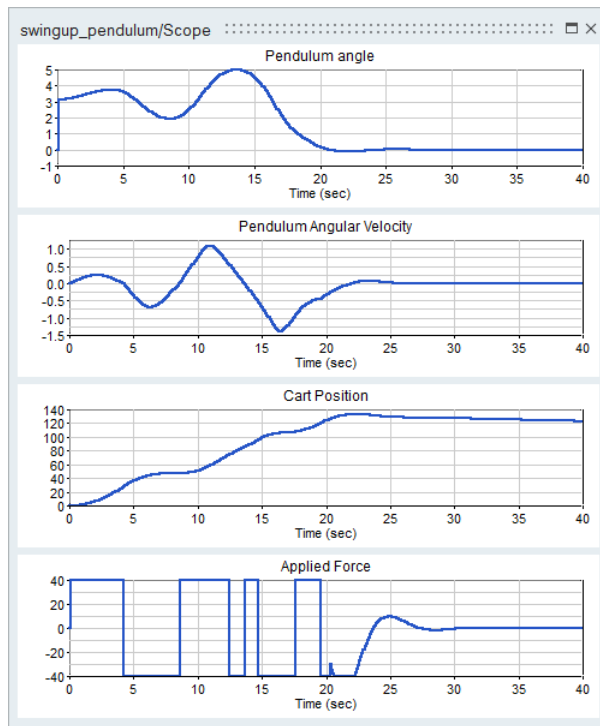


Figure 13.14: The co-simulation result displayed in **Activate**.



# Bibliography

- [1] Swinging up a pendulum by energy control, K. J. Astrom and K. Furuta, Automatica, Volume 36, pp 278-285, 2000.



## Chapter 14

# CoSimulation with Electromagnetic and Thermal Models

### 14.1 Introduction on Flux

Modeling and simulation of complex electromagnetic systems in engineering usually leads to hybrid systems of differential and algebraic system of equations with discrete-time equations. Such complex systems cannot often be modelled and simulated in one simulation tool. The system and control part is simulated in **Activate** and the electromagnetic and thermal parts are simulated in **Altair Flux**. **Flux** is a finite element software for low-frequency electromagnetic and thermal simulations. In order to simulate such multi-disciplinary models, CoSimulation is the principal solution. CoSimulation is a general approach for the joint simulation of models developed with different tools where each tool treats one part of a coupled problem. The data exchange (input and output variables, status information) between subsystems is restricted to communication points. In the time between two communication points, the subsystems are solved independently from each other by their individual solvers.

**Activate** provides an interface to **Flux** which allows reading sensor variables such as current, position, and rotor speed defined in the **Flux** model and applying control variables computed by **Activate** such rotor angle position.

In this chapter, the **Activate-Flux** interface and the **Flux** block for CoSimulation will be discussed<sup>1</sup>.

### 14.2 CoSimulation with Flux

Similar to the **Activate-MotionSolve** CoSimulation interface (13.1), a master-slave method is used to co-simulate between **Activate** and **Flux**. In a master-slave approach the slave simulates the sub-model whereas the master is responsible for both coordinating the overall simulation as well as transferring data. The slaves are the simulation tools, which are prepared to simulate their subtask. The slaves are able to communicate data, execute control commands and return status information. Several **Flux** blocks representing different **Flux** models can be used in an **Activate** model.

The CoSimulation interface lets the user simulate a complex system that includes one or more electro-

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<sup>1</sup>For more information of on **Flux**, interested users are referred to the **Flux** User's Guide.

magnetic and thermal systems (defined by **Flux** blocks) and several subsystems modeled with **Activate** blocks.

In the **Activate-Flux** interface, **Activate** is the master and **Flux** is slave. The CoSimulation interface has been implemented in a block called **Flux** which is available in the `Activate/CoSimulation` palette, as shown in Figure 14.1.



Figure 14.1: **Flux** block in **Activate**.

In order to perform the simulation, it is necessary to have **Flux** installed on the machine. The installation path of **Flux** should be provided to **Activate** in the `Preferences` of **Activate**, as shown in Figure 14.2.

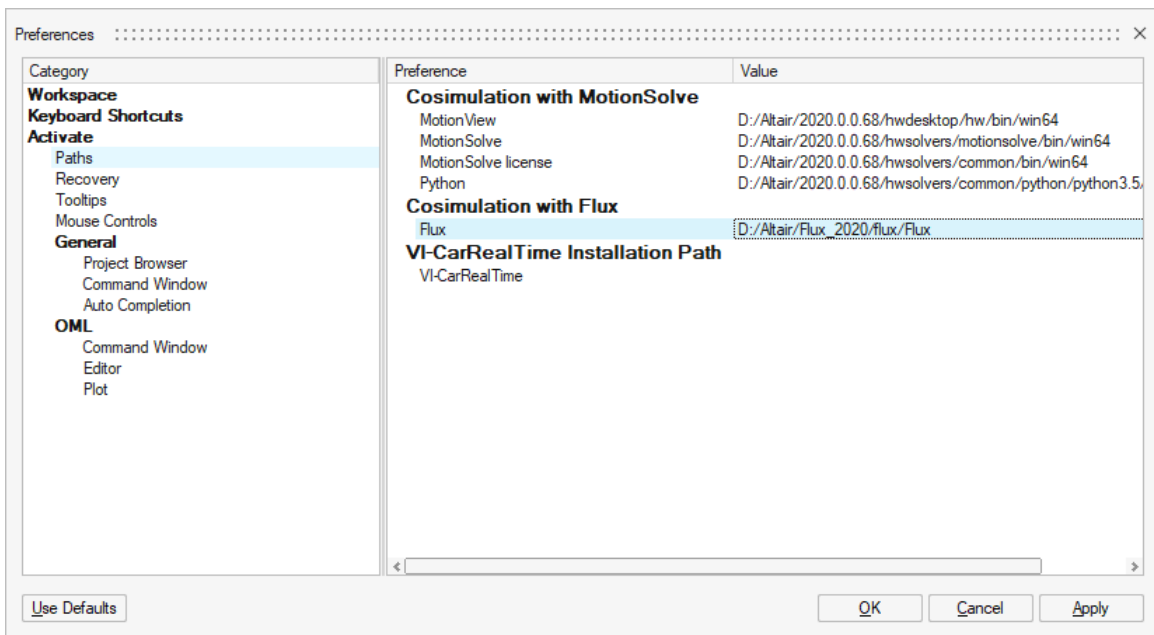


Figure 14.2: Defining the **Flux** installation path in **Activate**.

In **Flux**, once a project is created, the interface variables, i.e., input and output to **Activate** are defined. Then a CoSimulation interface is exported as an `*.f2sta` component<sup>2</sup>. The `.f2sta` component contains the **Flux** project information including the **Flux** version, the type of **Flux** solver, and the name of input and output ports. The information inside the `*.f2sta` file is loaded by **Activate** to setup the CoSimulation.

In order to explain the way a CoSimulation is performed, consider a model composed of two simulators as shown in Figure 14.3.

<sup>2</sup>The reader is referred to the **Flux** User's Guide for more details on exporting `.f2sta` components.

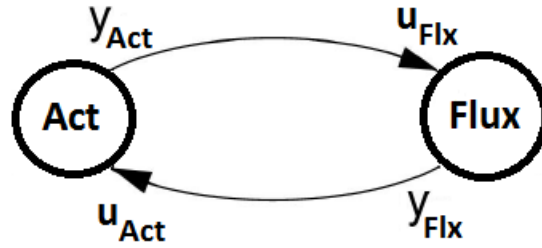


Figure 14.3: CoSimulation between **Activate** and **Flux**.

During the CoSimulation, after instantiation and initialization of the **Flux** solver, inputs to the **Flux** model is provided and the output of the **Flux** block is requested. Suppose that the output of each part is computed with following equations:

$$\begin{cases} Y_{Act} = F_{Act}(Y_{Flx}) \\ Y_{Flx} = G_{Flx}(Y_{Act}) \end{cases}$$

In order to co-simulate this system, this equation should be solved at each communication instant.  $F_{Act}$  and  $G_{Flx}$  functions are usually complex, non-linear, and time-dependent relationships, as a result, iterative methods are the only choice for solving such equations.

This method has been illustrated in Figure 14.4;

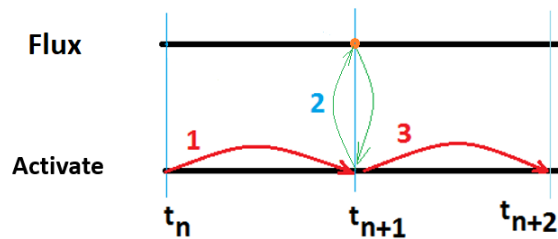


Figure 14.4: CoSimulation method used in **Activate**.

The **Activate** simulator is the master and advances the time. The following steps for the CoSimulation and time synchronization between **Activate** and **Flux** is done, as shown in Figure 14.4.

1. The **Activate** simulator updates the input of the **Flux** block at instant  $t_n$  and requests the **Flux** output at  $t_n$ .
2. The **Flux** block reads its inputs and performs either a complete computation or a simple extrapolation to compute the output at  $t_n$ .
3. The **Activate** simulator advances the time to  $t_{n+1}$  and repeats.

Usually in order to get a correct result, the communication step-size (time between two communication instants) should not be larger than the smallest time-constant of slaves. Often smaller communication step-size results in better convergence but longer simulation time.

The CoSimulation method shown in 14.4 is a rough representation of the CoSimulation method between **Activate** and **Flux**. There are subtle details about the way **Flux** block is called during the CoSimulation. Actually, each time the **Flux** engine performs a computation, the whole finite element computation is

done in **Flux** with the recent input received from **Activate**. But, each **Flux** engine computation is very costly in time. Then, it is crucial to reduce the number of computation of **Flux** to reduce the CoSimulation time.

In order to reduce the simulation time there are two solutions. The first way is done inside the **Flux** engine, i.e., the **Flux** engine does not perform the computation, if it is not necessary. In fact, a simple intuitive method to reduce the number of **Flux** computation is to not performing the computation inside **Flux**, if input values of the **Flux** block, provided by **Activate** have not changed within a range. In this way, a parameter in the **Flux** block GUI is used to define the percentage in input variations. The parameter '**Minimal input variation**' specifies the percentage of input variation between two consecutive calculations of **Flux**. If the **Flux** engine performs a computation at time  $t_n$ , the next **Flux** engine computation at  $t_{n+1}$  will be carried out only if the change in inputs exceeds the designated value. If the **Flux** engine calculation is not carried out, **Flux** block outputs are extrapolated based on the 'Output extrapolation order' value.

This method is useful when the signal incoming from **Activate** is almost steady or does not change very fast with respect to **Activate** signals. Note that this solution may become troublesome, if the **Flux** component contains time-dependent elements with fast time constants. Suppose there is a time dependent **Flux** model whose inputs provided by **Activate** are Constant values. If the '**Minimal input variation**' parameter is specified, since the block inputs does not change during the simulation, the **Flux** engine will perform the computation only once, which provides incorrect result. Because, since the **Flux** model is time dependent, there is no computation to show the signal variations inside the **Flux** component. If a minimum input variation parameter is specified, in order to force the **Flux** engine to perform a computation, the '**Maximum computation interval**' parameter is used. If this parameter is set to  $T_m$  value, the **Flux** engine performs at least one computation each  $T_m \pm 10\%$  seconds to ensure capturing the signal variations inside the **Flux** component. In order to disable the parameter '**Minimal input variation**', zero value or a very high value such as 1000% may be used. The parameter '**Maximum computation interval**' is ignored if the parameter '**Minimal input variation**' is not used. As a rule of thumb, the time step-size used by the **Flux** numerical solver would be a appropriate value for this this parameter.

The second way to reduce the simulation time is to reduce the number of calls to the **Flux** engine. In order to advance the time in **Activate** and take a step, the **Activate** simulator engine calls blocks in model several times for several reasons, e.g., at solver mesh-points, at discrete time-events, at zero-crossing time instants, at internal numerical solver points. At all of these instants, the **Flux** block is invoked and its outputs are read by **Activate**. For some of these invocations, it is useless to invoke the **Flux** engine to perform a computation and then update the outputs. For example, at internal updates required by the numerical solver, the updated **Flux** output is not required and an extrapolation of the previous outputs is sufficient. In the **Flux** block GUI, the *Call Flux at Activate event instants* checkbox in the **Flux Activation** tab 14.3.2 indicates if at discrete event time instants of the Activate simulator, the **Flux** engine computation is required. By default, the **Flux** block is invoked only at simulator mesh-points. If this checkbox is checked, the **Flux** block is called at mesh-points and at all discrete-time events of the simulator. If the checkbox is not checked, at discrete-event time instant, the **Flux** engine performs an extrapolation and no costly computation is done. Note that even if this extra point checkbox is checked, it does not mean that there will be a **Flux** computation at a point. Actually, it is the **Flux** engine who decides if a computation or an extrapolation should be done, based on the '**Minimal input variation**' and '**Maximum computation interval**' values.

It is important to indicate that when the **Flux** block is externally activated (i.e., **External Activation**' checkbox is checked), the '**Minimal input variation**' and '**Maximum computation interval**', as well



as the *Call Flux at Activate event instants* checkbox are ignored. In fact, if the block is externally activated, **Activate** forces the **Flux** engine to perform a computation, regardless of these parameters. The external activation is useful if, for example, a periodic update of the **Flux** block is desired. In the model shown in Figure 14.5, the Flux block is called at periodic interval of 0.01 seconds.

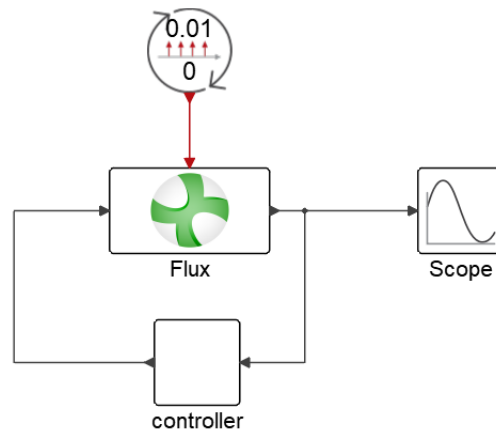


Figure 14.5: **Flux** block activated externally and called at periodic intervals.

## 14.3 Flux block parameters

The **Flux** block implements CoSimulation interface between **Activate** and **Flux**.

### 14.3.1 Parameters tab

- **Flux to Activate component file-name:** In this field, the name of the **Flux** component with extension `.f2sta` is given. The **Flux to Activate** component (`*.F2TSA`) is created by **Flux** and should have the same name as the **Flux** project. The file-name can be either given by an absolute path, or a relative path. If **Activate** and **Flux** models are in the same folder, no path is required. In the latter case, the absolute path is obtained from where the **Activate** model is located.
- **Reload button:** The information about the input and output of the **Flux** model, their names, and their sizes are all available in the (`*.f2sta`) file. Instead of filling the GUI fields manually, they can be filled automatically by pressing this button. Filling manually these fields is useful when the `*.f2sta` file is not still available and the user needs to build the **Activate** model before getting the `*.f2sta` file.
- **Multiplexed inputs/outputs:** Select vector or scalar mode for the input/output. Inputs and outputs to **Activate** can be defined as vector or scalars when the `*.f2sta` component is exported by **Flux**.
- **Number of input ports:** The number of input parameters stored in the `*.f2sta` file. This field is automatically filled if the Reload button is pressed.
- **Inputs:** Each row corresponds to an input port. The label of input as well as the size of the input should be provided here. These fields are automatically filled if the Reload button is pressed.

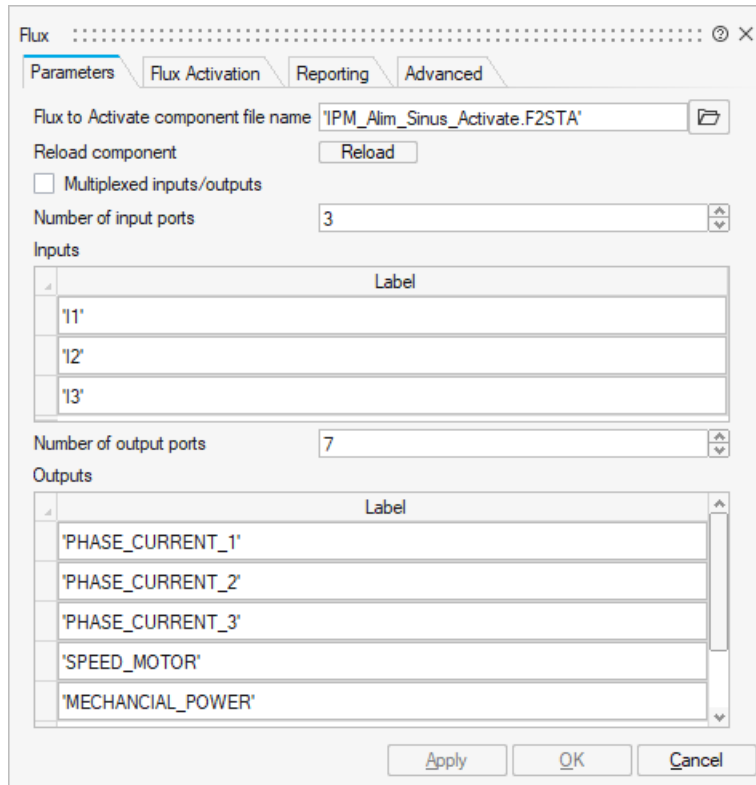


Figure 14.6: Graphical user interface of the **Flux** block: Parameters tab.

- **Number of output ports:** The number of output parameters stored in the \*.f2sta file. This field is automatically filled if the Reload button is pressed.
- **Outputs:** Each row corresponds to an output port. The label of output as well as the size of the output should be provided here. These fields are automatically filled if the Reload button is pressed.

### 14.3.2 Flux Activation tab

- **External activation:** Specifies whether the block receives an external activation or inherits its activation through its regular input ports. When 'External Activation' is selected, an additional activation port is added to the block. By default, external activation is not selected. If external activation is selected, the **Flux** engine only performs a computation at external event instants, *e.g.*, at periodic events defined by an external clock. If checked, '**Minimal input variation**', '**Maximum computation interval**', and '**Flux Activation**' are disabled and ignored by **Flux** and **Activate**.
- **Extra Flux calls at Activate event instants: Activate** only calls the **Flux** block at Activate solver mesh-point to update its output. If this checkbox is checked, the **Flux** block is also called at other instants such as discrete-time events during the CoSimulation. Extra points provides more detailed outputs especially at event instants.
- **Minimal input variation (%) to run Flux computation:** Specifies the percentage of input variation between two calculation steps under which a **Flux** calculation should not occur. When the

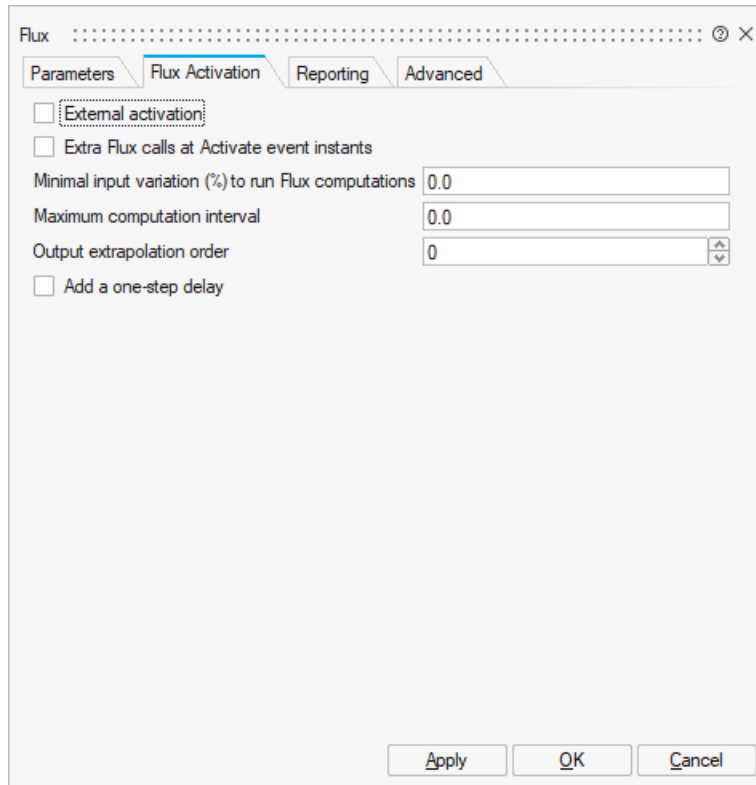


Figure 14.7: Graphical user interface of the **Flux** block: Flux Activation tab.

**Flux** calculation is not carried out, outputs are computed based on the 'Output extrapolation order' that is defined, therefore permitting several **Activate** time steps to be carried out without any **Flux** calculations.

- **Maximum computation interval:** If a non-zero '**Maximum computation interval**' is provided, and a non-zero '**Minimal input variation**' is specified, the **Flux** engine is forced to perform a computation not later than the '**Maximum computation interval**' since the last computation, even if the input values have not changed.
- **Output extrapolation order:** Choose the output extrapolation order (0 : zero-order hold extrapolation, 1 : linear extrapolation). If '**Minimal input variation**', '**Maximum computation interval**' parameters are defined and used, when the **Flux** engine does not perform the computation, the output of the **Flux** block is provided using extrapolation. In general, if the **Flux** simulator engine does not perform the complete calculation, *e.g.*, when the **Activate** simulator needs the Flux output at internal solver points (try points), **Flux** provides the output by extrapolation.
- **Add one-step delay:** Add a one-step delay to help avoid the algebraic loops caused by the **Flux** block.

### 14.3.3 Reporting tab

- **Subsampling (Reduced result storage):** N (which should be greater than 2) means one out of N data is stored in the result data file in **Flux**. This feature is available in **Flux** version 2018 and later.

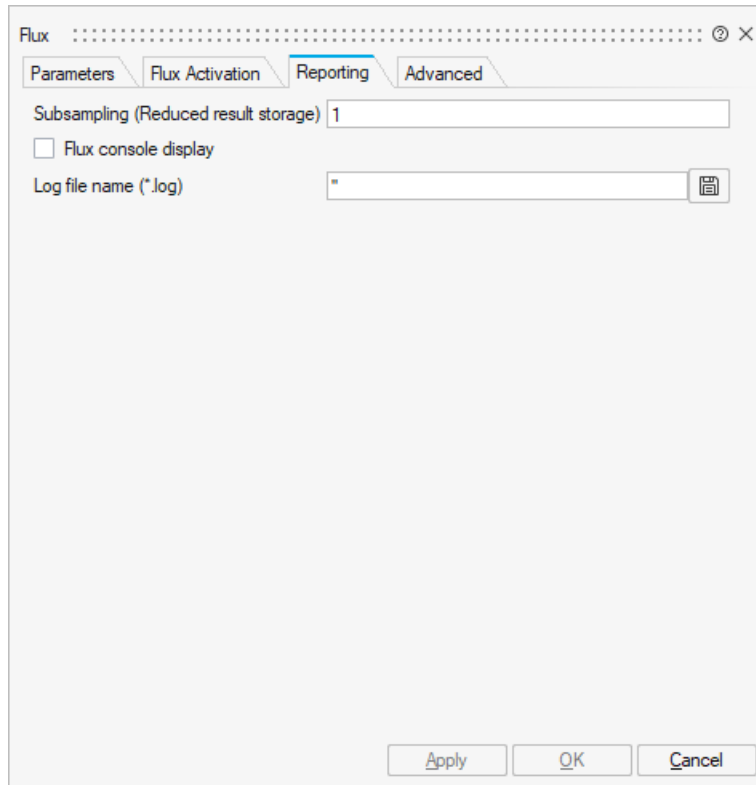


Figure 14.8: Graphical user interface of the **Flux** block: Report tab.

- **Flux console display:** Select the check box to turn on the **Flux** display console, though this may drastically slow down the computation.
- **Log file name (\*.log):** **Activate** messages as well as messages communicated from **Flux** to **Activate** are logged into this file. 'stderr' and 'stdout' are valid values that output log messages into the standard output.

#### 14.3.4 Advanced tab

- **Current Flux version:** Displays the **Flux** version in use. If the installation path of **Flux** is specified in **Preferences** of **Activate**, this field indicates the version of that installation. If no installation path is specified, the user may specify the **Flux** installation path by defining the environment variable 'INSTALLFLUX'. For example, in the OML command window of **Activate**, the user can enter: `setenv('INSTALLFLUX', 'D:/AltairSoftwares/Flux-2020/flux/Flux')`
- **Flux component version:** **Flux** version used to generate the coupling component, *i.e.*, the \*.f2sta file. This information is stored in the \*.f2sta file.
- **Flux solver:** Displays the **Flux** solver in use. This information is stored in the \*.f2sta file.
- **Numerical memory (MB):** Specifies the memory used for the various modeling actions. 3D mesh and solving processes (Flux 2D-3D) include a high memory demand. Memory allocation is a function of the application type (real/complex) and matrix size for the solving process. The default value of this information is stored in the \*.f2sta file.

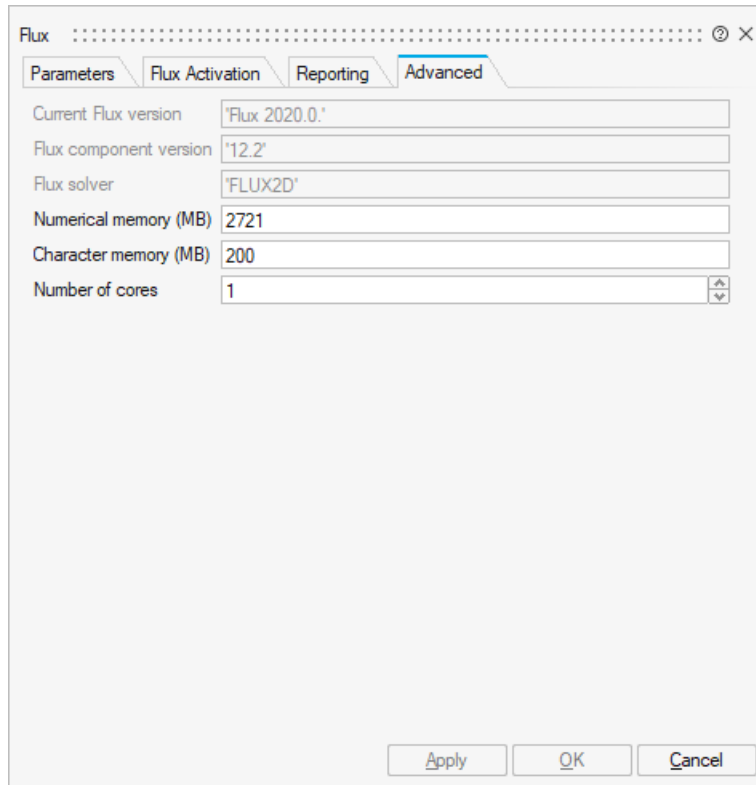


Figure 14.9: Graphical user interface of the **Flux** block: Advanced tab.

- **Character memory (MB):** Character memory is used for storing entity names such as parameters, transformations, regions, and project names presented in the directory. The default value of this information is stored in the `*.f2sta` file.
- **Number of cores:** Number of cores used by **Flux** during CoSimulation. This feature is available in **Flux** version 2018 and above. In Flux-2D, only 1 core is used.

## 14.4 Example: Brushless AC embedded permanent magnet motor

Consider the model in Figure 14.10 which is a model for a brushless AC embedded permanent magnet motor. The motor is derived by Activate sine wave signals.

The simulation result is shown in Figure 14.11.

## 14.5 Example: Externally activated block

The Cosimulation with **Flux** block is usually slow due to numerous calls to the **Flux** engine. This CoSimulation of the example in Figure 14.10 takes about 190 seconds. In order to reduce the calls to the **Flux** engine, the Flux block can be activated with an external clock, instead of the Activate numerical solver. In Figure 14.12, the model in Figure 14.10, is Activated externally by a clock with the period of 0.001 seconds. In this model, the Flux block perform a computation every one milli-second.

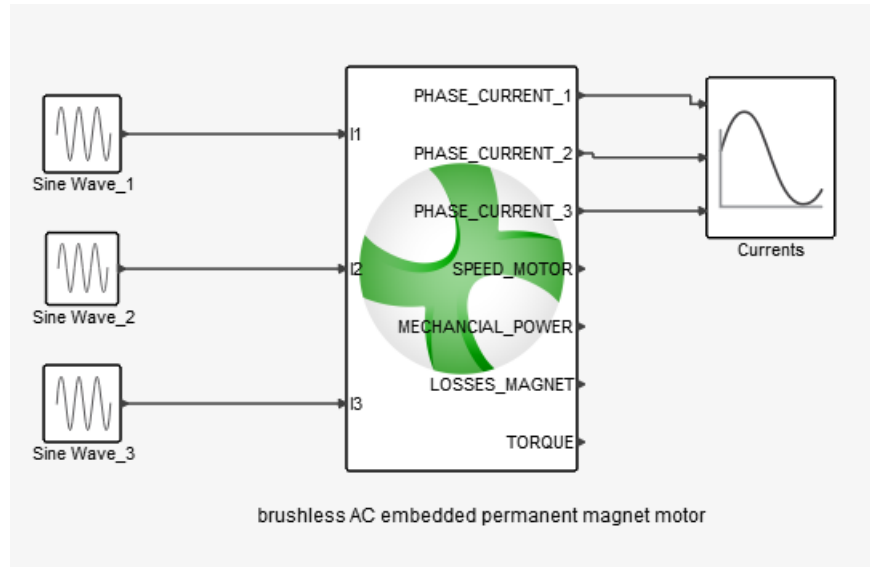


Figure 14.10: The brushless AC motor

Now the simulation only takes 60 seconds. The reason is the reduced number of calls to Flux. The only drawback of using external activation is a result which is sampled at clock event instants, as shown in Figure 14.13. The sampled result (blue curve) is superposed on the result obtained in Figure 14.11 (red curve).

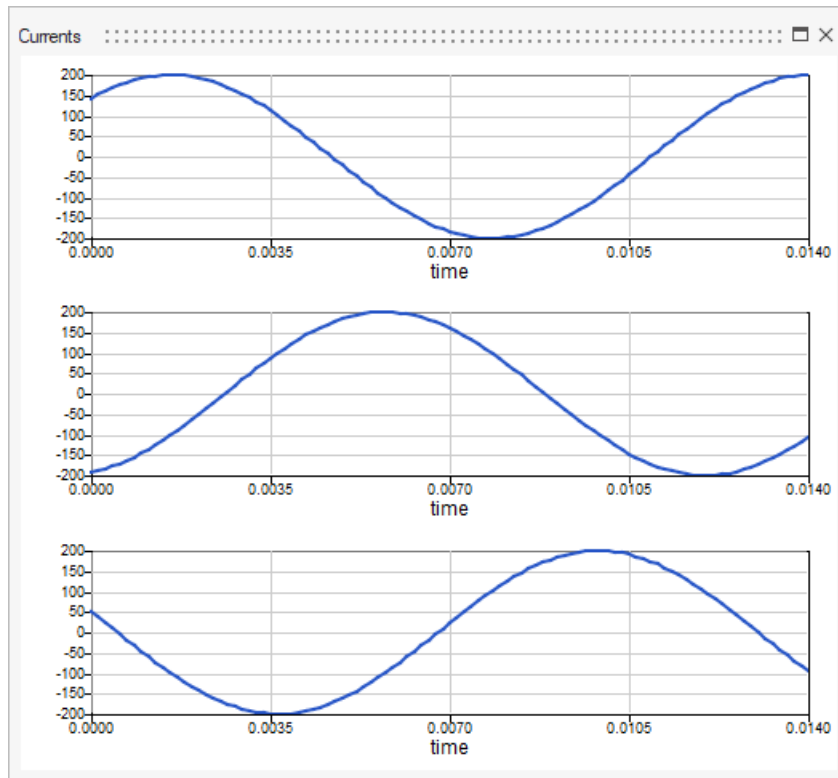


Figure 14.11: The brushless AC motor: Simulation result.

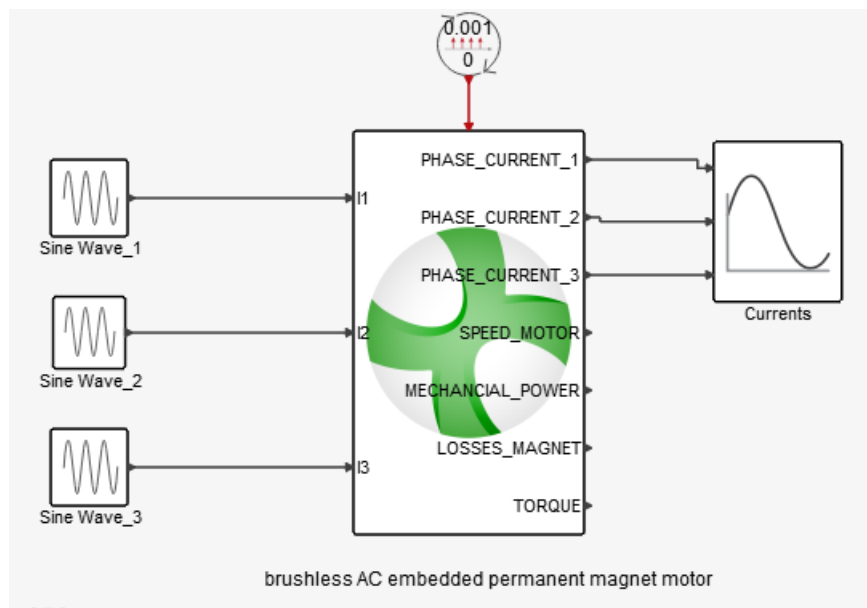


Figure 14.12: The brushless AC motor: externally activated by a clock.

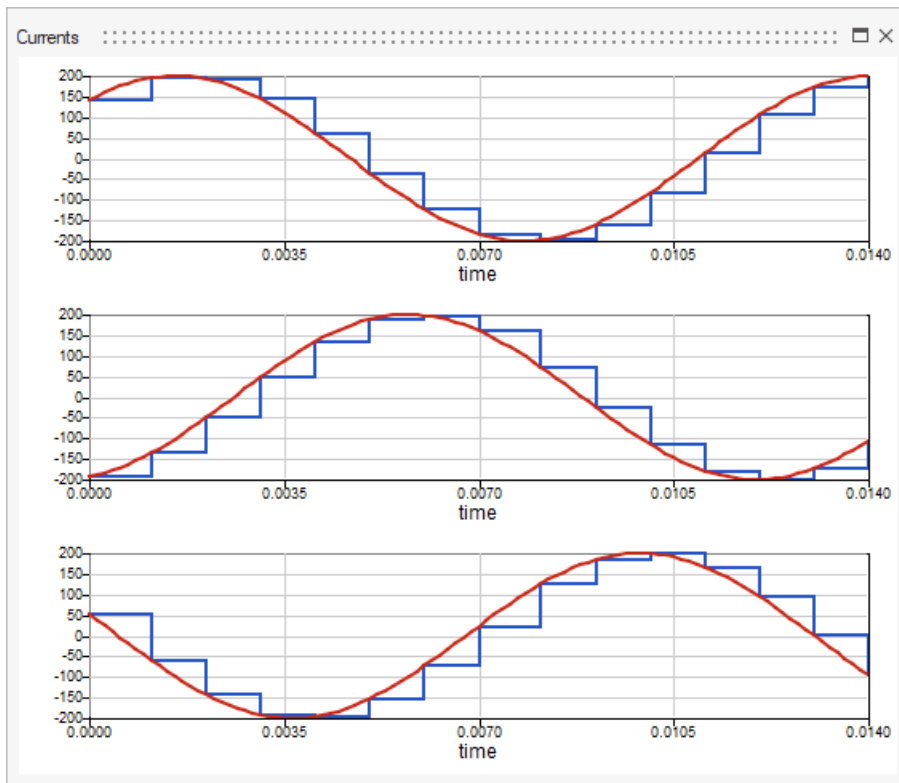


Figure 14.13: The brushless AC motor: Simulation result.



# Chapter 15

## HyperSpice in Altair Activate

### 15.1 Introduction

This chapter is dedicated to users who want to model and simulate any electronic circuits inside its system description. To cover this field, Activate is proposing an environment and a simulator which is based on the SPICE language. Without being a specialist in electronic, a designer can easily describe and/or get electronic designs in order to simulate them as a black-box.

#### 15.1.1 What is exactly SPICE ?

SPICE (Simulation Program with Integrated Circuit Emphasis) is a software dedicated to simulating analog circuits with the highest accuracy. The language used as input of a SPICE simulator is a textual representation of the electronic circuit. The language itself is called the SPICE language. As it is a textual representation, there is no specific needs to define a circuit. Moreover descriptions can be shared easily without any restrictions.

#### 15.1.2 Why using SPICE ?

The first goal of SPICE is to keep the best accuracy for the simulation results. Over the time SPICE became a reference by silicon foundries as well as analog circuit designers. Even for digital simulations, timing analysis and power consumptions, SPICE simulation has become essential for building libraries and propose accurate results.

### 15.2 HyperSpice™

HyperSpice™ is a high performance circuit simulator using SPICE syntax. Its language is based on the Spice3 spice standard circuit description language. His architecture is based on a hierarchical view of circuits description and is using a complex elaboration which offers the possibility to simulate huge circuit providing accuracy and good simulation speed while consuming less memory. HyperSpice is a matrix based simulator. The solving is made by using a specific solver using the latest powerful techniques of the market for the system resolution.

HyperSpice is able to perform simulation in three modes:

- DC : Operating point simulation
- Transient: time domain simulation
- AC: frequency response simulation

In terms of device models, HyperSpice is using the Berkeley public models. Devices models level 1, 2 and 3 as well as Bsim3 are supported by HyperSpice.

Below is the list of all supported models:

Devices	AC/DC and Transient simulation
Resistor	X
Capacitor	X
Inductor	X
Mutual inductances	X
Transmission lines	X
Lossy Transmission lines	X
Vsources	X
Isources	X
Voltage Controlled Current Sources	X
Voltage Controlled Voltage Sources	X
Current Controlled Current Sources	X
Current Controlled Voltage Sources	X
Voltage Switches	X
Current Switches	X
S-Parameters	X
Diodes	X
BJTs	X
MOSs	Level 1, 2, 3 and Bsim 3
JFETs	Not Supported yet
MESFETs	Not supported yet

HyperSpice is taking as input a netlist, written in the Spice syntax. A quick documentation of Spice components is described below, but the user can refer also to the public documentation on

[http://bwrcs.eecs.berkeley.edu/Classes/IcBook/SPICE/UserGuide/elements\\_fr.html](http://bwrcs.eecs.berkeley.edu/Classes/IcBook/SPICE/UserGuide/elements_fr.html)

HyperSpice has also been written to be compatible with other Spice simulators such as Spice3, NGSpice, LTSpice and PSpice as well as some commercial Spice syntax.

## 15.3 Spice Language

### 15.3.1 Units

Spice language is defining different symbols associated to different unit factors. These characters can be used for any setting values. The table below is representing the associated symbols with their representation:

V	Deg	Db	f	Hz	H	u	k	M	m	n	p	s
Volt	Degree	Decibel	Fento	Hertz	Henri	Micro	Kilo	Mega	Milli	Nano	Pico	Second

### 15.3.2 Comments

To create a comment in Spice, we can use the symbol '\*' to set the complete line as comment. However if a partial comment is needed, it is possible to use the symbol '!'. This symbol means that the comment is starting from the symbol until the end of line. For example:

```
* This line is a entire comment line
```

```
R1 1 0 1k ! This line is a resistor of 1K, but after the ! it is a comment
```

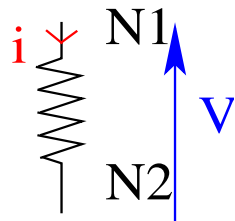
An important thing to take into account is that the Spice language is considering the first line as a comment. The user should either declare a comment for the first line or consider that the first line will be ignored.

### 15.3.3 Resistor

#### Spice Syntax

```
RXXXXXXXX N1 N2 VALUE
```

#### Graphical representation



```
RXXXXXXXX N1 N2 <VALUE> <MNAME> <L=LENGTH> <W=WIDTH> <TEMP=T>
```

```
RLOAD 2 10 10K RMOD 3 7 RMODEL L=10u W=1u
```

#### Equation:

$$V = R * i$$

#### Examples:

```
R1 N1 N2 100
```

```
RC1 12 17 1K
```

N1 and N2 are the two element nodes. VALUE is the resistance (in ohms) and may be positive or negative but not zero.

This is the more general form of the resistor presented in section 6.1, and allows the modeling of temperature effects and for the calculation of the actual resistance value from strictly geometric information and the specifications of the process. If VALUE is specified, it overrides the geometric information and defines the resistance. If MNAME is specified, then the resistance may be calculated from the process information in the model MNAME and the given LENGTH and WIDTH. If VALUE is not specified, then MNAME and LENGTH **must** be specified. If WIDTH is not specified, then it is taken from the default width given in the model. The (optional) TEMP value is the temperature at which this device is to operate, and overrides the temperature specification on the .OPTION control line.

The resistor model consists of process-related device data that allow the resistance to be calculated from geometric information and to be corrected for temperature. The parameters available are:

name	parameter	units	default	example
TC1	first order temperature coefficient.	$\Omega / ^\circ\text{C}$	0.0	-
TC2	second order temperature coefficient.	$\Omega / ^\circ\text{C}^2$	0.0	-
RSH	sheet resistance	$\Omega / \mu\text{m}$	-	50
DEFW	default width	meters	1e-6	2.e-6
NARROW	narrowing due to side etching	meters	0.0	1.e-7
TNOM	parameter measurement temperature	$^\circ\text{C}$	27	50

The sheet resistance is used with the narrowing parameter and L and W from the resistor device to determine the nominal resistance by the formula:

$$R = RSH * \frac{L - \text{Narrow}}{W - \text{Narrow}}$$

DEFW is used to supply a default value for W if one is not specified for the device. If either RSH or L is not specified, then the standard default resistance value of 1k is used. TNOM is used to override the circuit-wide value given on the .OPTIONS control line where the parameters of this model have been measured at a different temperature. After the nominal resistance is calculated, it is adjusted for temperature by the formula:

$$R(T) = R(T_0) [1 + TC_1(T - T_0) + TC_2(T - T_0)^2]$$

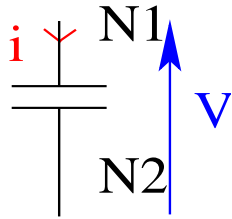
### 15.3.4 Capacitor

#### Spice Syntax

CXXXXXXXX N+ N- VALUE

CXXXXXXXX N1 N2 <VALUE> <MNAME> <L=LENGTH> <W=WIDTH>

## Graphical representation



### Examples:

```
Cap1 13 0 1n
```

```
CBYP 13 0 1UF COSC 17 23 10U
```

```
CLOAD 2 10 10P CMOD 3 7 CMODEL L=10u W=1u
```

N+ and N- are the positive and negative element nodes, respectively. VALUE is the capacitance in Farads.

The (optional) initial condition is the initial (time-zero) value of capacitor voltage (in Volts). Note that the initial conditions (if any) apply 'only' if the UIC option is specified on the .TRAN control line.

This is the more general form of the Capacitor presented in section 6.2, and allows for the calculation of the actual capacitance value from strictly geometric information and the specifications of the process. If VALUE is specified, it defines the capacitance. If MNAME is specified, then the capacitance is calculated from the process information in the model MNAME and the given LENGTH and WIDTH. If VALUE is not specified, then MNAME and LENGTH **must** be specified. If WIDTH is not specified, then it is taken from the default width given in the model. Either VALUE or MNAME, LENGTH, and WIDTH may be specified, but not both sets.

The capacitor model contains process information that may be used to compute the capacitance from strictly geometric information.

name	parameter	units	default	example
CJ	junction bottom capacitance	F/meters <sup>2</sup>	-	5.e-5
CJSW	junction sidewall capacitance	F/meters	-	2.e-11
DEFW	default device width	meters	1.e-6	2.e-6
NARROW	narrowing due to side etching	meters	0.0	1.e-7

The capacitor has a capacitance computed as:

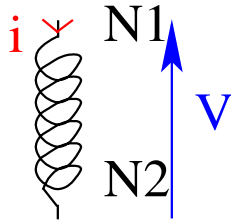
$$Cap = CJ(Length - Narrow)(Width - Narrow) + 2 * CJSW(Length + Width - 2 * Narrow)$$

## 15.3.5 Inductor

### Spice Syntax

```
LYYYYYYY N+ N- VALUE <IC=INCOND>
```

## Graphical representation



### Examples:

```
LLINK 42 69 1UH
```

```
LSHUNT 23 51 10U IC=15.7MA
```

N+ and N- are the positive and negative element nodes, respectively. VALUE is the inductance in Henries.

The (optional) initial condition is the initial (time-zero) value of inductor current (in Amps) that flows from N+, through the inductor, to N-. Note that the initial conditions (if any) apply only if the UIC option is specified on the .TRAN analysis line.

Equation:

$$V = L \frac{di}{dt}$$

## 15.3.6 Mutual Inductors

### Spice syntax

```
KXXXXXXXX LYYYYYYY LZZZZZZZ VALUE
```

### Examples:

```
K43 LAA LBB 0.999
```

```
KXFRMR L1 L2 0.87
```

The K device is coupled Mutual Inductors. LYYYYYYY and LZZZZZZZ are the names of the two coupled inductors, and VALUE is the coefficient of coupling, K, which must be greater than 0 and less than or equal to 1. Using the 'dot' convention, place a 'dot' on the first node of each inductor.

The mutual inductance is equal to:

$$M = \sqrt{L_1 * L_2}$$

## 15.3.7 Mosfet

### Spice syntax

```
MXXXXXXXX ND NG NS NB MNAME <L=VAL> <W=VAL> <AD=VAL> <AS=VAL>  
+ <PD=VAL> <PS=VAL> <NRD=VAL> <NRS=VAL> <OFF>
```

+ <IC=VDS, VGS, VBS> <TEMP=T>

### Examples:

```
M1 24 2 0 20 TYPE1
```

```
M31 2 17 6 10 MODM L=5U W=2U
```

```
M1 2 9 3 0 MOD1 L=10U W=5U AD=100P AS=100P PD=40U PS=40U
```

ND, NG, NS, and NB are the drain, gate, source, and bulk (substrate) nodes, respectively. MNAME is the model name.

L and W are the channel length and width, in meters.

AD and AS are the areas of the drain and source diffusions, in  $m^2$ . Note that the suffix U specifies microns ( $1e-6$  m) and P square-microns ( $1e-12$   $m^2$ ). If any of L, W, AD, or AS are not specified, default values are used. The use of defaults simplifies input file preparation, as well as the editing required if device geometries are to be changed.

PD and PS are the perimeters of the drain and source junctions, in meters. RD and NRS designate the equivalent number of squares of the drain and source diffusions; these values multiply the sheet resistance RSH specified on the .MODEL control line for an accurate representation of the parasitic series drain and source resistance of each transistor.

PD and PS default to 0.0 while NRD and NRS to 1.0.

OFF indicates an (optional) initial condition on the device for dc analysis. The (optional) initial condition specification using IC=VDS, VGS, VBS is intended for use with the UIC option on the .TRAN control line, when a transient analysis is desired starting from other than the quiescent operating point. See the .IC control line for a better and more convenient way to specify transient initial conditions.

The (optional) TEMP value is the temperature at which this device is to operate, and overrides the temperature specification on the .OPTION control line. The temperature specification is ONLY valid for level 1, 2, 3, and 6 MOSFETs, not for level 4 or 5 (BSIM) devices.

## MOSFET Models (NMOS/PMOS)

SPICE provides several MOSFET device models, which differ in the formulation of the I-V characteristic. The variable LEVEL specifies the model to be used.

The models are declared using the syntax:

```
.MODEL MOSMNAME <PMOS|NMOS> LEVEL=<level number> <PARAMETER=<VALUE>>
```

At least for a MOS model the user should select the level number. For this he has to choose among the following models:

Level number	Mos type
0, 1	Mos Level 1
2	Mos Level 2
3	Mos Level 3
13	Bsim1
39	Bsim2
49, 53	Bsim3 v3.3(.2)

The DC characteristics of the level 1 through level 3 MOSFETs are defined by the device parameters VTO, KP, LAMBDA, PHI and GAMMA. These parameters are computed by SPICE if process parameters (NSUB, TOX, ...) are given, but user-specified values always override. VTO is positive (negative) for enhancement mode and negative (positive) for depletion mode N-channel (P-channel) devices. Charge storage is modeled by three constant capacitors, CGSO, CGDO, and CGBO which represent overlap capacitances, by the nonlinear thin-oxide capacitance which is distributed among the gate, source, drain, and bulk regions, and by the nonlinear depletion-layer capacitances for both substrate junctions divided into bottom and periphery, which vary as the MJ and MJSW power of junction voltage respectively, and are determined by the parameters CBD, CBS, CJ, CJSW, MJ, MJSW and PB.

Charge storage effects are modeled by the piecewise linear voltages-dependent capacitance model proposed by Meyer. The thin-oxide charge-storage effects are treated slightly different for the LEVEL=1 model. These voltage-dependent capacitances are included only if TOX is specified in the input description and they are represented using Meyer's formulation.

There is some overlap among the parameters describing the junctions, e.g. the reverse current can be input either as IS (in A) or as JS (in  $A/m^2$ ). Whereas the first is an absolute value the second is multiplied by AD and AS to give the reverse current of the drain and source junctions respectively. This methodology has been chosen since there is no sense in relating always junction characteristics with AD and AS entered on the device line; the areas can be defaulted. The same idea applies also to the zero-bias junction capacitances CBD and CBS (in F) on one hand, and CJ (in  $F/m^2$ ) on the other. The parasitic drain and source series resistance can be expressed as either RD and RS (in ohms) or RSH (in  $\Omega/sq.$ ), the latter being multiplied by the number of squares NRD and NRS input on the device line.

SPICE level 1, 2, 3 parameters:



	name	parameter	units	default	example
1	LEVEL	model	index	-	1
2	VTO	zero-bias threshold voltage (VT0)	V	0.0	1.0
3	KP	transconductance parameter	$A/V^2$	2.0e-5	3.1e-5
4	GAMMA	bulk threshold parameter ()	$V^{1/2}$	0.0	0.37
5	PHI	surface potential ()	V	0.6	0.65
6	LAMBDA	channel-length modulation	1/V	0.0	0.02
		MOS1 and MOS2 only			
7	RD	drain ohmic resistance	$\Omega$	0.0	1.0
8	RS	source ohmic resistance	$\Omega$	0.0	1.0
9	CBD	zero-bias B-D junction capacitance	F	0.0	20fF
10	CBS	zero-bias B-S junction capacitance	F	0.0	20fF
11	IS	bulk junction saturation current (IS)	A	1.0e-14	1.0e-15
12	PB	bulk junction potential	V	0.8	0.87
13	CGSO	gate-source overlap capacitance per meter channel width	F/m	0.0	4.0e-11
14	CGDO	gate-drain overlap capacitance per meter channel width	F/m	0.0	4.0e-11
15	CGBO	gate-bulk overlap capacitance per meter channel length	F/m	0.0	2.0e-10
16	RSH	drain and source diffusion sheet resistance	$\Omega/q$	0.0	10.0
17	CJ	zero-bias bulk junction bottom cap. per sq-meter of junction area	$F/m^2$	0.0	2.0e-4
18	MJ	bulk junction bottom grading coeff.	-	0.5	0.5
19	CJSW	zero-bias bulk junction sidewall cap. per meter of junction perimeter	F/m	0.0	1.0e-9
20	MJSW	bulk junction sidewall grading coeff.	-	0.50(level1) 0.33(level2,3)	
21	JS	bulk junction saturation current per sq-meter of junction area	$A/m^2$		1.0e-8
22	TOX	oxide thickness	meter	1.0e-7	1.0e-7
23	NSUB	substrate doping	1/cm3	0.0	4.0e15
24	NSS	surface state density	$1/cm^2$	0.0	1.0e10
25	NFS	fast surface state density	$1/cm^2$	0.0	1.0e10
26	TPG	type of gate material: +1 opp. to substrate -1 same as substrate 0 Al gate	-	1.0	
27	XJ	metallurgical junction depth	meter	0.0	1
28	LD	lateral diffusion	meter	0.0	0.8
29	UO	surface mobility	$cm^2/Vs$	600	700

30	UCRIT	critical field for mobility degradation (MOS2 only)	V/cm	1.0e4	1.0e4
31	UEXP	critical field exponent in mobility degradation (MOS2 only)	-	0.0	0.1
32	UTRA	transverse field coeff. (mobility) (deleted for MOS2)	-	0.0	0.3
33	VMAX	maximum drift velocity of carriers	m/s	0.0	5.0e4
34	NEFF	total channel-charge (fixed and mobile) coefficient (MOS2 only)	-	1.0	5.0
35	KF	flicker noise coefficient	-	0.0	1.0e-26
36	AF	flicker noise exponent	-	1.0	1.2
37	FC	coefficient for forward-bias depletion capacitance formula	-	0.5	
38	DELTA	width effect on threshold voltage (MOS2 and MOS3)	-	0.0	1.0
39	THETA	mobility modulation (MOS3 only)	1/V	0.0	0.1
40	ETA	static feedback (MOS3 only)	-	0.0	1.0
41	KAPPA	saturation field factor (MOS3 only)	-	0.2	0.5
42	TNOM	parameter measurement temperature	C	27	50

For more information on models see the Berkeley device models documentation.

### 15.3.8 BJT

#### Spice syntax

```
QXXXXXXXX NC NB NE <NS> MNAME <AREA> <OFF> <IC=VBE, VCE> <TEMP=T>
```

```
.MODEL MNAME NPN|PNP <Param=Value>
```

#### Examples:

```
Q23 10 24 13 QMOD IC=0.6
```

```
Q50A 11 26 4 20 MOD1
```

The BJT device is a Bipolar Junction Transistor.

NC, NB, and NE are the collector, base, and emitter nodes, respectively.

NS is the (optional) substrate node. If unspecified, ground is used.

MNAME is the model name, AREA is the area factor, and OFF indicates an (optional) initial condition on the device for the dc analysis. If the area factor is omitted, a value of 1.0 is assumed.

The (optional) initial condition specification using IC=VBE, VCE is intended for use with the UIC option on the .TRAN control line, when a transient analysis is desired starting from other than the quiescent operating point. See the .IC control line description for a better way to set transient initial conditions.

The (optional) TEMP value is the temperature at which this device is to operate, and overrides the temperature specification on the .OPTION control line.

## BJT Models (NPN/PNP)

The bipolar junction transistor model in SPICE is an adaptation of the integral charge control model of Gummel and Poon. This modified Gummel-Poon model extends the original model to include several effects at high bias levels. The model automatically simplifies to the simpler Ebers-Moll model when certain parameters are not specified. The parameter names used in the modified Gummel-Poon model have been chosen to be more easily understood by the program user, and to reflect better both physical and circuit design thinking.

The dc model is defined by the parameters IS, BF, NF, ISE, IKF, and NE which determine the forward current gain characteristics, IS, BR, NR, ISC, IKR, and NC which determine the reverse current gain characteristics, and VAF and VAR which determine the output conductance for forward and reverse regions.

Three ohmic resistances RB, RC, and RE are included, where RB can be high current dependent. Base charge storage is modeled by forward and reverse transit times, TF and TR, the forward transit time TF being bias dependent if desired, and nonlinear depletion layer capacitances which are determined by CJE, VJE, and MJE for the B-E junction, CJC, VJC, and MJC for the B-C junction and CJS, VJS, and MJS for the C-S (Collector-Substrate) junction.

The temperature dependence of the saturation current, IS, is determined by the energy-gap, EG, and the saturation current temperature exponent, XTI. Additionally base current temperature dependence is modeled by the beta temperature exponent XTB in the new model. The values specified are assumed to have been measured at the temperature TNOM, which can be specified on the .OPTIONS control line or overridden by a specification on the .MODEL line.

The BJT parameters used in the modified Gummel-Poon model are listed below. The parameter names used in earlier versions of SPICE2 are still accepted.

### Modified Gummel-Poon BJT Parameters

	name	parameter	units	default	example	area
1	IS	transport saturation current	A	1.0e-16	1.0e-15	*
2	BF	ideal maximum forward beta	-	100	100	
3	NF	forward current emission coefficient	-	1.0	1	
4	VAF	forward Early voltage	V	infinite	200	
5	IKF	corner for forward beta high current roll-off	A	infinite	0.01	*
6	ISE	B-E leakage saturation current	A	0	1.0e-13	*
7	NE	B-E leakage emission coefficient	-	1.5	2	
8	BR	ideal maximum reverse beta	-	1	0.1	
9	NR	reverse current emission coefficient	-	1	1	
10	VAR	reverse Early voltage	V	infinite	200	
11	IKR	corner for reverse beta high current roll-off	A	infinite	0.01	*
12	ISC	leakage saturation current	A	0		8
13	NC	leakage emission coefficient	-	2	1.5	
14	RB	zero bias base resistance	$\Omega$	0	100	*
15	IRB	current where base resistance falls halfway to its min value	A	infinite	0.1	*
16	RBM	minimum base resistance at high currents	$\Omega$	RB	10	*
17	RE	emitter resistance	$\Omega$	0	1	*
18	RC	collector resistance	$\Omega$	0	10	*
19	CJE	B-E zero-bias depletion capacitance	F	0	2pF	*

20	VJE	B-E built-in potential	V	0.75	0.6	
21	MJE	B-E junction exponential factor	-	0.33	0.33	
22	TF	ideal forward transit time	sec	0	0.1ns	
23	XTF	coefficient for bias dependence of TF	-	0		
24	VTF	voltage describing VBC dependence of TF	V	infinite		
25	ITF	high-current parameter for effect on TF	A	0		*
26	PTF	excess phase at freq=1.0/(TF*2PI) Hz	deg	0		
27	CJC	B-C zero-bias depletion capacitance	F	0	2pF	*
28	VJC	B-C built-in potential	V	0.75	0.5	
29	MJC	B-C junction exponential factor	-	0.33	0.5	
30	XCJC	fraction of B-C depletion capacitance connected to internal base node	-	1		
31	TR	ideal reverse transit time	sec	0	10ns	
32	CJS	zero-bias collector-substrate capacitance	F	0	2pF	*
33	VJS	substrate junction built-in potential	V	0.75		
34	MJS	substrate junction exponential factor	-	0	0.5	
35	XTB	forward and reverse beta temperature exponent	-	0		
36	EG	energy gap for temperature effect on IS	eV	1.11		
37	XTI	temperature exponent for effect on IS	-	3		
38	KF	flicker-noise coefficient	-	0		
39	AF	flicker-noise exponent	-	1		
40	FC	coefficient for forward-bias depletion capacitance formula	-	0.5		
41	TNOM	Parameter measurement temperature	°C	27	50	

### 15.3.9 JFET

#### Spice Syntax

JXXXXXXX ND NG NS MNAME <AREA> <OFF> <IC=VDS, VGS> <TEMP>

#### Examples:

J1 7 2 3 JM1 OFF

The JFET device is a Junction Field-Effect Transistors (JFETs). ND, NG, and NS are the drain, gate, and source nodes, respectively. MNAME is the model name, AREA is the area factor, and OFF indicates an (optional) initial condition on the device for dc analysis. If the area factor is omitted, a value of 1.0 is assumed.

The (optional) initial condition specification, using IC=VDS, VGS is intended for use with the UIC option on the .TRAN control line, when a transient analysis is desired starting from other than the quiescent operating point. See the .IC control line for a better way to set initial conditions.

The (optional) TEMP value is the temperature at which this device is to operate, and overrides the

temperature specification on the .OPTION control line.

## JFET Models (NJF/PJF)

The JFET model is derived from the FET model of Shichman and Hodges. The dc characteristics are defined by the parameters VTO and BETA, which determine the variation of drain current with gate voltage, LAMBDA, which determines the output conductance, and IS, the saturation current of the two gate junctions. Two ohmic resistances, RD and RS, are included. Charge storage is modeled by nonlinear depletion layer capacitances for both gate junctions which vary as the -1/2 power of junction voltage and are defined by the parameters CGS, CGD, and PB.

Note that in Spice3f and later, a fitting parameter B has been added. For details, see [9].

	name	parameter	units	default	example	area
1	VTO	threshold voltage (VT0)	V	-2.0	-2.0	
2	BETA	transconductance parameter	$A/V^2$	1.0e-4	1.0e-3	*
3	LAMBDA	channel-length modulation	$1/V$	0	1.0e-4	
4	RD	drain ohmic resistance	$\Omega$	0	100	*
5	RS	source ohmic resistance	$\Omega$	0	100	*
6	CGS	zero-bias G-S junction capacitance (Cgs)	F	0	5pF	*
7	CGD	zero-bias G-D junction capacitance (Cgs)	F	0	1pF	*
8	PB	gate junction potential	V	1	0.6	
9	IS	gate junction saturation current (IS)	A	1.0e-14	1.0e-14	*
10	B	doping tail parameter	-	1	1.1	
11	KF	flicker noise coefficient	-	0		
12	AF	flicker noise exponent	-	1		
13	FC	coefficient for forward-bias	-	0.5		
14	TNOM	parameter measurement temperature	$^{\circ}\text{C}$	27	50	

## 15.3.10 Switches

### V-Switch

#### Spice syntax

```
SXXXXXXX N+ N- NC+ NC- MODEL <ON><OFF>
```

#### Examples:

```
.MODEL smodel1 RON=100K
```

```
Switch1 1 2 10 0 smodel1
```

The S- device is an ideal voltage switch. Nodes 1 and 2 are the nodes between which the switch terminals are connected. The model name is mandatory while the initial conditions are optional nodes, 3 and 4 are the positive and negative controlling nodes respectively.

## Switch Model (SW)

The switch model allows an almost ideal switch to be described in SPICE.

The switch is not quite ideal, in that the resistance cannot change from 0 to infinity, but must always have a finite positive value. By proper selection of the on and off resistances, they can be effectively zero and infinity in comparison to other circuit elements.

The parameters available are:

name	parameter	units	default
VT	threshold voltage	Volts	0.0
VH	hysteresis voltage	Volts	0.0
RON	on resistance		1.0
ROFF	off resistance		1/ <i>GMIN</i> *

\*(See the .OPTIONS control line for a description of GMIN, its default value results in an off-resistance of 1.0e+12 ohms.)

The use of an ideal element that is highly nonlinear such as a switch can cause large discontinuities to occur in the circuit node voltages.

A rapid change such as that associated with a switch changing state can cause numerical roundoff or tolerance problems leading to erroneous results or timestep difficulties.

The user of switches can improve the situation by taking the following steps:

First, it is wise to set ideal switch impedances just high or low enough to be negligible with respect to other circuit elements. Using switch impedances that are close to “ideal” in all cases aggravates the problem of discontinuities mentioned above. Of course, when modeling real devices such as MOS-FETS, the on resistance should be adjusted to a realistic level depending on the size of the device being modeled.

If a wide range of ON to OFF resistance must be used in the switches ( $ROFF/RON > 1e+12$ ), then the tolerance on errors allowed during transient analysis should be decreased by using the .OPTIONS control line and specifying TRTOL to be less than the default value of 7.0. When switches are placed around capacitors, then the option CHGTOL should also be reduced. Suggested values for these two options are 1.0 and 1e-16 respectively. These changes inform SPICE3 to be more careful around the switch points so that no errors are made due to the rapid change in the circuit.

## C-Switch

### Spice syntax

```
WYYYYYYY N+ N- VNAME MODEL <ON><OFF>
```

### Examples:

```
.MODEL switchmod1 IT=1e-3
```

```
W1 1 2 vclock switchmod1
```

The W-device is a Current controlled switch. Nodes 1 and 2 are the nodes between which the switch terminals are connected. The model name is mandatory while the initial conditions are optional. The controlling current is that through the specified voltage source. The direction of positive controlling

current flow is from the positive node, through the source, to the negative node.

## Switch Model (CSW)

The switch model allows an almost ideal switch to be described in SPICE.

The switch is not quite ideal, in that the resistance cannot change from 0 to infinity, but must always have a finite positive value. By proper selection of the on and off resistances, they can be effectively zero and infinity in comparison to other circuit elements.

The parameters available are:

name	parameter	units	default
IT	threshold current	Amps	0.0
IH	hysteresis current	Amps	0.0
RON	on resistance		1.0
ROFF	off resistance		$1/GMIN^*$

\*(See the .OPTIONS control line for a description of GMIN, its default value results in an off-resistance of  $1.0e+12$  ohms.)

The use of an ideal element that is highly nonlinear such as a switch can cause large discontinuities to occur in the circuit node voltages. A rapid change such as that associated with a switch changing state can cause numerical roundoff or tolerance problems leading to erroneous results or timestep difficulties. The user of switches can improve the situation by taking the following steps:

First, it is wise to set ideal switch impedances just high or low enough to be negligible with respect to other circuit elements. Using switch impedances that are close to "ideal" in all cases aggravates the problem of discontinuities mentioned above. Of course, when modeling real devices such as MOS-FETS, the on resistance should be adjusted to a realistic level depending on the size of the device being modeled.

If a wide range of ON to OFF resistance must be used in the switches ( $ROFF/RON > 1e+12$ ), then the tolerance on errors allowed during transient analysis should be decreased by using the .OPTIONS control line and specifying TRTOL to be less than the default value of 7.0. When switches are placed around capacitors, then the option CHGTOL should also be reduced. Suggested values for these two options are 1.0 and  $1e-16$  respectively. These changes inform SPICE3 to be more careful around the switch points so that no errors are made due to the rapid change in the circuit.

## 15.3.11 Transmission Lines

### Lossless Transmission Lines

#### Spice syntax

```
TXXXXXXX N1 N2 N3 N4 Z0=VALUE <TD=VALUE> <F=FREQ <NL=NRMLN>>  
+ <IC=V1, I1, V2, I2>
```

#### Examples:

```
T1 1 0 2 0 Z0=50 TD=10NS
```

The T device is a Lossless Transmission Lines.

N1 and N2 are the nodes at port 1.

N3 and N4 are the nodes at port 2.

Z0 is the characteristic impedance.

The length of the line may be expressed in either of two forms. The transmission delay, TD, may be specified directly (as TD=10ns, for example). Alternatively, a frequency F may be given, together with NL, the normalized electrical length of the transmission line with respect to the wavelength in the line at the frequency F. If a frequency is specified but NL is omitted, 0.25 is assumed (that is, the frequency is assumed to be the quarter-wave frequency). Note that although both forms for expressing the line length are indicated as optional, one of the two must be specified.

Note that this element models only one propagating mode. If all four nodes are distinct in the actual circuit, then two modes may be excited. To simulate such a situation, two transmission-line elements are required.

The (optional) initial condition specification consists of the voltage and current at each of the transmission line ports. Note that the initial conditions (if any) apply 'only' if the UIC option is specified on the .TRAN control line.

Note that a lossy transmission line (see below) with zero loss may be more accurate than the lossless transmission line due to implementation details.

## Lossy Transmission Lines

### Spice syntax

```
OXXXXXXXX N1 N2 N3 N4 MNAME
```

### Examples:

```
O23 1 0 2 0 LOSSYMOD
```

```
OCONNECT 10 5 20 5 INTERCONNECT
```

The O device is a Lossy Transmission Lines. This is a two-port convolution model for single-conductor lossy transmission lines.

N1 and N2 are the nodes at port 1

N3 and N4 are the nodes at port 2.

Note that a lossy transmission line with zero loss may be more accurate than the lossless transmission line due to implementation details.

## Lossy Transmission Line Model (LTRA)

The uniform RLC/RC/LC/RG transmission line model (referred to as the LTRA model henceforth) models a uniform constant-parameter distributed transmission line.

The RC and LC cases may also be modeled using the URC and TRA models. However, the newer LTRA model is usually faster and more accurate than the others.

The operation of the LTRA model is based on the convolution of the transmission line's impulse re-



sponses with its inputs.

The LTRA model takes a number of parameters, some of which must be given and some of which are optional.

name	parameter	units/type	default	example
R	resistance/length		0.0	0.2
L	inductance/length	henrys/unit	0.0	9.13e-9
G	conductance/length	mhos/unit	0.0	0.0
C	capacitance/length	farads/unit	0.0	3.65e-12
LEN	length of line		no default	1.0
REL	breakpoint control	arbitrary unit	1	0.5
ABS	breakpoint control		1	5
NOSTEPLIMIT	don't limit timestep to less than line delay	flag	not set	set
NOCONTROL	don't do complex timestep control	flag	not set	set
LININTERP	use linear interpolation	flag	not set	set
MIXEDINTERP	use linear when quadratic seems bad		not set	set
COMPACTREL	special reltol for history compaction	flag	RELTOL	1.0e-3
COMPACTABS	special abstol for history compaction		ABSTOL	1.0e-9
TRUNCNR	use Newton-Raphson method for timestep control	flag	not set	set
TRUNCDONTCUT	don't limit timestep to keep impulse-response errors low	flag	not set	set

The following types of lines have been implemented so far:

RLC (uniform transmission line with series loss only), RC (uniform RC line), LC (lossless transmission line), and RG (distributed series resistance and parallel conductance only).

Any other combination will yield erroneous results and should not be tried.

The length LEN of the line must be specified.

NOSTEPLIMIT is a flag that will remove the default restriction of limiting time-steps to less than the line delay in the RLC case.

NOCONTROL is a flag that prevents the default limiting of the time-step based on convolution error criteria in the RLC and RC cases. This speeds up simulation but may in some cases reduce the accuracy of results.

LININTERP is a flag that, when specified, will use linear interpolation instead of the default quadratic interpolation for calculating delayed signals.

MIXEDINTERP is a flag that, when specified, uses a metric for judging whether quadratic interpolation is not applicable and if so uses linear interpolation; otherwise it uses the default quadratic interpolation.

TRUNCDONTCUT is a flag that removes the default cutting of the time-step to limit errors in the actual calculation of impulse-response related quantities.

COMPACTREL and COMPACTABS are quantities that control the compaction of the past history of values stored for convolution. Larger values of these lower accuracy but usually increase simulation speed. These are to be used with the TRYTOCOMPACT option, described in the .OPTIONS section.

TRUNCNR is a flag that turns on the use of Newton-Raphson iterations to determine an appropriate timestep in the timestep control routines. The default is a trial and error procedure by cutting the previous timestep in half.

REL and ABS are quantities that control the setting of breakpoints.

The option most worth experimenting with for increasing the speed of simulation is REL. The default value of 1 is usually safe from the point of view of accuracy but occasionally increases computation time. A value greater than 2 eliminates all breakpoints and may be worth trying depending on the nature of the rest of the circuit, keeping in mind that it might not be safe from the viewpoint of accuracy. Breakpoints may usually be entirely eliminated if it is expected the circuit will not display sharp discontinuities. Values between 0 and 1 are usually not required but may be used for setting many breakpoints.

COMPACTREL may also be experimented with when the option TRYTOCOMPACT is specified in a .OPTIONS card. The legal range is between 0 and 1. Larger values usually decrease the accuracy of the simulation but in some cases improve speed. If TRYTOCOMPACT is not specified on a .OPTIONS card, history compaction is not attempted and accuracy is high.

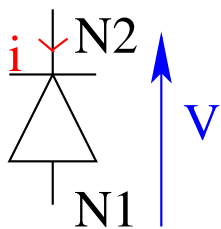
NOCONTROL, TRUNCDONTCUT and NOSTEPLIMIT also tend to increase speed at the expense of accuracy.

### 15.3.12 Diode

#### Spice syntax

```
.MODEL MNAME DIO <Param=value>  
  
DXXXXXXX N1 N2 MNAME <AREA>> <OFF> <IC=VD> <TEMP>
```

#### Graphical representation



#### Examples:

```
DBRIDGE 2 10 DIODE1
```

```
DCLMP 3 7 DMOD 3.0 IC=0.2
```

The D device is a junction diode. N1 and N2 are the positive and negative nodes, respectively. N1 is the Anode and N2 is the Cathode.

MNAME is the model name, AREA is the area factor, and OFF indicates an (optional) starting condition on the device for dc analysis. If the area factor is omitted, a value of 1.0 is assumed.

The (optional) initial condition specification using IC=VD is intended for use with the UIC option on the

.TRAN control line, when a transient analysis is desired starting from other than the quiescent operating point.

The (optional) TEMP value is the temperature at which this device is to operate, and overrides the temperature specification on the .OPTION control line.

### Diode Model (D)

The DC characteristics of the diode are determined by the parameters IS and N. An ohmic resistance, RS, is included. Charge storage effects are modeled by a transit time, TT, and a nonlinear depletion layer capacitance which is determined by the parameters CJO, VJ, and M. The temperature dependence of the saturation current is defined by the parameters EG, the energy and XTI, the saturation current temperature exponent.

The nominal temperature at which these parameters were measured is TNOM, which defaults to the circuit-wide value specified on the .OPTIONS control line. Reverse breakdown is modeled by an exponential increase in the reverse diode current and is determined by the parameters BV and IBV (both of which are positive numbers).

	name	parameter	units	default	example	area
1	IS	saturation current	A	1.0e-14	1.0e-14	*
2	RS	ohmic resistance	$\Omega$	0	10	*
3	N	emission coefficient	-	1	1.0	
4	TT	transit-time	sec	0	0.1ns	
5	CJO	zero-bias junction capacitance	F	0	2pF	*
6	VJ	junction potential	V	1	0.6	
7	M	grading coefficient	-	0.5	0.5	
8	EG	activation energy	eV	1.11	1.11 Si 0.69 Sbd 0.67Ge	
9	XTI	saturation-current temp. exp	-	3.0	3.0jn 2.0Sbd	
10	KF	flicker noise coefficient	-	0		
11	AF	flicker noise exponent	-	1		
12	FC	coefficient for forward-bias depletion capacitance formula	-	0.5		
13	BV	reverse breakdown voltage	V	infinite	40.0	
14	IBV	current at breakdown voltage	A	1.0e-3		
15	TNOM	parameter measurement temperature	$^{\circ}\text{C}$	27	50	

### 15.3.13 S-Parameter blocks

#### Spice syntax

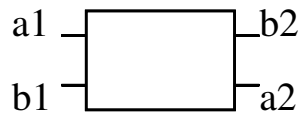
```
.MODEL MNAME SPARAM <PARAM=VALUE>
```

```
YXXXXXX MNAME PARAM: PARAM=VALUE PIN: PIN1 PIN2 .. PINn
```

### Examples:

```
.model SPARAMM SPARAM  
  
ysparam SPARAMM param:  
+ force_passivity=1  
+ sparamfile=lowpassfilter2.s2p  
+ pin:  
+ 2 0 3 0
```

### Graphical representation



$$S_{11} = \frac{b1}{a1} \Big|_{a2=0} \quad S_{12} = \frac{b1}{a2} \Big|_{a1=0}$$

$$S_{21} = \frac{b2}{a1} \Big|_{a2=0} \quad S_{22} = \frac{b2}{a2} \Big|_{a1=0}$$

$$\begin{bmatrix} b1 \\ b2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a1 \\ a2 \end{bmatrix}$$

A Yelem device which is declared with a model named SPARAM is called an S-Parameter block.

Param name	iValue
IM	LC (Linear Cartesian) LC (Linear Cartesian) VF (Vector fitting) CS (Cubic spline) CSL(Cubic Spline Log)
EX	1 (Activate) or 0 (Disactivate)
SPARAMFILE	Filename of the SnP S-Parameter data
FORCE_PASSIVITY	1(Activate or 0 (Disactivate)

## 15.3.14 Linear Dependent Sources

### Voltage-Controlled Voltage Sources (Esrc)

#### Spice syntax

```
EXXXXXXX N+ N- NC+ NC- VALUE
```

#### Examples:

```
E1 2 3 14 1 2.0
```

N+ is the positive node, and N- is the negative node. NC+ and NC- are the positive and negative controlling nodes, respectively. VALUE is the voltage gain.

## Current-Controlled Current Sources (Fsrc)

### Spice syntax

```
FXXXXXXXX N+ N- VNAME VALUE
```

### Examples:

```
F1 13 5 VSENS 5
```

N+ and N- are the positive and negative nodes, respectively. Current flow is from the positive node, through the source, to the negative node. VNAME is the name of a voltage source through which the controlling current flows. The direction of positive controlling current flow is from the positive node, through the source, to the negative node of VNAME. VALUE is the current gain.

## Voltage-Controlled Current Sources (Gsrc)

### Spice syntax

```
GXXXXXXXX N+ N- NC+ NC- VALUE
```

### Examples:

```
G1 2 0 5 0 0.1MMHO
```

N+ and N- are the positive and negative nodes, respectively. Current flow is from the positive node, through the source, to the negative node. NC+ and NC- are the positive and negative controlling nodes, respectively. VALUE is the transconductance (in mhos).

## Current-Controlled Voltage Sources (Hsrc)

### Spice syntax

```
HXXXXXXXX N+ N- VNAME VALUE
```

### Examples:

```
HX 5 17 VZ 0.5K
```

N+ and N- are the positive and negative nodes, respectively. VNAME is the name of a voltage source through which the controlling current flows. The direction of positive controlling current flow is from the positive node, through the source, to the negative node of VNAME. VALUE is the transresistance (in ohms).

## 15.3.15 Linear Independent Sources

### Voltage Sources

#### Spice Syntax

```
VXXXXXXXX N+ N- <DC >DC/TRAN Value> <AC <AC Mag < AC Phase>>>
```

#### Examples:

VCC 10 0 DC 6

VIN 13 2 0.001 AC 1 SIN(0 1 1MEG)

VIN 2 0 PWL 0 0 10n 0 11n 3.3 25n 3.3 26n 0 R

N+ and N- are the positive and negative nodes, respectively. Note that voltage sources need not be grounded.

Voltage sources, being used for circuit excitation, are the 'ammeters' for SPICE, that is, zero valued voltage sources may be inserted into the circuit for the purpose of measuring current. They of course have no effect on circuit operation since they represent short-circuits.

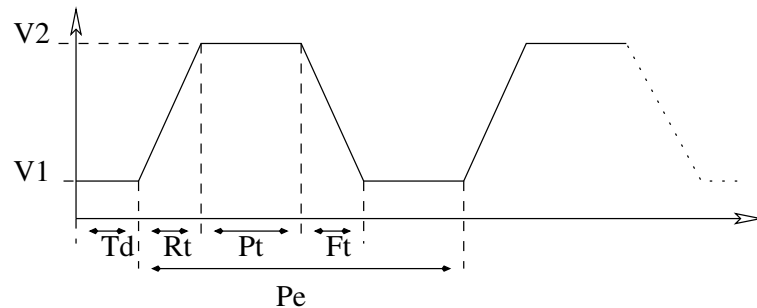
DC/TRAN is the dc and transient analysis value of the source. If the source value is zero both for dc and transient analyses, this value may be omitted. If the source value is time-invariant (e.g., a power supply), then the value may optionally be preceded by the letters DC.

ACMAG is the ac magnitude and ACPHASE is the ac phase. The source is set to this value in the ac analysis. If ACMAG is omitted following the keyword AC, a value of unity is assumed. If ACPHASE is omitted, a value of zero is assumed. If the source is not an ac small-signal input, the keyword AC and the ac values are omitted.

Any independent source can be assigned a time-dependent value for transient analysis. If a source is assigned a time-dependent value, the time-zero value is used for dc analysis.

**Pulse**

PULSE(V1 V2 TD TR TF PW PER)



**Examples:**

VIN 3 0 PULSE(-1 1 2NS 2NS 2NS 50NS 100NS)

parameter	default value	units
V1 (initial value)	-	Volts or Amps
V2 (pulsed value)	-	Volts or Amps
TD (delay time)	0.0	seconds
TR (rise time)	TSTEP	seconds
TF (fall time)	TSTEP	seconds
PW (pulse width)	TSTOP	seconds
PER(period)	TSTOP	seconds

A single pulse so specified is described by the following table:

time	value
0	V1
TD	V1
TD+TR	V2
TD+TR+PW	V2
TD+TR+PW V2	V1
TSTOP	V1

Intermediate points are determined by linear interpolation.

### Sinusoidal

SIN(V0 VA FREQ TD THETA)

### Examples:

VIN 3 0 SIN(0 1 100MEG 1NS 1E10)

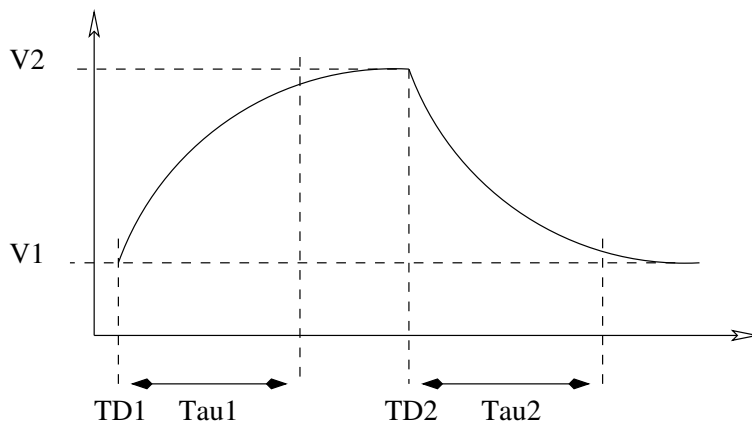
parameters	default value	units
VO (offset)		Volts or Amps
VA (amplitude)		Volts or Amps
FREQ (frequency)	1/TSTOP	Hz
TD (delay)	0.0	seconds
THETA (damping factor)	0.0	1/seconds

The shape of the waveform is described by:

time	value
0 to TD	V0
TD to TSTOP	$V0 + VA \exp\left(\frac{-(t-TD)}{THETA}\right) \sin(2\pi * Freq(t + TD))$

### Exponential

EXP(V1 V2 TD1 TAU1 TD2 TAU2)



### Examples:

VIN 3 0 EXP(-4 -1 2NS 30NS 60NS 40NS)

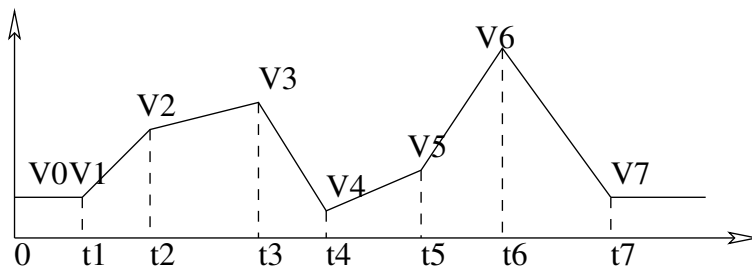
parameter	default value	units
V1 (initial value)	-	Volts or Amps
V2 (pulsed value)	-	Volts or Amps
TD1 (rise delay time)	0.0	seconds
TAU1 (rise time constant)	TSTEP	seconds
TD2 (fall delay time)	TD1+TSTEP	seconds
TAU2 (fall time)	TSTEP	seconds

The shape of the waveform is described by the following table:

time	value
0 to TD1	V1
TD1 to TD2	$V1 + (V2 - V1)[1 - \exp\frac{-(t-TD1)}{Tau1}]$
TD2 to Tstop	$V1 + (V2 - V1)[1 - \exp\frac{-(t-TD1)}{Tau1}] + (V1 - V2)[1 - \exp\frac{-(t-TD2)}{Tau2}]$

### Piece-Wise Linear

PWL(T1 V1 <T2; V2 T3 V3 T4 V4 ...>)



### Examples:

VCLOCK 7 5 PWL(0 -7 10NS -7 11NS -3 17NS -3 18NS -7 50NS -7)

Each pair of values (Ti, Vi) specifies that the value of the source is Vi (in Volts or Amps) at time=Ti. The value of the source at intermediate values of time is determined by using linear interpolation on the input values.

### Pattern Function

PATTERN VHI VLO < TDELAY TRISE TFALL TSAMPLE BITS R >

### Examples:

VPATTERN 1 0 PATTERN 5 0 0 10p 10p 5n 0 1 1 R

Generating a digital like source. The VHI/VHO are used to select the voltages of the digit and the pattern is used for representing the signal.

### Current Sources

#### Spice Syntax

IYYYYYYY N+ N- <<DC <DC/TRAN Value>> <AC <AC Mag <AC Phase>>>

### Examples:



ISRC 1 0 1m

N+ and N- are the positive and negative nodes, respectively. Positive current is assumed to flow from the positive node, through the source, to the negative node. A current source of positive value forces current to flow out of the N+ node, through the source, and into the N- node.

DC/TRAN is the dc and transient analysis value of the source. If the source value is zero both for dc and transient analyses, this value may be omitted. If the source value is time-invariant (e.g., a power supply), then the value may optionally be preceded by the letters DC.

ACMAG is the ac magnitude and ACPHASE is the ac phase. The source is set to this value in the ac analysis. If ACMAG is omitted following the keyword AC, a value of unity is assumed. If ACPHASE is omitted, a value of zero is assumed. If the source is not an ac small-signal input, the keyword AC and the ac values are omitted.

### 15.3.16 Hierarchy declaration

#### Block sub-circuit declaration

In Spice a block can be defined by using the keyword .SUBCKT to define a block or a black box. The syntax of the declaration is as follows:

```
.SUBCKT BNAME PIN1 PIN2 ... PINn <Param1=value> .. <Paramn=value>
* Circuit Description of the subckt *
.ENDS
```

Between these two keywords (.SUBCKT and .ENDS), the user can define any devices, instances or redefine again new blocks.

For example, an inverter can be declared as :

```
.SUBCKT INVERTER IN OUT VDD GND
.MODEL TN NMOS level=3
.MODEL TP PMOS level=3
* MOS instantiation D G S B
m1 OUT IN GND GND TN w=0.25u l=0.25u
m2 OUT IN VDD VDD TP w=0.250u l=0.25u
.ENDS
```

Optional parameters can be declared to set some parameters which are considered as new parameters for the scope of the block.

If a parameter MYPAR=1 is declared on the block definition, it means that inside the block MYPAR is equal to 1, but outside it is not defined.

#### Block sub-circuit instantiation

In Spice, an instance of a block can be defined by using the keyword X/x using the line syntax below:

```
Xname NODE1 NODE2 ... NODEn SUBCKTNAME <Param1=value> .. <Paramn=value>
```

Optional parameters can be used to overwrite the parameters that have been defined for the "SUBCKTNAME" block definition.

### 15.3.17 Parameters

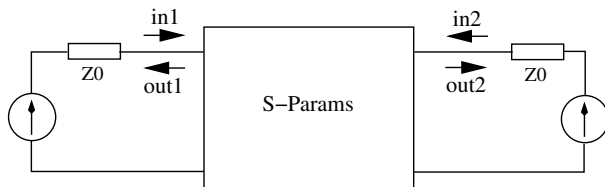
It is possible to define parameters through a command line as follow:

```
.PARAM PARAMNAME=VALUE
```

This syntax can be used as a define statement. It is used when a generic value has to be defined several times.

### 15.3.18 S-Parameter extraction

HyperSpice offers the possibility to extract the S-Parameter data of a circuit. S-Parameters are considered to be the ratio measured between the inflective and the reflective waveforms on each ports of the inputs of the S-Parameter block. The scheme is the following one:



The first step of the S-Parameter extraction is to select the ports inside the circuit that we will be taken into account to make the extraction.

To select a port the user has to put a voltage port declaration. This is needed to select clearly the nodes which will be the S-Parameter block input/output. For this the following syntax is proposed:

```
V<n> PIN_NAME 0 iport=<NUM PORT> rport=<Z0>
```

The user will have to put as many Voltage port declaration as port number of the S-Parameter block. The 'n' number is just an increasing number. The PIN\_NAME value is the circuit node on which a measure will be made and will be considered as a port of the S-Parameter block.

The "iport" is the index port number of the port which will be seen in S-Parameter results.

Rport parameter is the parameter used to specify the impedance which will be used to make the measures. By default this number have to be equal to 50 ohms.

These Voltage ports are just needed so to specify the port input/outputs. We will also have to specify the frequency list which will be used to make the simulation. For this, the user will have to set the AC simulation and give the frequency range of the simulation by using the command:

```
.AC DEC <Nb frequency divider> <Starting frequency> <Stop frequency>
```

The user will have to add the following command to produce the results as outputs of the several simulations

```
.SEXTRACT <output filename>
```

The command will inform the simulator that results of S-Parameter extraction will have to be put on the filename called "output filename". The file will contain data as for an example of 2x2 ports S-Parameter block:

```
\# Hz S M A R 50 ! S(1,1) S(2,1) S(1,2) S(2,2)  
1.0000000000000000E+00 3.3336841518187355E-01 5.8793299979049041E-01 ...
```

```
1.2589254117941673E+00 3.3338893175136475E-01 7.4011447062501234E-01 ...  
...
```

## 15.3.19 Options

### Setting DC algorithm

```
.option DCALGO=<GMIN | GRAMP | VRAMP>
```

Force the DC solver to use a specific algorithm for operating point computation.

GMIN is a method to add a impedance on nodes in order to facilitate convergence

GRAMP and VRAMP are methods to facilitate convergence on increasing voltage on sources

### Force an Initial condition value

The command .IC can be used to force an initial condition on a node.

```
.IC V(node)=0.5
```

For example on the line below, the command is forcing the simulator to use an initial value of 0.5 volt for the node "node".

```
.NODESET V(node)=<value>
```

Performs a 2-pass DC analysis :

- First pass ac .IC (force initial conditions)
- Second pass starts with first pass solution but stops enforcing initial conditions.

```
.GUESS V(node)=<value>
```

Performs first DC iteration with specified initial conditions, but subsequent iterations are made without .GUESS initial conditions.

### Fixing step (STEP)

```
.option STEP=value
```

This option can be used to force the simulator to make fixed step for the transient simulation. It should be used on certain conditions as it is changing the accuracy of the results. By default the fixed step is disabled.

### Setting a max step (HMAX)

```
.option HMAX=value
```

By default the simulator is using a variable time step which is internally determined according to the step computation. In some conditions the user can force the simulation to take a maximum value for a step computation.

### Setting a relative tolerance (RELTOL)

```
.option RELTOL=value
```

This option can be used to change convergence tolerances for all devices. The default value is 1e-3

### Setting Simulation Temperature

```
.option TEMP=value
```

This option can be used to change the temperature of the simulation. By default the temperature is set to 300.15 K.

### .option VNTOL

```
.option VNTOL=value
```

This option can be used to change the convergence tolerance for node voltages only. The default value is set to 1e-6V

### .option ITOL

```
.option ITOL=value
```

This option can be used to change the convergence tolerance for node voltages only. The default value is set to 1e-5A

### .option BE

```
.option BE
```

This option can be used to change the integration scheme method. By default HyperSpice is managing the integration method internally according to the solver results. It is possible to fix it to a more simple scheme like BE, to facilitate convergence on some circuits.

## 15.4 Known Issues

```
Error: Warning: No ground node is defined
```

This message means that ground node (implicit node '0') isn't connected to any device, resulting in an unsolvable equation set. At least one device should be connected to ground node. The user should then declare at least the following command:

```
Vgnd gnd 0 0
```

## 15.5 SpiceCustomBlock

In **Activate** the user can enter and simulate Spice netlists using the `SpiceCustomBlock` block [15.1](#) located in `CustomBlocks` palette.



Figure 15.1: SpiceCustomBlock available in CustomBlocks palette.

In the block GUI the netlist, as well as input and output nodes of the netlist, can be defined. See Fig. [15.2](#).

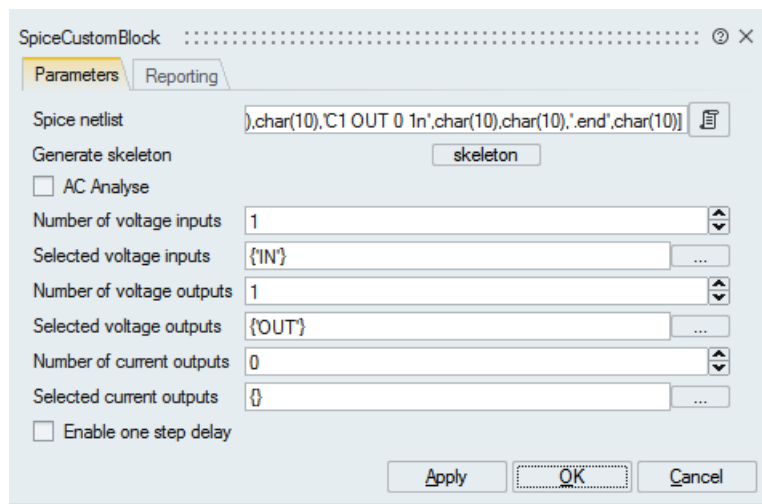



Figure 15.2: GUI of the SpiceCustomBlock block.

### Netlist editor

In `SpiceCustomBlock`, the user can edit the Spice netlist. In order to open the editor, click on the icon . The netlist editor ([15.3](#)) opens and the user can edit the Spice netlist.

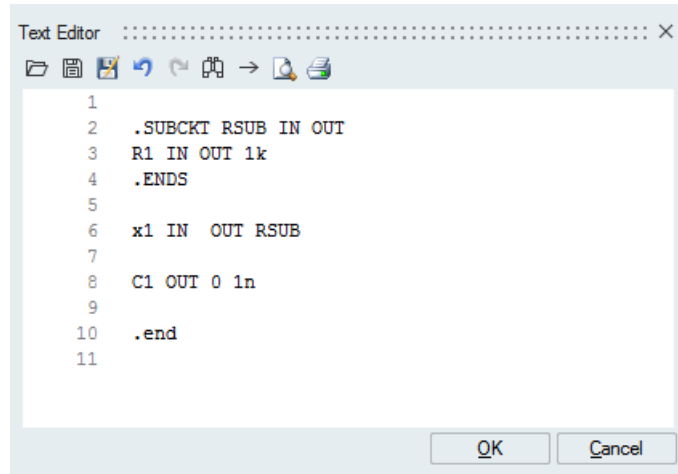


Figure 15.3: Netlist editor.

As an example, suppose we need to simulate a simple RC circuit, as shown in Fig.15.4.

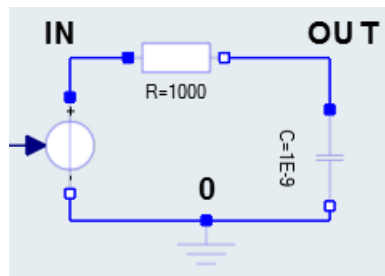



Figure 15.4: The simple RC circuit to be simulated using SpiceCustomBlock.

The corresponding netlist of this circuit is shown in Fig.15.3. This circuit is composed of two components **R** and **C** and three nodes **IN**, **OUT**, **0**. The input signal voltage is supposed to be defined by standard **Activate** blocks. In order to allow communication between Spice netlist and other standard **Activate** blocks, input and output nodes of the netlist should be selected. click on the icon  , to display the port selection GUI, as shown in Fig.15.5.

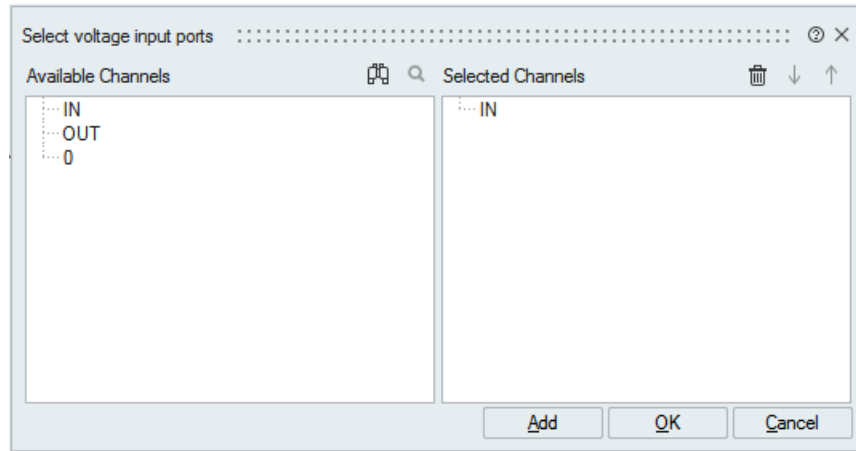


Figure 15.5: input/output port selection interface.

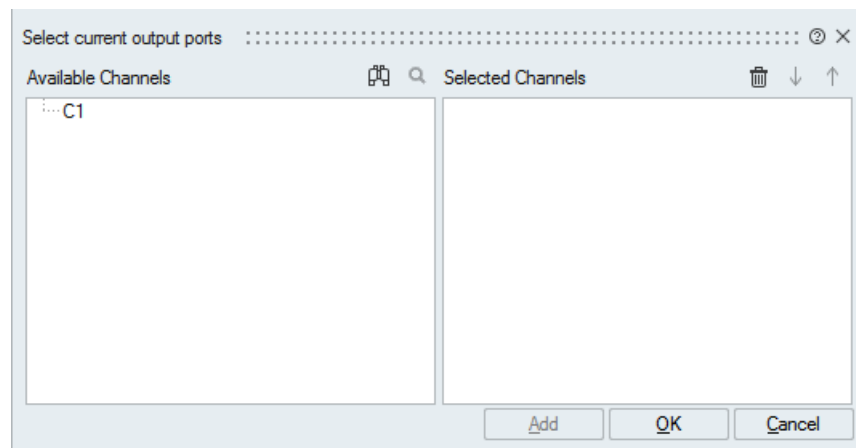


Figure 15.6: Selection of components for current output port.

## Input/output port selection

SpiceCustomBlock allows for choosing three types of input and output ports.

1. input voltage port: If a node is chosen as input, the voltage of that node is provided by the corresponding input port of the SpiceCustomBlock. In the RC example, the port **IN** is an input, see Fig.15.5.
2. output voltage port: If a node is chosen as output, the voltage signal of that node is available at the corresponding output port of the SpiceCustomBlock. This signal is then available inside **Activate** for example to be displayed by Scopes. In the RC example, the port **OUT** can be chosen as an output port to be displayed.
3. output current port: The block can also provide the current passing through components. In order to choose current signal, the corresponding component should be chosen, see Fig.15.6. For example in order to choose the current passing through a resistor, that resistor should be selected in the GUI.

Once input and output voltage nodes are selected, click on the OK button and the corresponding input and output ports will be added to the block.

A complete model for the RC can be made by defining the input signal, *i.e.*, the voltage signal at node **IN**, and connecting the output port of the SpiceCustomBlock to a Scope block. In order to compare the results, the circuit corresponding to the RC circuit is made with Modelica blocks in **Activate** and is displayed in the Scope, as shown in Fig.15.7. The circuit is excited by a sinusoid signal. The output voltage is shown in Fig.15.8. Note that output voltages of the Spice netlist and the Modelica RC circuit are identical and superposed in Fig.15.8.

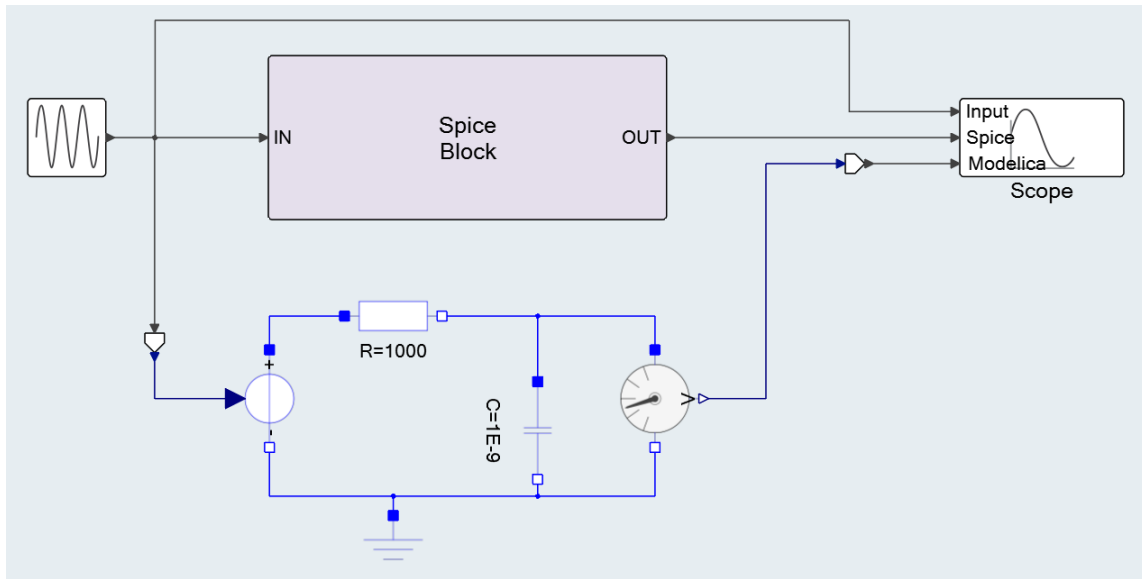


Figure 15.7: The complete Spice model and circuit counterpart in Modelica.

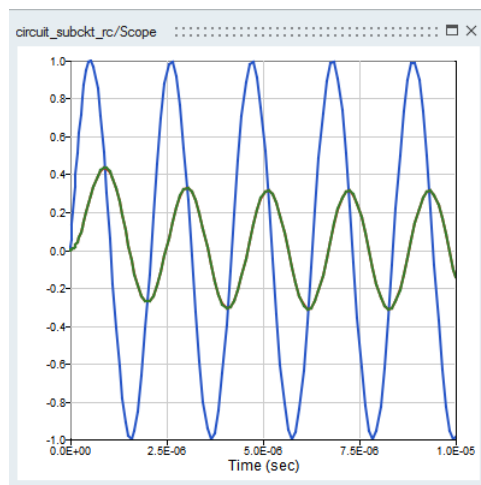


Figure 15.8: Simulation result of 15.7 in **Activate**.



## AC analysis

SpiceCustomBlock has an important feature to perform the frequency analysis of a Spice netlist. In order to obtain the frequency response of a Spice netlist, check on the **AC Analysis** checkbox in the block GUI, see Fig.15.2. If this checkbox is selected, an input Frequency port is added to the block. This input defines the frequencies at which the Spice circuit should be excited. For each frequency the output frequency response is computed. Note that in the AC analysis the input and output ports are complex numbers.

Consider this netlist representing an low-pass filter circuit.

```
*LPFILTER.CIR - SIMPLE RC LOW-PASS FILTER
*
VS 1 0 AC 1 SIN(1 1 2KHZ)
*
R1 1 2 1K
C1 2 0 0.032UF
*
* ANALYSIS
.AC DEC 5 10 10MEG
.TRAN 5US 500US
*
* VIEW RESULTS
.PRINT AC VM(2) VP(2)
.PRINT TRAN V(1) V(2)
*
.PROBE
.END
```

In order to obtain the frequency response of this circuit, an Activate model is built, as shown in Fig.15.9.

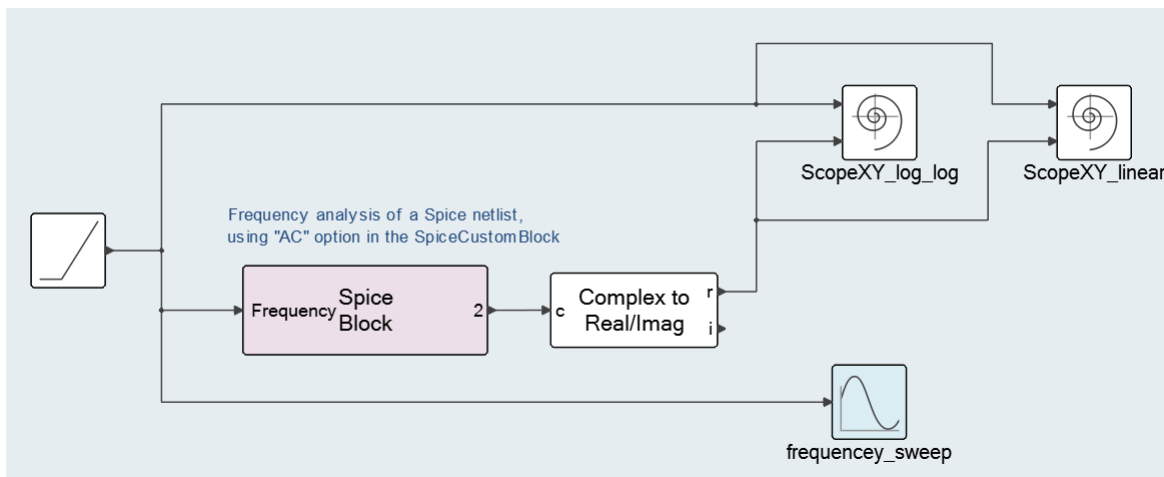


Figure 15.9: An Activate model to compute the frequency response of a Spice netlist.

The SpiceCustomBlock block GUI is shown in Fig.15.10. The block has one only output port (*i.e.*, node 2). The block has no input port.

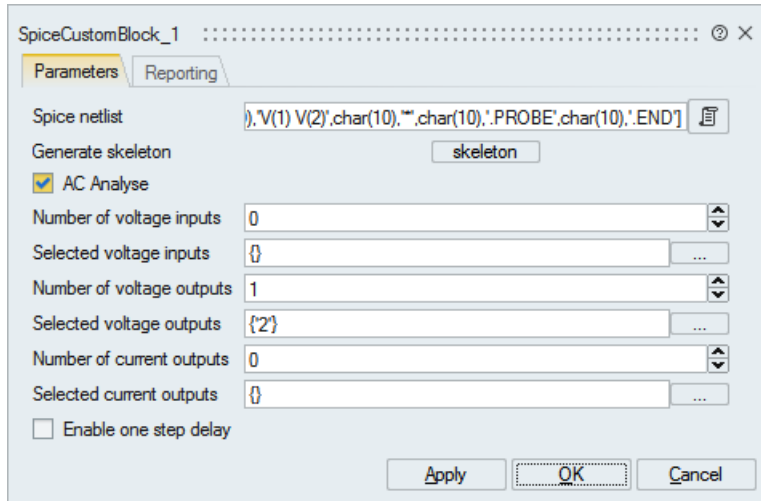


Figure 15.10: An Activate model to compute the frequency response of a Spice netlist.

A **Ramp** block has been used to sweep the frequency. The frequency is started from 1Hz and is increased with a rate of 1KHz/Sec. The output of the ramp block is connected to the Frequency input of the SpiceCustomBlock block. The filter output (*i.e.*, node 2) is complex, so a **Complex to Real/Imag** block is used to separate the complex and imaginary parts of the frequency response. The real part of output frequency response with respect to the input frequency is shown in Fig.15.11. Note that frequency axis is display on a logarithmic scale.

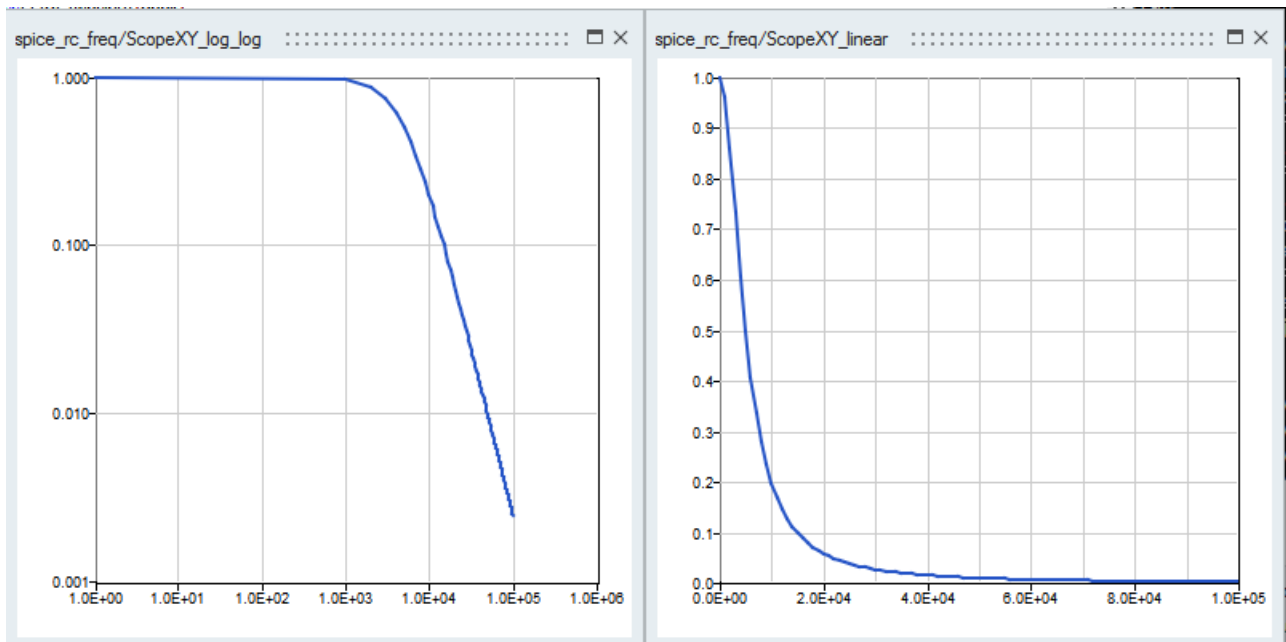


Figure 15.11: Frequency response of the low-pass filter for the model in Fig.15.9. Left plot is in Log-Log scale and the right plot is Linear-Log scale

## Reporting

When a simulation of a Spice netlist fails, it is helpful to generate a logfile to find out the reason of the failure. In order to create a logfile, click on the **Reporting** tab in the block GUI as shown in Fig. 15.12.

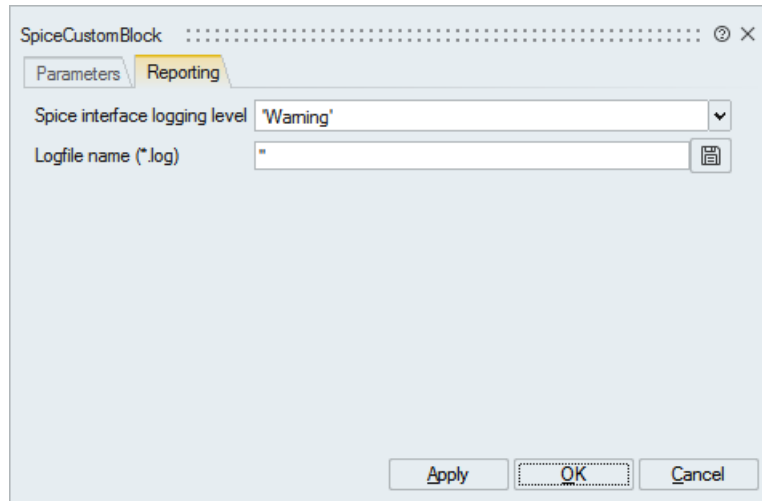


Figure 15.12: GUI of the SpiceCustomBlock block for the report generator.

In the Spice interface logging level, the user can choose one of `Nothing`, `Error`, `Warning`, `Info`.

1. `Nothing`: No message is logged at all. This turns off the logger.
2. `Error`: Only error messages are logged.
3. `Warning`: Only `Warning` and error messages are logged.
4. `Info`: Every message emitted by the HyperSpice solver is logged.

The log filename should be specified. If a relative path is used, the file is created next to the model.



## Chapter 16

# Hybrid Automata in Altair Activate

### 16.1 Abstract

Hybrid automaton is a standard model for describing a hybrid system. A hybrid automaton is a state machine augmented with differential equations and is generally represented by a graph composed of vertices and edges where vertices represent continuous activities and edges represent discrete transitions. Modeling a hybrid automaton with large number of vertices may be difficult, time-consuming and error prone using standard modules in modeling and simulation environments such as **Activate**. In this section, we present the `Automaton` block used for modeling and simulation of hybrid automata.

### 16.2 Introduction

Hybrid systems define a very wide class of dynamical systems that involve the interaction of heterogeneous data types and dynamics especially the interaction of continuous-time dynamics described by differential equations, with discrete dynamics described by a finite state under automata or other models of computation. Examples include mechanical systems with collisions, circuits with ideal diodes and switches, chemical processes controlled by valves or pumps, and most importantly, embedded computation systems where digital devices interact with an analog environment.

Early works on formal models for hybrid systems includes phase transition systems and hybrid automata. A hybrid automaton is a state machine augmented with differential equations. It is a standard model for describing a hybrid system. Hybrid automata come in several forms: The Alur-Henzinger hybrid automaton is a popular model that was developed primarily for algorithmic analysis of hybrid systems model checking [1, 2, 3]. In this document we will explain the way **Activate** environment can be used to model and simulate hybrid systems, and in particular, hybrid automata.

Hybrid automata have been introduced by Xavier Nicollin *et al.* in [5] and an analysis of linear hybrid automata are given in [2, 4]. They modeled a hybrid system as a finite automaton that is expanded with a set of real valued variables. These variables can be tested and modified at transitions. At an automaton mode the value of variables change continuously with time according to evolution laws which are associated with the mode. The transition relations are specified by guarded commands; the activities by differential equations; and the invariants by logical formulas. We now present the relevant definition of a hybrid automaton from Henzinger's theory [4] that will be used in the construction of our

formal semantics for **Activate** automaton block.

$H$  is a tuple which is defined as:

$H = (V, E, X, F, Invariant, Initial, Jump)$ , where

- $V$  is the set of control modes  $\{v_1, \dots, v_M\}$  where  $M$  is the number of control modes;
- $E$  is the set of control switches which identify the source mode and a destination mode during a transition.
- $X$  is a set of real-valued variables  $\{x_1, \dots, x_N\}$  where  $N$  is the dimension of  $H$ .  $\dot{X}$  is first time derivative of  $X$ ;
- $F$  is a predicate over  $\{X, \dot{X}\}$  assigned to each control mode; *Invariant* defines the admissible range for  $\{X, \dot{X}\}$  in each mode;
- *Initial* defines initial values of  $\{X, \dot{X}\}$  in each mode.
- *Jump* is an edge labeling function which assigns a predicate over  $\{X, \dot{X}\}$  to each edge.

As an example consider the model of the bouncing ball in (16.1).

$$\begin{cases} \dot{x} = v \\ \dot{v} = -g \end{cases} \quad (16.1)$$

The model of this bouncing ball has been implemented in **Activate** as shown in Fig. 16.1.

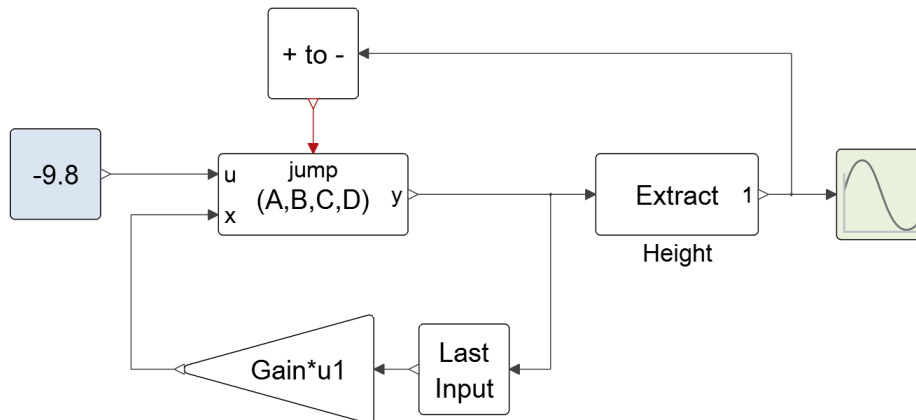


Figure 16.1: The model of a control system in **Activate**

The output of the block `JumpStateSpace` is a vector of size two consisting of  $[x, v]^t$ . Whenever the condition  $(x < 0)$  is satisfied, an event is generated by the `Zero-crossing (+ to -)` block. This event makes the `Jump` block do a reinitialization, *i.e.*,  $v := -0.9v$  and  $x := x$ . In Fig. 16.1, the `Clock` block generates periodic events (activation signals) to activate the `Scope` block which displays  $x$  and  $v$ .

This hybrid system can also be modeled as an automaton with a single control mode. The graphical representation of this automaton is depicted in Fig. 16.2. In this model, there are two variables  $x$  and  $v$  whose initial values are given at  $t = 0$ . There are also two event sources; a zero-crossing event and a periodic time event. Only the zero-crossing event causes discontinuity in variables.

Although it is often possible to model hybrid automata with generic **Activate** blocks, it is not an easy and efficient way to develop a hybrid automaton especially when the number of control modes is high. In a hybrid automaton composed of several control modes, at any time instant, only one mode or

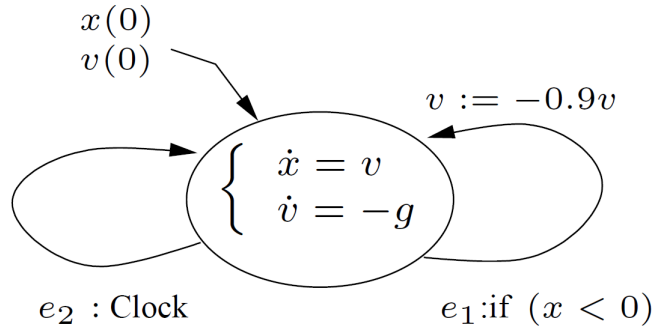


Figure 16.2: Automaton for the bouncing ball

subsystem is active and the others should stay inactive. In **Activate**, even though it is possible to generate an activation signal to activate conditionally a subsystem via `If-Then-Else` blocks, the variables are still present in the state vector of the model and reduce the performance of the numerical solver. Another difficulty is in detecting the jumps; in each control mode of the hybrid automaton, zero-crossing functions should be evaluated for monitoring the jump conditions. If we develop a hybrid automaton with several subsystems activated by `If-Then-Else` blocks, all zero-crossing functions are active and it will be difficult to distinguish the zero-crossing functions that have to be monitored. Using switches and selector blocks, to activate or deactivate the zero-crossings, increases the model size and reduces the simulation speed.

**Activate** is a hybrid system simulator which means that **Activate** can simulate mixed continuous-time and discrete-time subsystems. Developing hybrid automata containing large number of modes and continuous-time states [using generic **Activate** blocks] is a cumbersome, time-consuming, and difficult task. In this section, we present the `Automaton` block used for modeling and simulation of hybrid automata. This block provides a modular way to model a general hybrid automaton in **Activate**. This block provides an interface to model and encode the graphical representation of an automaton to be used by the simulator and consequently by the numerical solver in a completely transparent way.

### 16.3 Automaton Block

The automaton block provides a switching mechanism between subsystems corresponding to control modes of the automaton. Subsystems are constructed in such a way that they receive the state vector as input from the automaton block and compute the flow and jump functions. The state variables are defined only in the automaton block and not in the subsystems.

Suppose that a hybrid automaton consists of  $M$  control modes. The continuous-time dynamics in mode ' $i$ ' is defined with DAE ( $0 = F_i(\dot{x}, x, u)$ ) where  $i \in \{1 \dots M\}$  and the dimension of  $x$  is  $N$  ( $N \geq 0$ ) for any  $i \in \{1 \dots M\}$ , as shown in Fig. 16.3. Suppose that in control mode ' $i$ ', there are  $Z_i$  jump conditions indicating jumps toward other modes. The jump conditions are defined by  $Jump_{ij}(\dot{x}, x, u)$  functions where  $j \in \{1 \dots Z_i\}$ . When a jump function becomes positive a mode transition will happen. When a transition to mode ' $k$ ' happens state vector  $x$  is reset to  $Reset_k(\dot{x}, x, u)$ , i.e.,  $x_l = Reset_{kl}(\dot{x}, x, u)$  for  $l \in \{1 \dots N\}$ . Note that the reset function is identical for all incoming edges of mode  $k$ . In order to develop an automaton containing a mode with multiple reset functions, the value of the current and previous active modes should be used. These values are available at the first output port of the block.

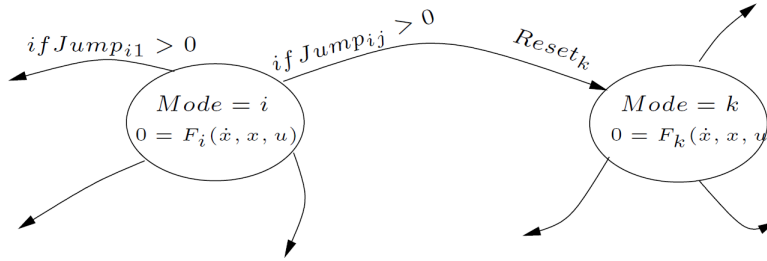


Figure 16.3: A graphical representation of an automaton.

For such a hybrid automaton, we can use the `Automaton` block of **Activate**. This block has the following input/output ports, see Fig. 16.4.

- **Output 1:** The first output port is a vector of size two consisting of the current and the previous active control modes.
- **Output 2:** The second output port is a vector of size  $2N$  providing the state vector and its first time derivative, *i.e.*,  $[x, \dot{x}]^t$ .
- **Inputs:** The automaton block has  $M$  vector input ports corresponding to  $M$  modes or subsystems of the automaton. The input port ' $i$ ' which is the output of the  $i^{th}$  subsystem of the automaton defines the dynamic behavior in the control mode ' $i$ '. This input port is a vector of size  $2N + Z_i$ , as indicated in (16.2). The diagram in Fig.16.4 shows the way subsystems are connected to the automaton block: Using an automaton block defined with  $M$  modes and  $N$  continuous-time states. The automaton block has therefore  $M$  input ports, two output ports of size 2 and  $2N$ , and an event output port.

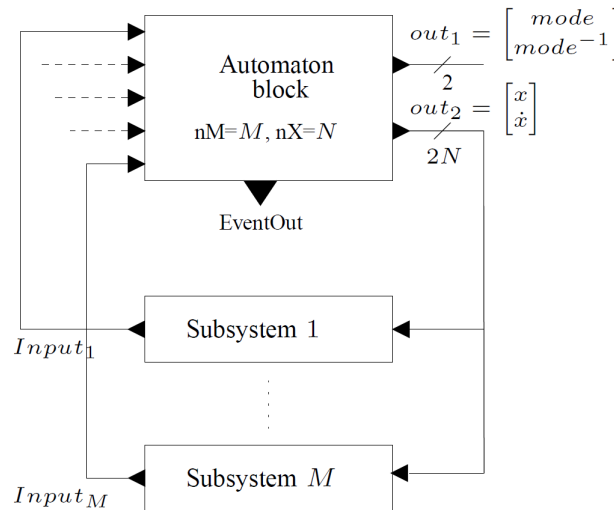


Figure 16.4: Automaton with M modes and N ports

$$Input_i = \begin{bmatrix} F_i(\dot{x}, x, u) \\ Reset_i(\dot{x}, x, u) \\ Jump_i(\dot{x}, x, u) \end{bmatrix} \quad (16.2)$$



- The first  $N$  elements of the  $Input_i$  are the continuous-time dynamics. The dynamics of the system in the control mode ' $i$ ' is described by a smooth index-1 DAE (i.e.,  $0 = F_i(\dot{x}, x, u, t)$ ).
  - The next  $N$  elements of  $Input_i$  are the values used to reset the continuous-time states when a transition to control mode ' $i$ ' is activated.
  - The next  $Z_i$  elements of  $Input_i$  are the jump or zero-crossing functions. If the  $j^{th}$  zero-crossing function of mode ' $i$ ' crosses zero with negative to positive direction, a transition to ' $j^{th}$ ' destination mode happens.
- **Event Output:** This is an event output port, which is activated whenever a mode transition happens. This event is useful when an event is needed to activate or initialize a part of the subsystem not included in the internal dynamics of the automaton block.

In order to parametrize the Automaton block, the user should double click on the block. Then the block dialog opens and the user can enter the block parameters, as shown in Fig. 16.5.

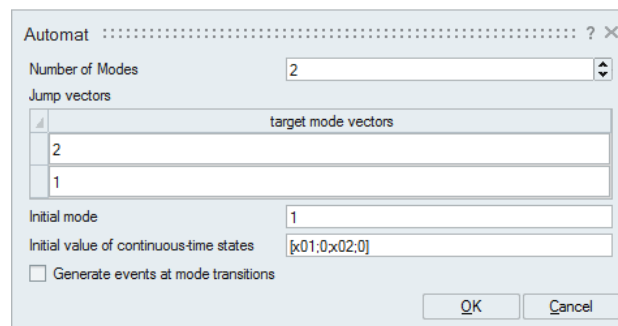


Figure 16.5: Block dialog of the Automaton block

In the block dialog, the number of control modes, the initial control mode and the initial value of continuous-time states at the beginning of the simulation should be given.  $Xproperties$  vector, coded in an  $M \times N$  matrix, indicates whether a continuous-time state is algebraic or differential in each control mode. If a state is differential, its  $Xproperty$  should be set to "+1", otherwise it should be "-1".  $Xproperty$  can be given as a  $1 \times N$  vector if  $Xproperties$  of the states in all modes are identical.

The  $Jump$  fields express the mode transition information. Suppose that all control modes are labeled from 1 to  $M$ . Then, in the field corresponding to control mode ' $i$ ', destination modes of mode ' $i$ ' are defined in a vector.  $j^{th}$  element of this vector gives the destination mode when  $j^{th}$  jump function ( $Jump_{ij}(\dot{x}, x, u)$ ) becomes positive. For example, if in the field of the mode '2', the user defines [1; 3; 4], it means that in mode '2', there are three active jump functions. When, for example, the third jump function becomes positive, a mode transition to mode '4' will be activated.

In order to illustrate the utility of this block for simplifying the development of hybrid automata models, the rest of this section is devoted to the implementation of two automata examples in **Activate**.

### 16.3.1 Example 1: Pendulum Swing Up

The inverted pendulum is a popular system for illustrating control problems. In [6], the hybrid control strategy for swinging up a pendulum from a downward to an upright position has been presented. Here we will explain the way the pendulum and the control strategy can be modeled (and thus simulated) in **Activate**, using a hybrid automaton block.

Let  $x_1$  be the angle between the vertical axis and the pendulum,  $x_2$  the angular velocity, and  $u$  the applied acceleration, which we consider as the control signal. Then, the dynamics of the system is given by

$$0 = f(\dot{x}, x, u) = \begin{cases} -\dot{x}_1 + x_2 \\ -\dot{x}_2 + ml(g \sin(x_1) - u \cos(x_1))/J \end{cases} \quad (16.3)$$

where  $g, m, l, J$  are the acceleration of gravity, the mass, the length, and the moment of inertia of the pendulum. It can easily be shown that the input control force  $u$  defined with (16.4)

$$u = \frac{\sin(x_1) + x_1 + x_2}{\cos(x_1)}g \quad (16.4)$$

brings the pendulum to the upward position with a single swing. Since the force of the pivot ( $u$ ) is limited to  $[-u_m, u_m]$ , the control force defined with (16.4) cannot be used. As a result, in order to bring the pendulum up, more than one swing may be needed.

A swing-up control strategy suggested by Astrom and Furuta is based on a bang-bang control strategy controlling the energy of the pendulum [6]. Assuming zero potential energy in the upward position, the total energy of the pendulum  $E$  is given by:

$$E = J\frac{x_2^2}{2} + mgl(\cos(x_1) - 1) \quad (16.5)$$

Starting from a negative energy in down position, the control strategy should bring up the pendulum to zero energy. To change the energy as fast as possible, the magnitude of the control signal should be as large as possible. The pendulum energy will become zero, if

$$\frac{dE^2}{dt} = -mluEx_2 \cos(x_1) < 0 \quad (16.6)$$

This condition is fulfilled with the control law (16.7), which drives  $\|E\|$  to zero.

$$\begin{aligned} u &= u_m \text{sign}(\beta) \\ \beta(x_1, x_2) &= Ex_2 \cos(x_1) \end{aligned} \quad (16.7)$$

The control law (16.7) results in chattering. Chattering can be avoided by using a *saturation* function instead of the *sign* function, *i.e.*, the control law (16.8).

$$u = u_m \text{sat}(k\beta) = \begin{cases} -u_m & k\beta < -1 \\ k u_m \beta & -1 < k\beta < 1 \\ u_m & 1 < k\beta \end{cases} \quad (16.8)$$

The parameter  $k$  determines the region where the strategy is linear. In practice,  $k$  is determined by the noise levels on the measured signals. When the pendulum arrives near the upward position, *i.e.*,  $\cos(x_1) > c_1$ , the nonlinear controller (16.4) is applied to take over the control of the pendulum. This pendulum system can be modeled with four control modes as shown in Fig. 16.6. Since following each jump the states are re-initialized to last value of states in the previous mode, the initialization equations are not shown.

The block dialog use to parametrize the Automaton block for this automaton is given in Fig. 16.5. The **Activate** model of this system is given in Fig. 16.7. There are four subsystems handled by this

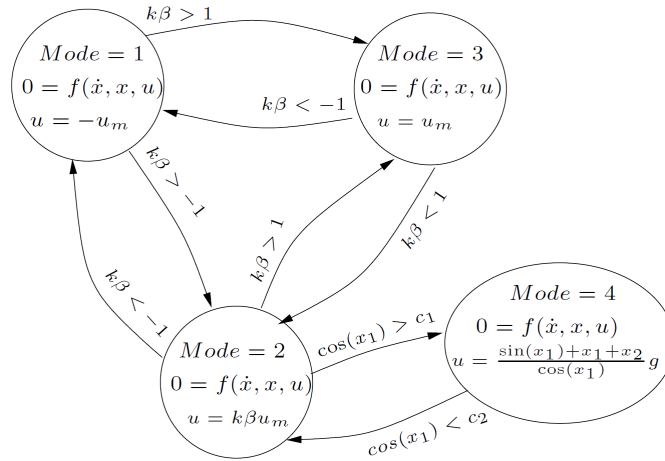


Figure 16.6: Hybrid control strategy to swing up and stabilize an inverted pendulum on a cart

automaton block. As an example, the superblock SB2 corresponding to the control mode-2 is given in Fig. 16.8.

The simulation result for a pendulum with  $mgJ/J = 1$ ,  $u_m = g/4$ , and  $x(0) = [\pi, 0]^t$  is shown in Fig. 16.9. The evolution of control modes and the pendulum angle and angular velocity ( $x_1$  and  $x_2$ ) are given in the first and the second subplots, respectively.

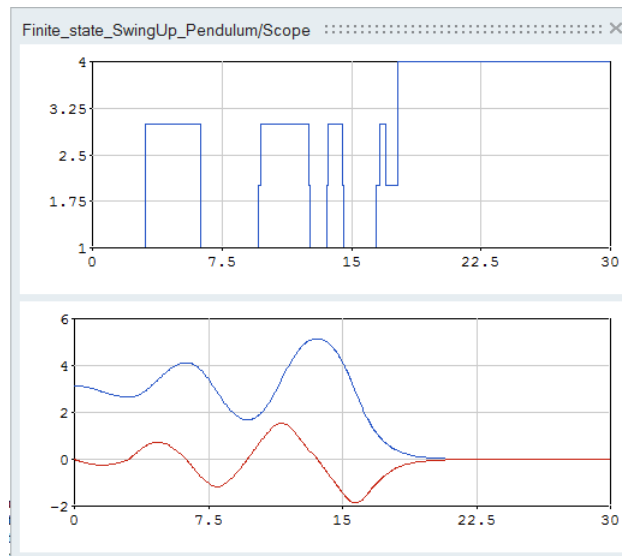


Figure 16.9: Simulation result for the model of Fig. 16.7

### 16.3.2 Example 2: DC/DC Buck Converter

Another hybrid system that shows an interesting dynamical behavior is the DC/DC buck converter whose schematic circuit diagram is shown in Fig. 16.10. A buck converter is a step-down DC to DC power supply composed of two electronic switches (a transistor and a diode), an inductor, and a capacitor [7].

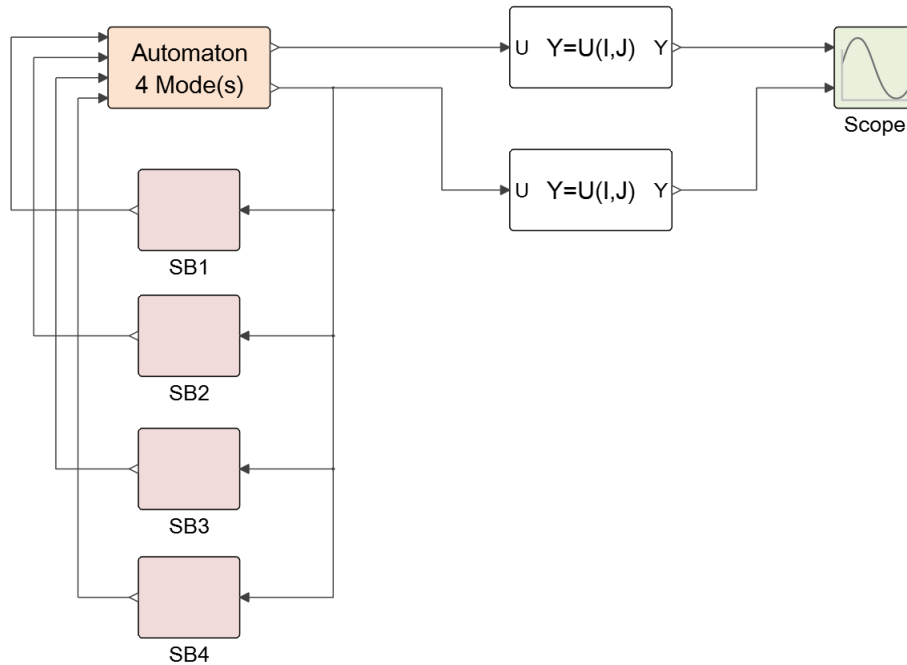


Figure 16.7: A model for swing up a pendulum (Finite\_state\_SwingUp\_Pendulum.scm)

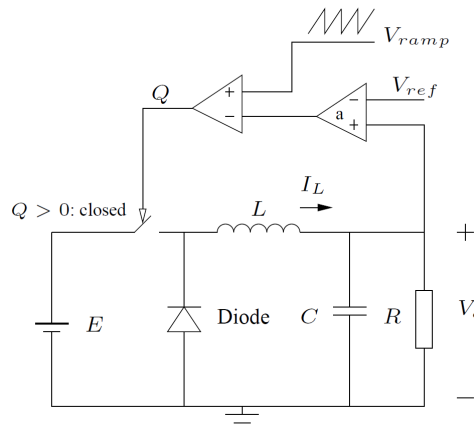


Figure 16.10: Voltage-mode controlled Buck DC/DC converter

The operation of the buck converter is fairly simple. It alternates between connecting the inductor to source voltage to store energy in the inductor and discharging the inductor into the load. Depending on the switches' status, the circuit has three configurations, thus three modes of operations. The first mode is active when the switching condition  $Q > 0$  is satisfied (see (16.9d) or Fig. 16.10). In this case, the switch is closed and the diode is conducting. The second case is when the switch is open and the current in the inductor is positive. The positive current allows the diode to conduct. The last case is when in the second mode, the current through the inductor falls to zero. In this case, the inductor is completely discharged and the diode does not conduct. In order to implement this system with the automaton block, an automaton graph with three control modes is drawn; see Fig. 16.11. In each mode a different linear continuous-time dynamics defines the behavior of the circuit. Taking the state vector

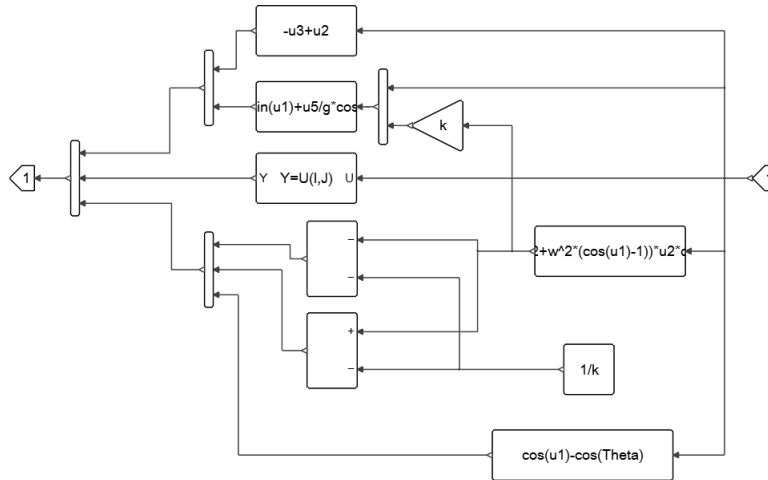


Figure 16.8: Superblock SB2 in the model of Fig. 16.7

as  $x = [V_o, I_L]^t$ , we get (16.9a-16.9c). Note that the Xproperties of the states evolve:  $x_2$  is a differential variable in mode 1 and mode 2, and an algebraic variable in mode 3.

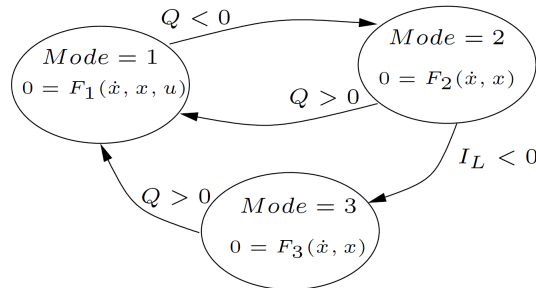


Figure 16.11: Hybrid automaton for a DC/DC Buck converter

$$0 = F_1(\dot{x}, x, u) = \begin{cases} RC\dot{x}_1 + x_1 - Rx_2 \\ L\dot{x}_2 + x_1 - E \end{cases} \quad (16.9a)$$

$$0 = F_2(\dot{x}, x) = \begin{cases} RC\dot{x}_1 + x_1 - Rx_2 \\ L\dot{x}_2 + x_1 \end{cases} \quad (16.9b)$$

$$0 = F_3(\dot{x}, x) = \begin{cases} RC\dot{x}_1 + x_1 \\ x_2 \end{cases} \quad (16.9c)$$

$$Q = V_{ramp} - (x_1 - V_{ref})a \quad (16.9d)$$

The hybrid automaton of Fig. 16.11 has been implemented in **Activate** as given in Fig. 16.12.

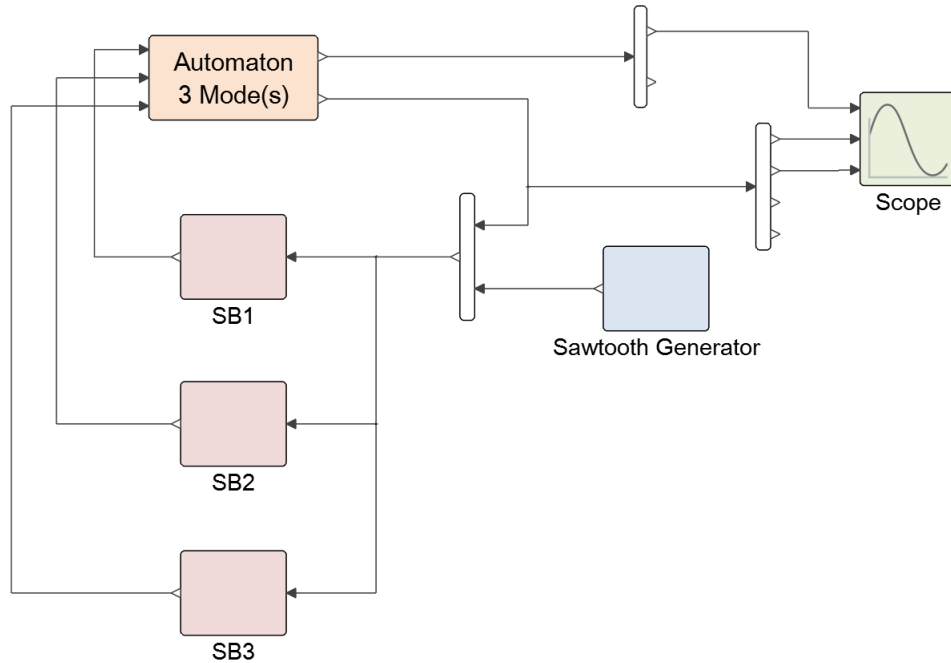


Figure 16.12: Model (Finite\_state\_Buck.scm) for automaton given in Fig. 16.11

For the circuit, we have used the parameter values proposed in [8].

$$\begin{aligned}
 L &= 20mH, & C &= 47\mu F, & R &= 110\Omega, \\
 a &= 8.4, & V_U &= 8.2v, & V_L &= 3.8v, \\
 V_{ref} &= 11.3v, & E &= 20v, & T &= 400\mu S
 \end{aligned}
 \tag{16.10}$$

Depending on the value of  $R$ , the circuit operates in two or three operating modes. The simulation result is given in Fig. 16.13. The first subplot is the evolution of the control mode during the simulation, the second subplot shows the output voltage, and the third subplot is the current through the inductor. It is interesting to note that the circuit exhibits a chaotic behavior as explained in [8, 9].



Figure 16.13: Simulation result for the model of Fig. 16.12

### 16.3.3 Example 3: Sticky balls

In this example ([10]), we consider two (sticky) masses on a flat frictionless table. As shown in Fig. 16.14. Each of the balls is attached to a spring with distinct spring coefficient. If both balls are free, they will swing back and forth. However, if the two balls hit each other, they stick together. The stickiness is assumed to be exponentially decaying after they hit, and the decay speed is controlled by the parameter "Stickiness Decay" So after a while, they will separate.

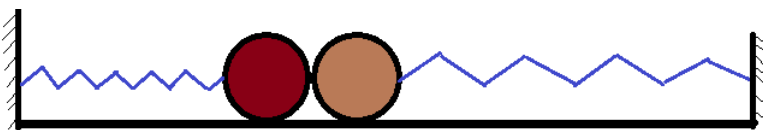


Figure 16.14: Schematic of two sticky balls

The dynamical equation describing the movement of two masses are given in 16.11 and 16.12.

$$\begin{cases} \dot{x}_1 &= v_1 \\ m_1 \dot{v}_1 &= -k_1(x_1 - a_1) \end{cases} \quad (16.11)$$

$$\begin{cases} \dot{x}_2 &= v_2 \\ m_2 \dot{v}_2 &= -k_2(x_2 - a_2) \end{cases} \quad (16.12)$$

where  $x_1$ ,  $x_2$ ,  $v_1$ , and  $v_2$  are position and velocity of masses. Whenever the condition ( $x_1 \geq x_2$ )

becomes true two balls stick together. The dynamical equation of the stuck masses is given in 16.13.

$$\begin{cases} \dot{x}_3 = v_3 \\ m_3 \dot{v}_3 = -k_3(x_3 - a_3) \end{cases} \quad (16.13)$$

where  $x_3$  and  $v_3$  are position and velocity of the stuck balls and

$$\begin{cases} m_3 = m_1 + m_2 \\ k_3 = k_1 + k_2 \\ a_3 = \frac{k_1 a_1 + k_2 a_2}{a_1 + a_2} \end{cases} \quad (16.14)$$

The stickiness force is defined with the equation

$$\dot{s} = -Cs$$

where  $C$  is the decay constant. At the sticking instant ( $t_e$ ),  $x_3$ ,  $v_3$ , and  $s$  are reinitialized to

$$\begin{cases} x_3(t_e) = x_1 \\ v_3(t_e) = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \\ s(t_e) = s_0 \end{cases} \quad (16.15)$$

where  $s_0$  is a constant. The stuck balls is subjected to two forces, the stickiness  $s$  and the separation force which equal to

$$F = k_1(x_1 - a_1) - k_2(x_2 - a_2)$$

Once  $s < F$  becomes true, two balls separate. At the separation instant, states are reinitialized as follows.

$$\begin{cases} x_1(t_e) = x_2(t_e) = x_3(t_e) \\ v_1(t_e) = v_2(t_e) = v_3(t_e) \end{cases} \quad (16.16)$$

The sticking balls can be modeled as a finite state system or an automation. There are two modes, separate and stuck as shown in Fig.16.15.

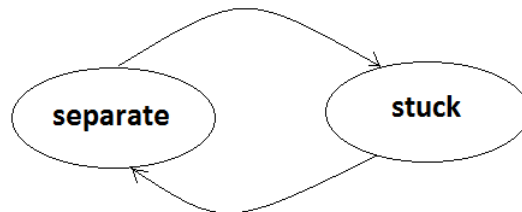


Figure 16.15: Hybrid automaton for sticky balls

In order to model this system in **Activate** with an automaton block, the automaton block should be parametrized as shown in Fig.16.16.



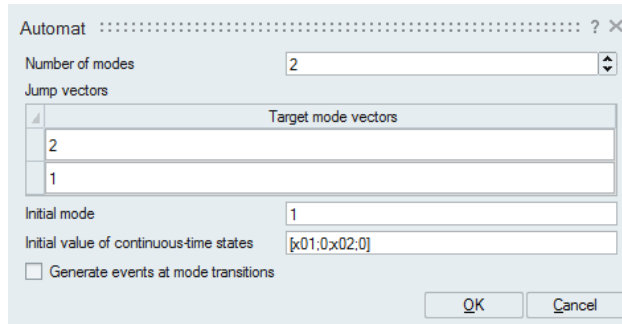


Figure 16.16: Parameterizing the Automaton block for the sticky masses model.

There are two modes and the continuous-time state size is four. Note that in this model number of states changes in each mode.

In the "separate" mode, the state vector  $X$  is composed of  $[x_1, v_1, x_2, v_2]$ . The residual vector delivered to the first port of the Automaton block is 16.17.

$$0 = \begin{cases} -\dot{x}_1 + v_1 \\ m_1 \dot{v}_1 + k_1(x_1 - a_1) \\ -\dot{x}_2 + v_2 \\ m_2 \dot{v}_2 + k_2(x_2 - a_2) \end{cases} \quad (16.17)$$

When a transition from "stuck" mode to "separate" model happens, the state vector  $X$  is reinitialized to  $[x_3, v_3, x_3, v_3]$ . The zero crossing function in the separate mode is:

$$g_1 = x_1 - x_2 \quad (16.18)$$

In the "stuck" mode, the state vector  $X$  is composed of  $[x_3, v_3, s, s]$ . The residual vector delivered to the second port of the Automaton block is 16.19. The number of states of the block cannot change, as a result, in mode "stuck" we need a dummy state ( $q$ ).

$$0 = \begin{cases} \dot{x}_3 = v_3 \\ m_3 \dot{v}_3 = -k_3(x_3 - a_3) \\ \dot{s} = -Cs \\ \dot{q} = -q \end{cases} \quad (16.19)$$

When a transition from "separate" mode to "stuck" model happens, the state vector  $X$  is reinitialized to  $[x_1, \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}, s_0, 1]$ . The zero crossing function in the separate mode is :

$$g_2 = k_1(x_1 - a_1) - k_2(x_2 - a_2) - s \quad (16.20)$$

The model is built in **Activate** using the an automaton block, the block diagram of the system is shown in Fig.16.17. The simulation result illustrating the position of masses ( $x_1$  and  $x_2$ ) is given in Fig.16.18.

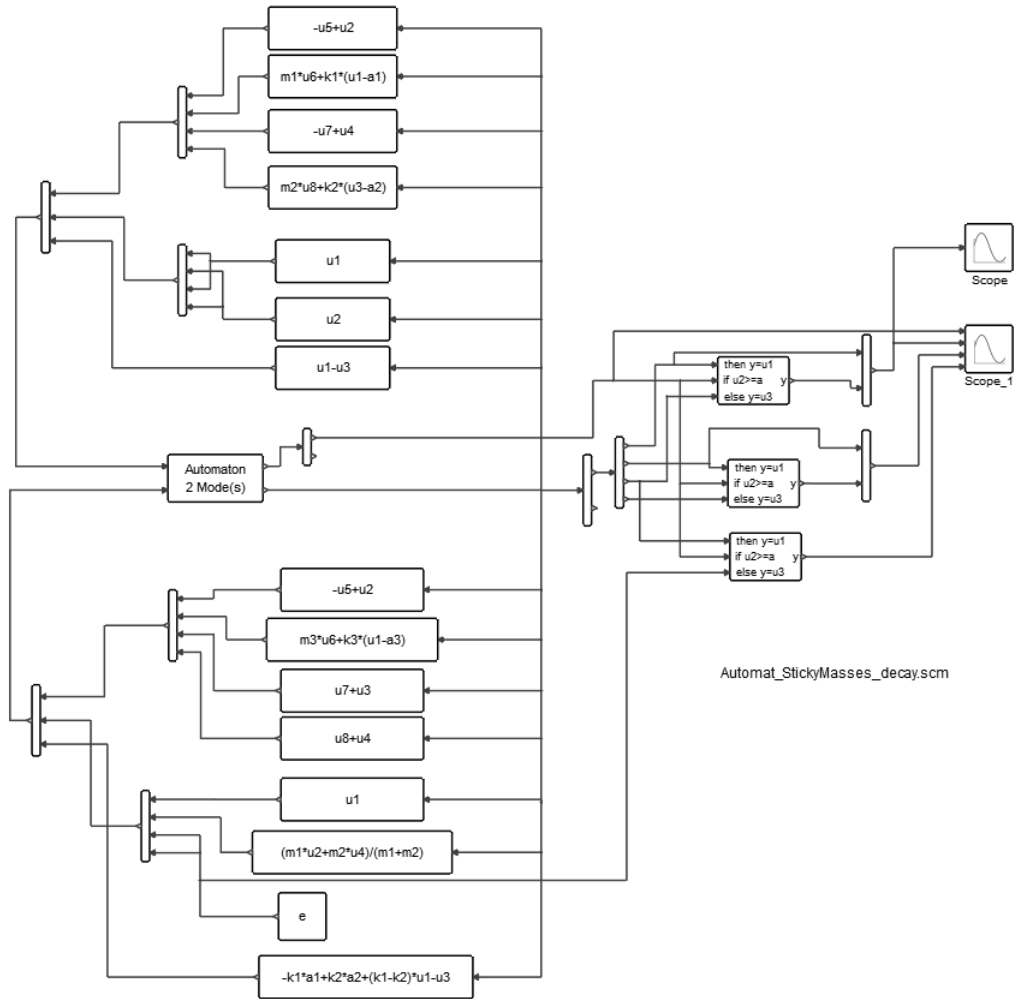


Figure 16.17: Complete model of sticky masses (Automat\_StickyMasses\_decay.scm)

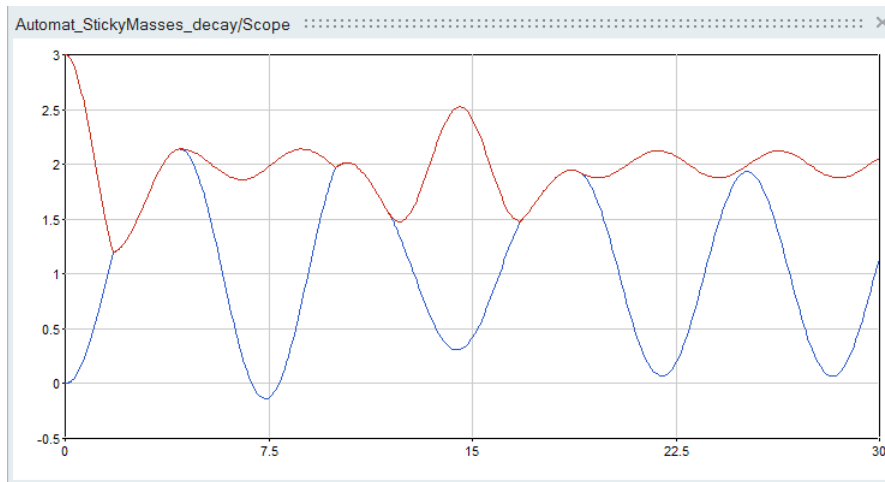


Figure 16.18: Position of masses.

## 16.4 conclusion

Hybrid automata are used to express hybrid dynamical systems. Although **Activate** is a general purpose hybrid system simulator, developing a hybrid automaton with several subsystems is difficult and time-consuming. That is why the `Automaton` block is useful to allow straightforward construction of hybrid automata. This block centralizes the control of the hybrid automaton and can simulate complex multi-mode DAEs based on hybrid automata. This block can model different continuous-time dynamics in each automaton mode.



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- [10] <http://sec.eecs.berkeley.edu/demo/StickyBall/StickyBall.htm>



## Chapter 17

# Interpolation and extrapolation methods in Activate

### 17.1 Introduction

Several blocks in **Activate** such as `Lookup table`, `Signal generator`, `FromHDF`, `FromText`, `FromMat`, `FromCSV`, and `Signal-in` generate continuous-time signals from discrete-time signals. The discrete-time signal variable provides the values of a signal  $Y_i$ , at different time instants  $X_i$ . The value of the signal at times other than elements of  $X_i$  are computed via interpolation or extrapolation. Several interpolation and extrapolation methods are available inside **Activate** blocks that will be explained in the following sections. In order to illustrate each method, the method is applied on this data-set whose x-y diagram is shown in Fig.17.1.

$$\begin{aligned} X &= [0, 1, 2, 4] \\ Y &= [4, 7, 6, 5] \end{aligned}$$

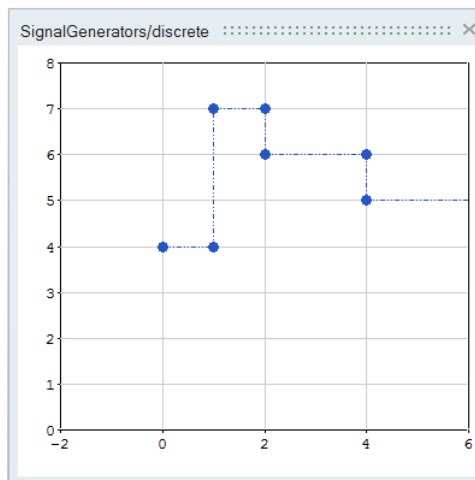


Figure 17.1: x-y diagram for test data set.

## 17.2 Interpolation methods

The values of the signal at time instants inside the range  $[X_1, X_N]$  are computed via interpolation. Several interpolation methods are available in **Activate** blocks:

- **ZeroOrder, ZeroOrder(floor, Point\_just\_below)**: The index of element in  $X$  which is equal or below  $x$  is chosen to compute the output, i.e.,

$$\text{for } X_i \leq x < X_{i+1}$$

$$y = Y_i$$

The interpolation result is shown in Fig.17.2.

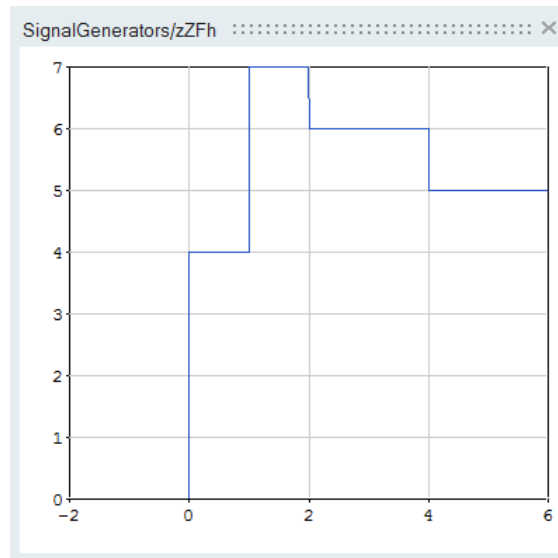


Figure 17.2: x-y diagram **ZeroOrder, ZeroOrder(floor)** methods.

- **ZeroOrder(ceil, Point\_just\_above)**: The index of element in  $X$  which is equal or above  $x$  is chosen to compute the output, i.e.,

$$\text{for } X_i < x \leq X_{i+1}$$

$$y = Y_{i+1}$$

The interpolation result is shown in Fig.17.3.



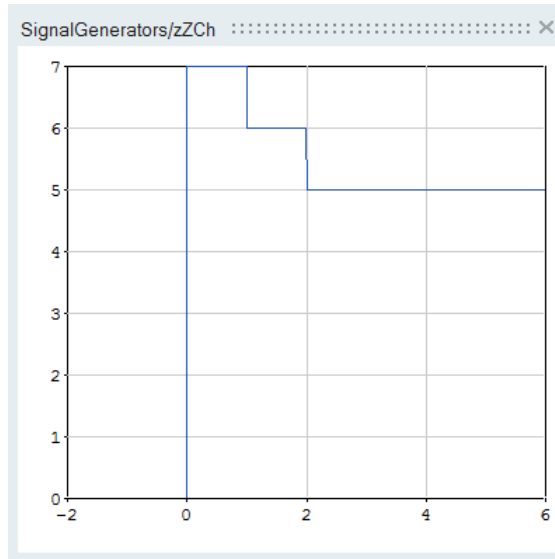


Figure 17.3: x-y diagram **ZeroOrder(ceil)** methods.

- **ZeroOrder(nearest, nearest\_poin):** The index of element in  $X$  which is nearest to  $x$  is chosen to compute the output, i.e.,

$$\text{for } \frac{X_{i-1} + X_i}{2} \leq x < \frac{X_i + X_{i+1}}{2}$$

$$y = Y_i$$

The interpolation result is shown in Fig.17.4.

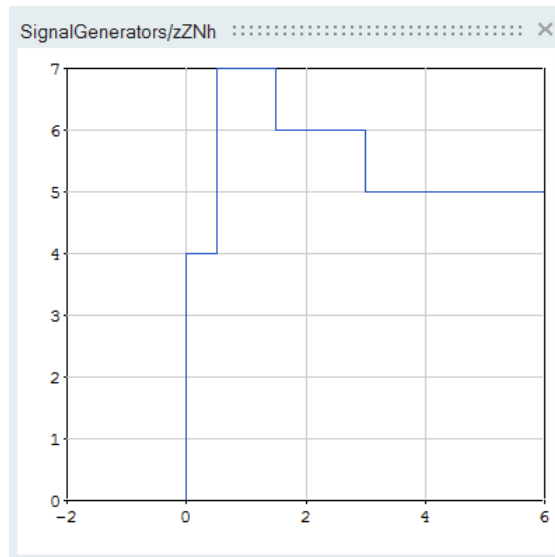


Figure 17.4: x-y diagram **ZeroOrder(nearest)** methods.

- **Linear.** This is the default method and performs a linear interpolation. If  $x$  matches one of  $X$  elements, the output is the corresponding element in the  $Y$  matrix. If no value matches, then

a linear interpolation is performed between two nearest elements of  $X$  to determine the output value, i.e.,

$$\text{for } X_i \leq x < X_{i+1}$$

$$y = Y_i + (x - X_i) \frac{Y_{i+1} - Y_i}{X_{i+1} - X_i}$$

The interpolation result is shown in Fig.17.5.

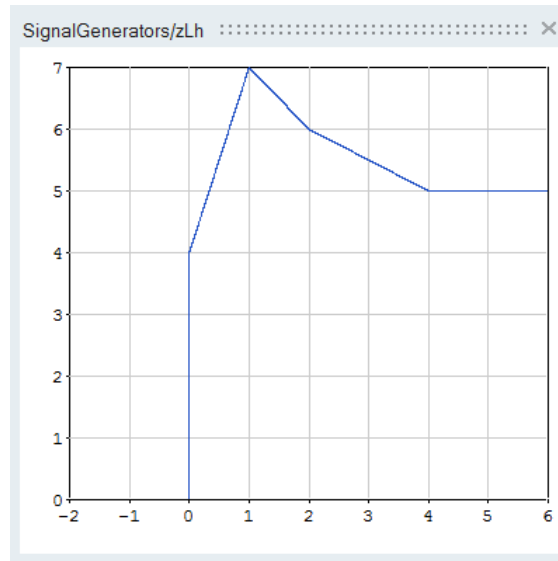


Figure 17.5: x-y diagram **Linear** methods.

- **Bilinear interpolation:** The bilinear interpolation is an extension of linear interpolation for interpolating functions of two variables on a rectilinear 2D grid. In this method, first an interpolation in one direction is performed, and then again in the other direction. Although each step is linear in the sampled values and in the position, the interpolation as a whole is not linear but rather quadratic in the sample location.
- **Trilinear interpolation:** The trilinear interpolation method is an interpolation on a 3-dimensional regular grid which approximates the value of an intermediate point within the local axial rectangular prism linearly, using data on the lattice points. The trilinear interpolation is an extension of linear interpolation for interpolating functions of three variables on a 3D grid. In this method, first an interpolation in one direction is performed, and then again in the other directions. Although each step is linear in the sampled values and in the position, the interpolation as a whole is not linear but rather cubic in the sample location.
- **Not\_a\_knot:** A cubic spline is computed by imposing following conditions (considering  $N$  points  $X_1, \dots, X_N$ ).

$$\begin{cases} S^{(3)}(X_2^-) &= S^{(3)}(X_2^+) \\ S^{(3)}(X_{N-1}^-) &= S^{(3)}(X_{N-1}^+). \end{cases}$$

where  $S^{(n)}$  is  $n^{th}$  derivative of the interpolated curve.

The interpolation result is shown in Fig.17.6.

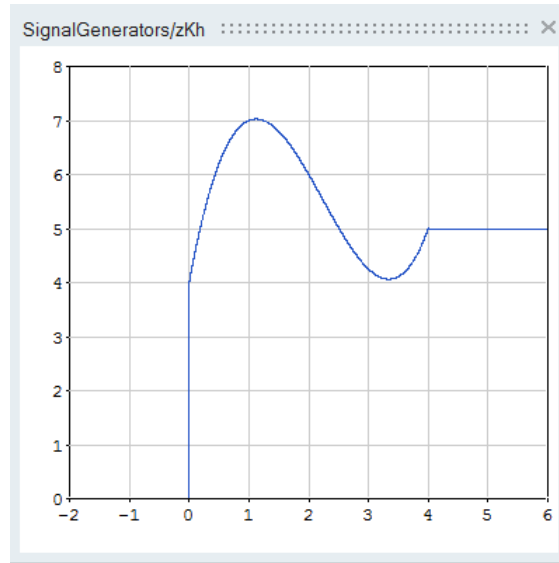


Figure 17.6: x-y diagram **Not\_a\_knot** methods.

- **Natural:** A cubic spline is computed by imposing following conditions:

$$\begin{cases} S^{(2)}(X_1) = 0 \\ S^{(2)}(X_N) = 0. \end{cases}$$

The interpolation result is shown in Fig.17.7.

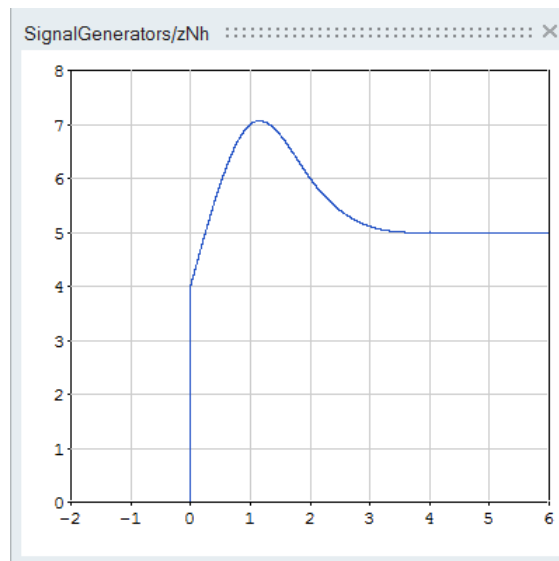


Figure 17.7: x-y diagram **Natural** methods.

- **Clamped\_To\_Zero:** A cubic spline is computed by imposing following conditions:

$$\begin{cases} S^{(1)}(X_1) = 0, \\ S^{(1)}(X_N) = 0. \end{cases}$$

The interpolation result is shown in Fig.17.8.

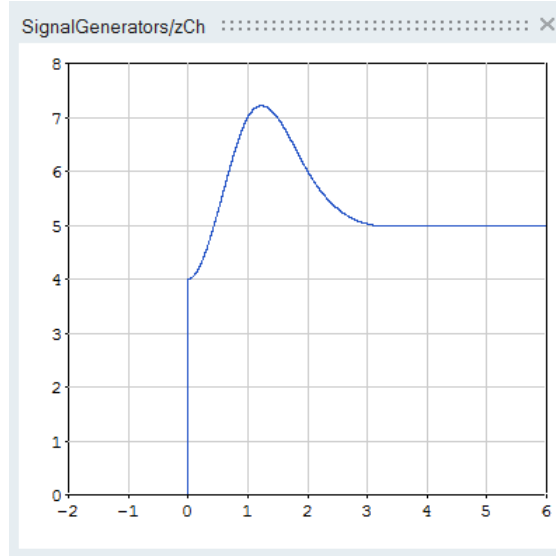


Figure 17.8: x-y diagram **Clamped\_To\_Zero** methods.

- **Akima interpolation method:** The Akima interpolation method <sup>1</sup> is a continuously differentiable sub-spline interpolation method built from piecewise third order polynomials. This method provides less ringing and overshoot than natural and not-a-knot spline algorithms. In this method only data from the next neighbor points is used to determine the coefficients of the interpolation polynomial. Since no functional form for the whole curve is assumed and only a small number of points is taken into account this method does not lead to unnatural values in the resulting curve.
- **Fritsch-Butland interpolation method:**  
The Fritsch-Butland interpolation method <sup>2</sup> is a continuously differentiable cubic interpolation method built from piecewise third order polynomials. In this method local first derivative approximations is used and guaranties to generate locally monotone interpolation curves.
- **Steffen interpolation method:**  
The Steffen interpolation method <sup>3</sup> guarantees the monotonicity of the interpolating function between the given data points. The minima and maxima can only occur exactly at the data points, and there can never be spurious oscillations between data points. The resulting curve is piecewise cubic on each interval with the slope at each grid point chosen to ensure monotonicity and prevent undesired oscillations. The interpolated function is piecewise cubic in each interval. The first-order derivative is guaranteed to be continuous everywhere, but the second derivative may be discontinuous.

<sup>1</sup>Hiroshi Akima, 'A New Method of Interpolation and Smooth Curve Fitting Based on Local Procedures,' Journal of the ACM, Vol. 17, No. 4, pp. 589-602, 1970

<sup>2</sup> F. N. Fritsch and J. Butland. 'A method for constructing local monotone piecewise cubic interpolants'. SIAM J. Sci. Statist. Comput., 5 (1984), pp. 300-304

<sup>3</sup>M.Steffen, "A simple method for monotonic interpolation in one dimension", Astron. Astrophys. 239, 443-450 (1990).

## 17.3 Extrapolation methods

The values of the signal at time instants outside of the range  $[X_1, X_N]$  are computed by extrapolation. Several extrapolation methods are available in **Activate** blocks. Extrapolation can be applied either at the left-side, i.e., for  $x < X_1$  or at the right-side, i.e., for  $x > X_N$ .

- **Zero**: This is the default method. In this case, the output is zero for time instants out of signal's range, i.e.,

$$\begin{cases} \text{for } x < X_1, & y = 0.0 \\ \text{for } x > X_N, & y = 0.0 \end{cases}$$

The left-side **Zero** extrapolation result is shown in Fig.17.9.

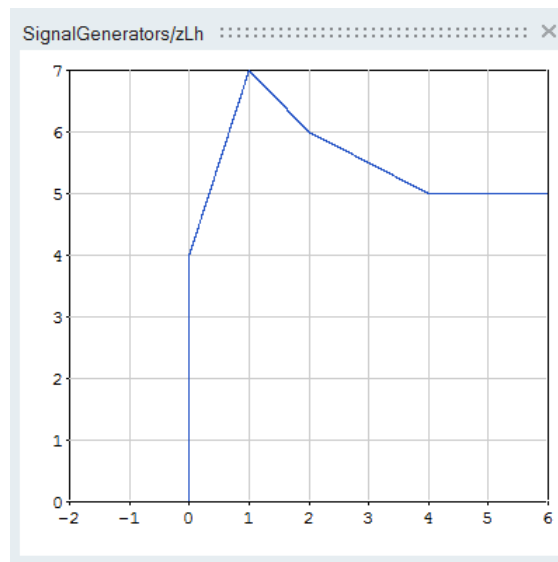


Figure 17.9: x-y diagram for left-side **Zero** extrapolation methods.

The left-side and right-side **Zero** extrapolation result is shown in Fig.17.10.

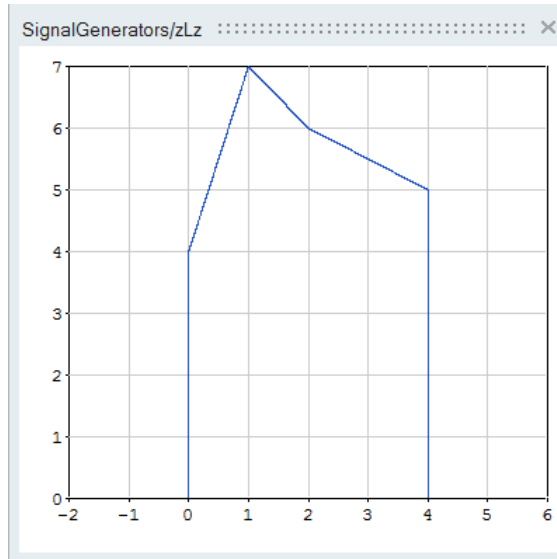


Figure 17.10: x-y diagram for left-side and right-side **Zero** extrapolation methods.

- **Hold:** The output is equal to the last element of the  $Y$  vector, i.e.,

$$\begin{cases} \text{for } x < X_1, & y = Y_1 \\ \text{for } x > X_N, & y = Y_N \end{cases}$$

The left-side and right-side **Hold** extrapolation result is shown in Fig. 17.11.

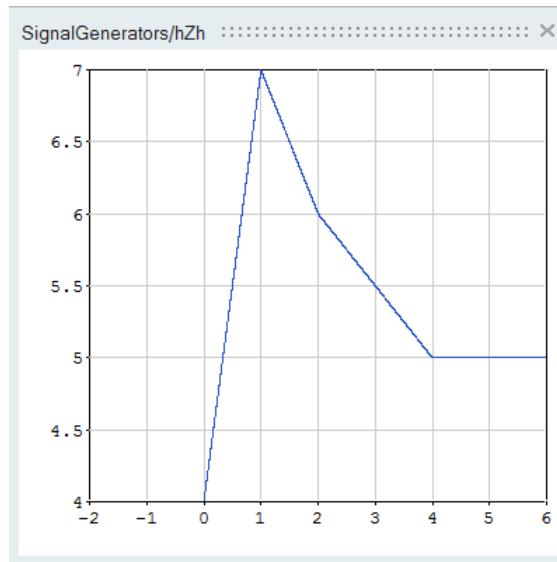


Figure 17.11: x-y diagram for left-side and right-side **Hold** extrapolation methods.

- **Extrapolation.** A linear extrapolation is performed using two left-most or right-most elements of the signal, i.e.,

$$\begin{cases} \text{for } x < X_1, & y = Y_1 + (x - X_1) \frac{Y_2 - Y_1}{X_2 - X_1} \\ \text{for } x > X_N, & y = Y_{N-1} + (x - X_{N-1}) \frac{Y_N - Y_{N-1}}{X_N - X_{N-1}}, \end{cases}$$

The left-side **Extrapolation** extrapolation result is shown in Fig.17.12.

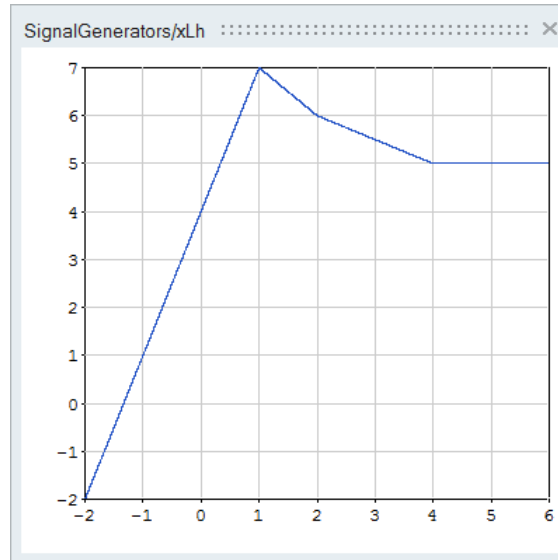


Figure 17.12: x-y diagram for left-side **Extrapolation** extrapolation methods.

The right-side **Extrapolation** extrapolation result is shown in Fig.17.13.

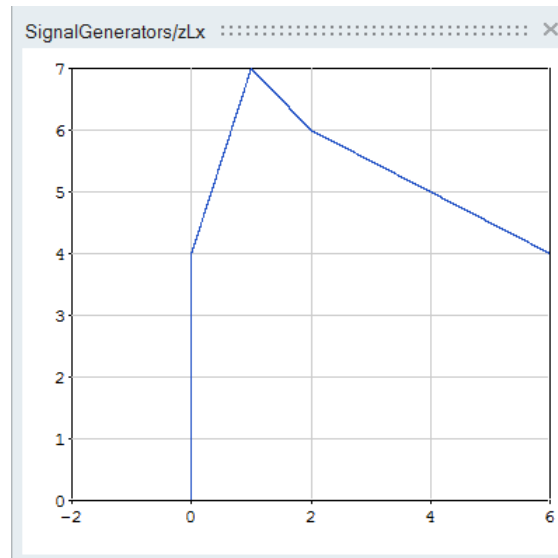


Figure 17.13: x-y diagram for right-side **Extrapolation** extrapolation methods.

- **Repeat.** The signal is repeated over the whole range of  $X$ . For  $x < X_1$  or  $x > X_N$

$$y = y(x - k(X_N - X_1))$$

where  $k$  is an integer number to satisfy the following condition.

$$X_1 \leq x - k(X_N - X_1) < X_N$$

The left-side **Repeat** extrapolation result is shown in Fig.17.14.

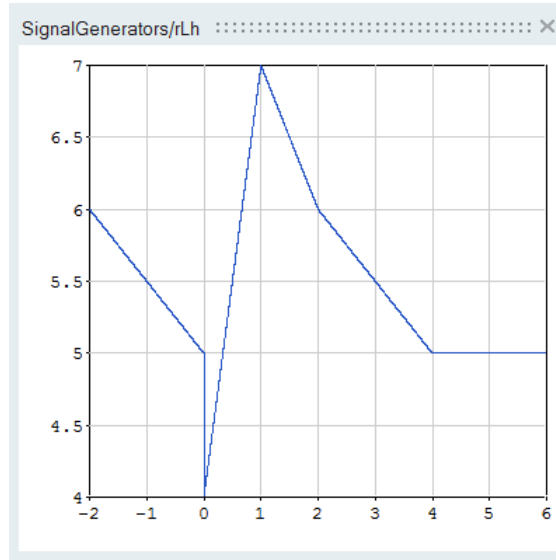


Figure 17.14: x-y diagram for left-side **Repeat** extrapolation methods.

The right-side **Repeat** extrapolation result is shown in Fig.17.15.

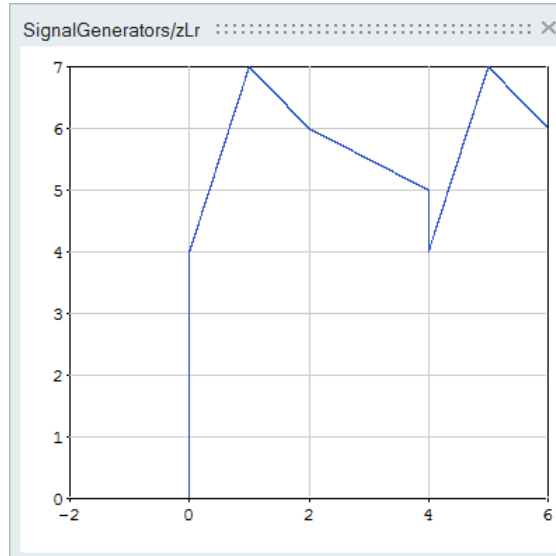


Figure 17.15: x-y diagram for right-side **Repeat** extrapolation methods.

### 17.3.1 Application - Behavior in blocks

The interpolation and extrapolation methods described above are available inside **Activate** blocks and are used to compute the value of the signal for inputs that are either before or after the data span.

In the following tables, all options and behaviors are described. The options can be set in the Advanced tabs of the blocks



Case <b>with</b> External Activation - Zero Order		
Selected Action	Left time span	Right time span
Zero	Zero	Zero
Hold	Hold	Hold
Extrapolation	Hold	Hold
Repeat	Repeat	Repeat

Case <b>without</b> External Activation - Zero Order		
Selected Action	Left time span	Right time span
Zero	Zero	Hold
Hold	Hold	Hold
Extrapolation	Extrapolation	Extrapolation
Repeat	Repeat	Repeat

Case <b>without</b> External Activation - Linear		
Selected Action	Left time span	Right time span
Zero	Zero	Zero
Hold	Hold	Hold
Extrapolation	Extrapolation	Extrapolation
Repeat	Repeat	Repeat



## Chapter 18

# SignalOut and SignalIn blocks in Altair Activate

### 18.1 Introduction

A signal is an object used to define time dependent function over a given time span. Signals are often defined and used in **Activate** but they are also accessible in the environment (**OML** workspace). Signals store time functions, mainly **Activate** simulation results, and also can provide input to **Activate** models. Signals also can be used for comparison and validation of simulation results, in particular in the QA process. Signals can also be used as reference inputs in **Activate** models.

Signals can either be created and queried in **OML** using table commands, or be created and accessed in the **Activate** environment using **SignalIn** and **SignalOut** blocks, respectively. A Signal is a table in the **OML** environment composed of the time (`time`) and data (`ch`) fields. Following commands are used to create a signal with name `mysg`.

- **mysg={}** : This command is used to create the empty signal `mysg` in **OML**.
- **mysg.time** : The `time` field is a 3\*N matrix where N is the number of time samples stored in the signals structure.
  - **time vector** : The first row of `mysg.time` matrix is the vector of the sampling time instants.
  - **event coding** : The second row of `mysg.time` matrix is the event-coding (synchronization information) of the corresponding sample. This coding will be explained in [18.2](#).
  - **event kind** : The third row of `mysg.time` matrix indicates the kind of the event that has triggered the sampling. The event kind will be explained in [18.2](#).
- **mysg.ch={}** : `mysg.ch` is a sub-table used for storing the set of data. Once `mysg.ch` created, data can be stored in the several channels. Each channel is a table composed of `type` and `data` fields.
- **mysg.chi.type** : Indicates the data-type of the  $i^{th}$  channel, i.e., `double`, `complex`, `int32`, `int32`, `int16`, `int8`, `uint32`, `uint16`, `uint8`.
- **mysg.chi.data** : Represents the data stored in the  $i^{th}$  channel and can be a matrix of size [Nrow, Ncol\*N]. In fact, the data matrices of samples of size [Nrow\*Ncol] are concatenated.

Example: Here is an example of a signals structure with two data sets.

```
B={};
B.time=[1 2 4];
B.ch={};
B.ch{1}={};
B.ch{1}.type='double';
B.ch{1}.data=[1 2 3];
B.ch{2}={};
B.ch{2}.type='int32';
B.ch{2}.data=[1 2 3 4 5 6;7 8 9 10 11 12];
```

Now you can use the "B" signal in a SignalIn block as we will see in following sections.

## 18.2 Activation information inside a signal object

Signal objects in **Activate**, contain specific information about how the SignalOut block has been activated. An activation signal specifies the time instants where the signal may potentially change value. For example a discrete-time signal, which in **Activate** is simply a piece-wise constant function, has an activation signal present at the points of potential discontinuities.

In general, an activation signal is a series of isolated points, called events, and intervals over time. If the activation signal contains only isolated points, then we say that the corresponding signal is discrete-time. A special case of a discrete-time signal is the periodic signal where the activation points are equally spaced. But the latter property does not imply that a discrete-time signal is periodic, a periodic signal must be declared as such. Activation intervals correspond to periods over which the signal evolves as a continuous-time function. The always-active signal is a special case of activation signal in which there exists a single activation interval covering the entire time period. A **Activate** signal in general is made of multiple activation intervals. Over each such period, the corresponding time function is supposed to be at least continuous and differentiable. Any point of discontinuity, in the value or the derivative of the function, corresponds to a specific event. Two events may have the same time, therefore a **Activate** signal may take on successive values at the same time instant. The time event that is not being used as an indexing mechanism can mean that based on the time values of the two events associated with two different signals, nothing can be concluded as to their synchronization. Different methods can be used for coding signals over continuous-time activation periods. The method used here samples the function and stores a list of time-value pairs, just as with event activations. An indicator however distinguishes between the two types of "events".

Signals are typically created in **Activate** by the SignalOut blocks and read by the SignalIn blocks. **Activate** signals are always considered asynchronous because each has its own activation signal. In order to preserve synchronization information between several data series, multiple data can be placed together into one signal. In other words, a signal may contain several data series and a single time vector. A Signal contains a time vector. For each time, the following information is associated:

Event-coding or synchronization information (accessed by `GetNevIn(block)` API) indicates the port[s] that has[ve ] been activated at the sampling time instants. remember binary coding of numbers, for example, if an event activates the SignalOut via the input port 1, its coding is 1. If an event activates the SignalOut via the input port 2, its coding is 2. if an event activates the SignalOut via the input port 2 and 1, its coding is 3.

Event-kind or simulation phase (accessed by `GetSimulationPhase(block)` API) provides information about the kind of activation. In the following table, different kinds of activation are listed.

Phase	Definition
PHASE_MESHPOINT_LEFTTL (numerical value=-1)	A Left-Limit of Discrete event: sampling that is triggered just before a discrete event happens. The sampling of this kind of point is not active by default in the SignalOut block.
PHASE_MESHPOINT (numerical value=0)	Mesh-point (continuous): A pure continuous sampling point, also known as a solver's mesh-point. The sampling of this kind of point is active by default in the SignalOut block.
PHASE_DISCRETE (numerical value=1)	discrete event: A discrete event has triggered the sampling, or this is the very first non-try call of the block. The sampling of this kind of point is active by default in the SignalOut block.
PHASE_TRY_MFX (numerical value=2), PHASE_TRY_MRLX (numerical value=3)	Try calls: Indicates the try points that a simulator or a solver may take. The sampling of this kind of point is not active by default in the SignalOut block.
PHASE_ZC_LEFTTL (numerical value=4)	Left-limit of Zero-crossing: A sample just before the zero-crossing event takes place. This is useful to register the states or outputs just before a zero-crossing event that may cause discontinuities. The sampling of this kind of point is not active by default in the SignalOut block.
PHASE_ZCROSS (numerical value=5)	Zero-crossing: A zero-crossing event triggered the sampling. The sampling of this kind of point is active by default in the SignalOut block.
PHASE_EXIT_INITIALIZATION (numerical value=6)	First-call: The block is being called for the first time after the initialization has finished. After this call the simulation is started.

### 18.3 SignalOut block

The SignalOut block<sup>1</sup> allows for creating a Signal in the **Activate** environment. The created signals then can be retrieved in the **OML** workspace using ordinary table operations. The Parameters Dialog of the SignalOut block is shown in Fig.18.1 and 18.2.

<sup>1</sup>Available in `SignalExporters` palette

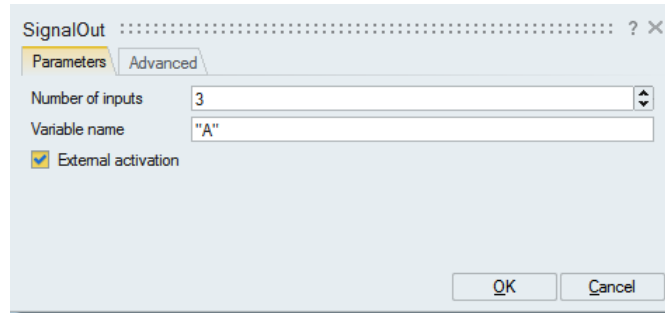


Figure 18.1: Parameters Dialog of the SignalOut block.

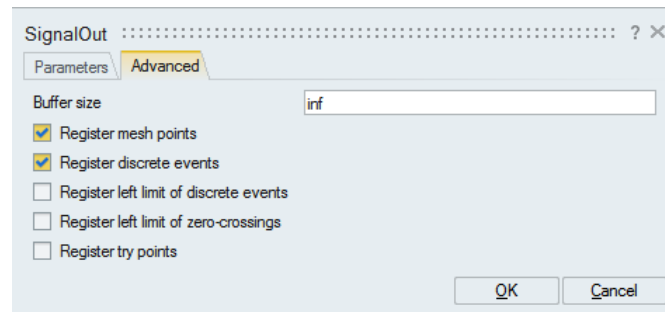


Figure 18.2: Parameters Dialog of the SignalOut block (advanced tab).

The block has several parameters:

- **Number of inputs:** Used for specifying the number of data sets to be stored. This represents the number of elements in the `ch` field that will be available in the generated Signals. The data-type of each channel can be different and is inherited from the source block.
- **Variable name:** Indicates the name of the signal generated in the **OML** environment.
- **External activation:** Once checked, the block samples its inputs only when the block is activated by the incoming events.
- **Buffer-size:** The buffer indicates the number of samples to be stored. If number of samples exceeds the size of the buffer, the earliest sample is replaced by the new one. If the user needs to keep all samples, the buffer-size should set to `inf` which means unlimited.
- **Sampling filter:** In many occasions, the user needs to register only specified kind of samples, e.g., only continuous-time samples or only discrete ones. These check boxes allow the user filtering out unnecessary samplings points. Note that the event kind `try` would not generate samples with increasing time samples. Note that in order to regenerate the signal by the `SignalIn` block, the time vector of the signal needs to be increasing.

## 18.4 Signalln block

The Signalln block<sup>2</sup> reproduces the data series stored in the importing signal in the **Activate** environment. The GUI of the SignalOut block is shown in Fig.18.3 and 18.4.

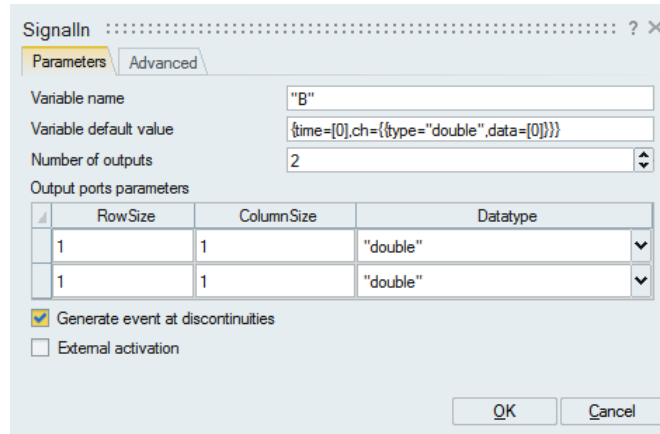


Figure 18.3: GUI of the signalln block.

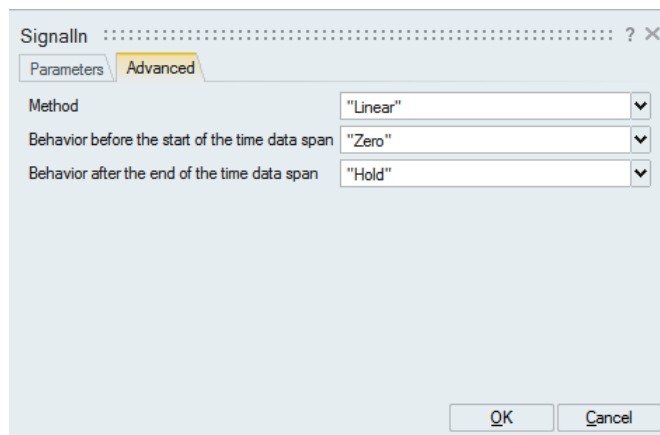


Figure 18.4: GUI of the signalln block (advanced tab).

The block has several parameters:

- **Variable name:** The name of the signal to be retrieved from **OML** environment.
- **Variable default value:** If the requested signal name is not present in the **OML** environment, this field allows providing a default value for the signal.
- **Number of outputs:** The Signalln block needs the number of `ch` channels in the signal variable. Furthermore, for each channel, the data-size and data-type should be known, before reading the signal variable. Otherwise an error signal will be emitted indicating the difference between the read signal and the given parameter in the GUI.
- **Activated by the external events (ignore time):** Once checked, all time information in the signal

<sup>2</sup>Available in `SignalImporters` palette

variable is ignored and external events trigger the output of the block. At each event trigger, a new sample is read.

- **Generate event at discontinuities:** This option allows generation events at sample times stored in the signal variable.
- **Method:** Indicates the interpolation method used for regeneration of the signal data series in the **Activate** environment. The acceptable values are:
  - Zero-order (order 0)
  - Linear (order 1)
  - Not-a-knot (order 3)
  - Natural (order 3)
  - Clamped to zero (order 3)

These methods have been explained in the Interpolation methods chapter(17). When one of the data types stored in the signal variable is not of type double, the interpolation method should be zero-order.

- **Behavior before the start of the time data span:** If the output value of a channel in a signal value is requested for a time instant before the start of the time span of the signal, this extrapolation method is used. The accepted values are:
  - Zero: Output is zero.
  - Extrapolation (Linear): The output is computed based on a linear extrapolation between first and second data samples (if they exist).
  - Hold: The output will be the value of the first data sample.
  - Repeat: The output is computed to produce a periodic signal, repeating the whole time span of the signal variable.

These methods has been explained in the Interpolation methods chapter(17).

- **Behavior after the start of the time data span:** If the output value of a channel in a signal value is requested for a time instant after the start of the time span of the signal, this extrapolation method is used. The accepted values are
  - Zero: Output is zero.
  - Extrapolation (Linear): The output is computed based on a linear extrapolation between two end data samples (if they exist).
  - Hold: The output will be the value of the final data sample.
  - Repeat: The output is computed to produce a periodic signal, repeating the whole time span of the signal variable.

These methods have been explained in the Interpolation Methods chapter (17).



# Chapter 19

## Animation

Currently **Activate** does not provide means to create sophisticated animations. The only block creating animations is the **Anim2D** block, which can be used to animate simple 2D graphical objects.

The block is based on **OML** functions to create figures and graphical objects, and define and modify their parameters. These functions can also be used directly by the user in an **OML** based block (for example the **OmlCustomBlock**) to create custom animations. In most cases however the **Anim2D** block is easier to use. Some behaviors, for example graphical objects changing color during simulation, are not supported by the block and custom animation must be used. The **OML** functions also give access to more properties such as the edge colors and thickness of some of the objects. The list of these **OML** functions can be found in **OML** documentations.

### 19.1 Anim2D block

The **Anim2D** block provided in the `SignalViewers` palette can be used to create 2D animations. Such animations are useful in particular to illustrate the behavior of simple mechanical and hydraulics systems.

The **Anim2D** block can be used to animate different types of graphical objects in a Figure (on a canvas). The block has two input activation ports and a variable number of regular input ports.

Block regular inputs can be associated with different object types:

1. Type 1 is a closed polyline and the input signal is an  $N \times 2$  matrix where each row represents the  $x, y$  coordinates of a point of the polyline.
2. Type 2 is similar to type 1 but the polyline is not closed.
3. Type 3 corresponds to circles and the input is an  $M \times 3$  matrix. Each row corresponds to a circle and contains the  $x, y$  coordinates of the center of the circle and its radius.
4. Type 4 corresponds to rectangles and the input is a  $K \times 4$  matrix. Each row corresponds to a rectangle and contains the  $x, y$  coordinates of the lower left corner of the rectangle, and its width and height.

The block parameters are:

- Number of input ports ( $n_{in}$ ), i.e., the number of input signals.

- Object types: vector of size  $nin$ . If all types are identical, a scalar may be used.
- Canvas geometry: vector of size 4.
- Color table: a matrix with three columns representing the RGB values of each color (valued between 0 and 1). A row including  $-1$ 's is considered transparent. The number of rows need not match the number of objects being animated. If the table is too small, colors are reused from the top of the table.
- Zoom factor setting: correspondence between Figure coordinate values and the number of points on screen.

When the block is activated through its first input activation port, the animation advances by moving all the objects to the positions specified by the current input signals. When it is activated through its second activation port, the objects are duplicated and a copy is frozen in place on the Figure in order to create snapshots visible at the end of the simulation (useful for example for exporting the Figure as an image).

Object types can be given negative values (-1 instead of 1, -2 instead of 2, etc.) if they should be excluded from snapshots. This is particularly useful for non-moving objects (such as background decorations) that need not be included in the snapshots multiple times.

## 19.2 Examples

### 19.2.1 Bouncing balls

The **Activate** bouncing balls demo contains the animation of an arbitrary number of balls bouncing (in 2D) against each other and against the walls, floor and ceiling. The information available for animation is the positions and the radii of the balls, and the positions of the walls, floor and ceiling. The coordinates  $x$  and  $y$  of the center of balls are given by signals, the rest are constant parameters.

The top diagram of the bouncing balls demo is shown in Fig. 19.1. The content of the Animation Super Block is illustrated in Fig. 19.2. The **Anim2D** block is used with two inputs. The first is of type rectangle used to display the walls, floor and ceiling. The second input is of type circle and animate the balls. See Fig. 19.3. The **Anim2D** block parameters are shown in Fig. 19.4.



Figure 19.1: The Bouncing balls block output vectors provide the  $x$  and  $y$  coordinates of the balls.

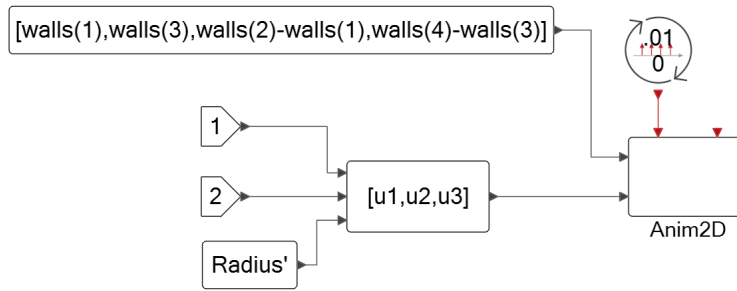


Figure 19.2: The content of the Animation Super Block.

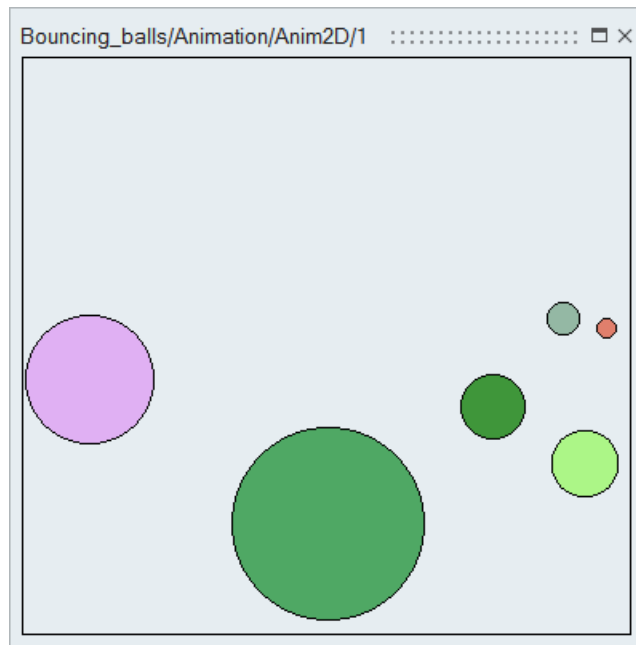


Figure 19.3: The animation Figure.

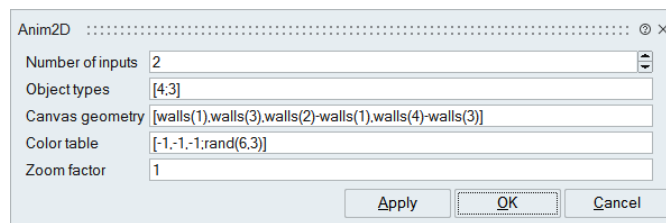


Figure 19.4: The parameters of the **Anim2D** block. The rectangle is not filled (its content is transparent) and the ball colors are chosen randomly.

## 19.2.2 Control of tank water levels

The model considered here, `level.scm`, is made up of two connected water containers. The first container is supplied with water by a constant flow pump which can be turned on and off. This container is connected via a pipe to a second container. The second container also has an exit pipe, which can be opened and closed by a valve. It is opened at random times and closed after a constant delay.

A simple controller is used to keep the level of water in the second container close to a given value while avoiding the water level in the first container to go over a maximum value.

Denoting the water levels in the two tanks respectively  $h_1$  and  $h_2$ , the flow  $f_1$  from container 1 to 2 can be expressed as follows

$$f_1 = \alpha \sqrt{2g(h_1 - h_2)}$$

and the flow out of the second container is

$$f_2 = \begin{cases} \beta \sqrt{2gh_2} & \text{if outflow valve is open} \\ 0 & \text{otherwise} \end{cases}$$

where the parameters  $\alpha$  and  $\beta$  depend on the pipes geometry. The water levels in the containers depend on the flows as follows

$$\begin{aligned} \dot{h}_1 &= \gamma(f_{in} - f_1) \\ \dot{h}_2 &= \delta(f_1 - f_2) \end{aligned}$$

where  $f_{in}$  is a constant  $f_0$  if the pump is on and 0 if it is off. The parameters  $\gamma$  and  $\delta$  depend on container surface areas.

The water levels are measured by two sensors. The sensor output  $y_i$  depends on  $h_i$ ,  $i = 1, 2$ :

$$\dot{y}_i = \tau(h_i - y_i), i = 1, 2.$$

The parameter  $\tau$  corresponds to the time response of the sensors.

The control is of type hysteresis where the pump is turned on when  $y_1$  is below the maximum allowed and  $y_2$  below  $h_{down}$ . It is turned off if  $y_1$  goes above the maximum allowed or  $y_2$  above  $h_{up}$ .

The full model is illustrated in Fig. 19.5.

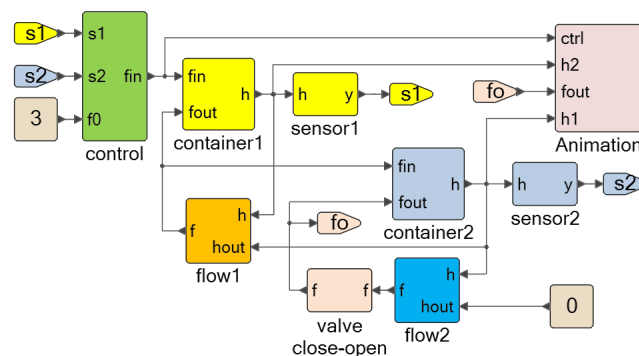


Figure 19.5: The two water container level control model.

The container and flow diagrams are given in Figs. 19.6 and 19.7. The Animation block content is illustrated in Fig. 19.8.

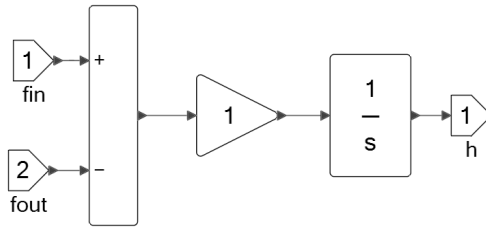


Figure 19.6: The diagram used to model the containers.

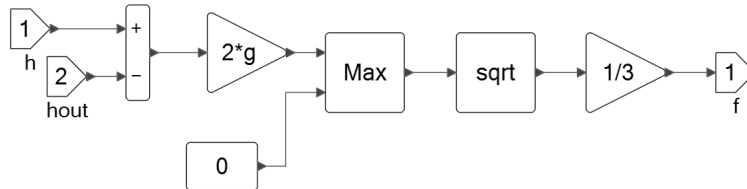


Figure 19.7: The diagram used to model flows.

The **Anim2D** block is used here to draw polylines and rectangles to illustrate not only the water levels in the containers but also the opening and closing of the exit valve and the state of the pump. A snapshot is given in Fig. 19.9.

### 19.2.3 Rolling disk on the ground

The demo `wheelsim.scm` contains an animation showing a wheel (disk) rolling on the ground. The 3D animation is obtained by projecting the 3D geometry into 2D using a projection matrix. The model top diagram is shown in Fig. 19.10 and the modified Animation block in Fig. 19.11. This modification is made to create snapshots of the wheel at regular times to obtain the picture shown in Fig. 19.12. The snapshot times correspond to events received by **Anim2D** block on its second input activation port.

One other change is required to make the snapshots visible: the type of the filled polyline representing the floor must be set to  $-1$ . Otherwise the floor is drawn at every step snapshot overshadowing all the previous snapshots of the wheel. This change is shown in Fig. 19.13.

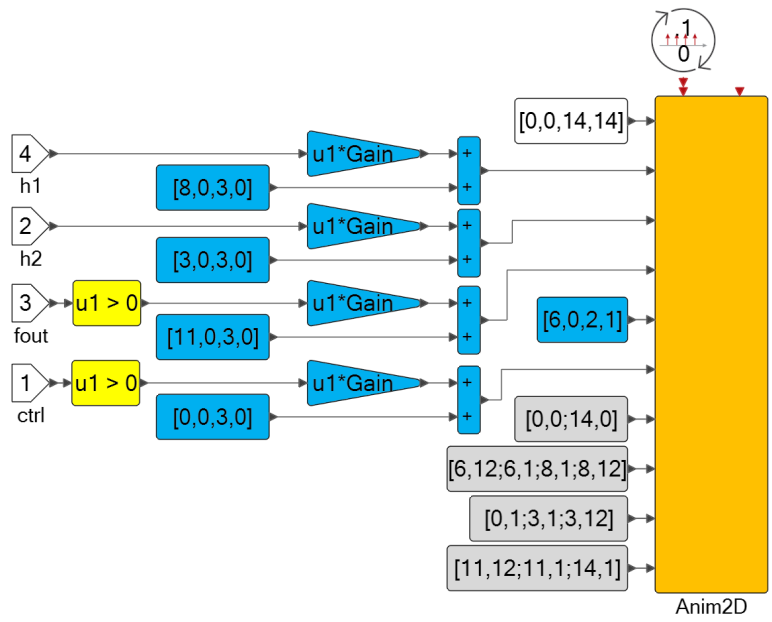


Figure 19.8: The content of the Animation block.

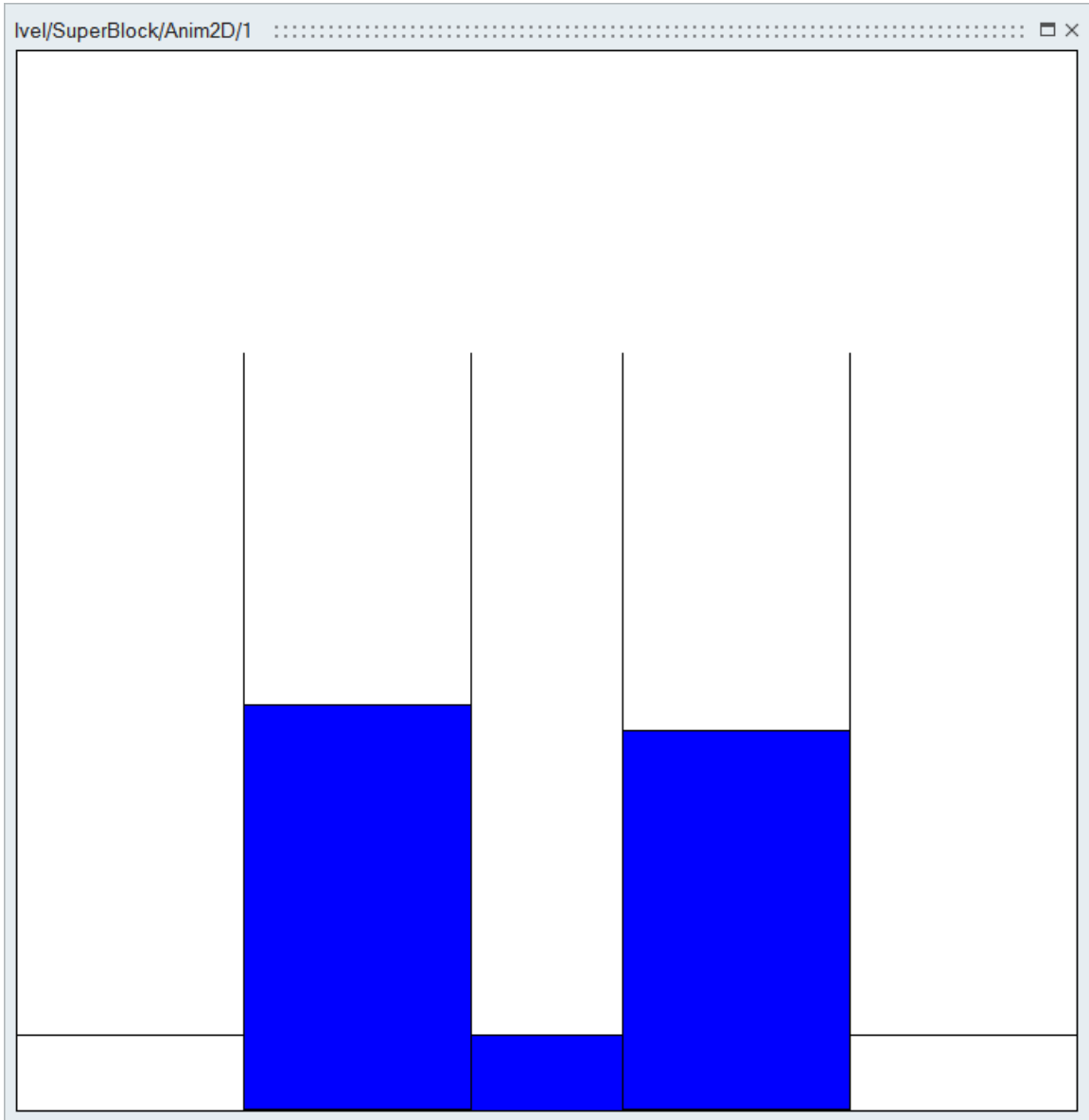


Figure 19.9: A snapshot of the animation. In this case the left and right rectangles are white because the flow in (controlled by pump) and flow out (controlled by the exit valve), are zero.

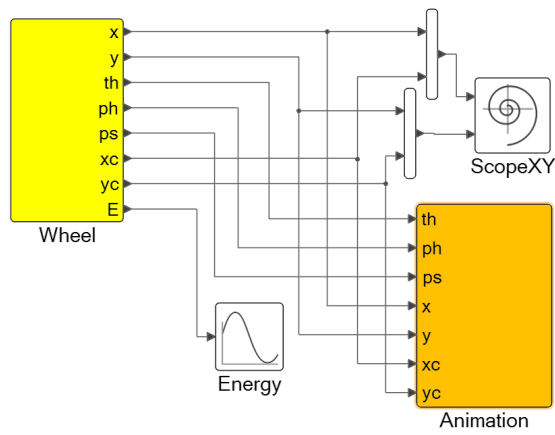


Figure 19.10: The top diagram of the wheelsim demo.

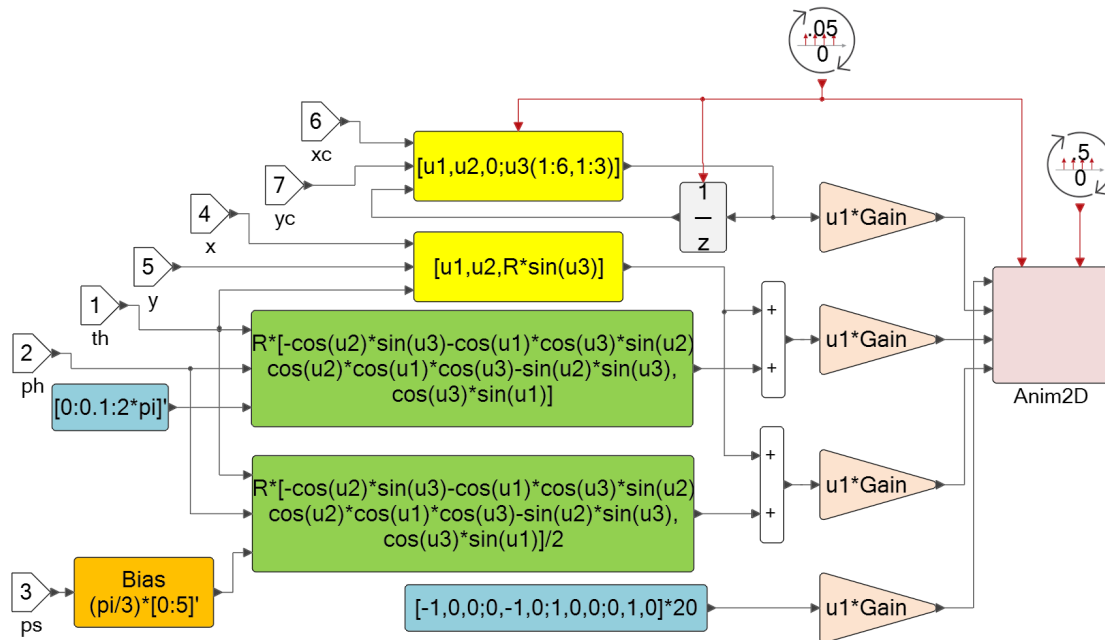


Figure 19.11: The content of the Animation block.



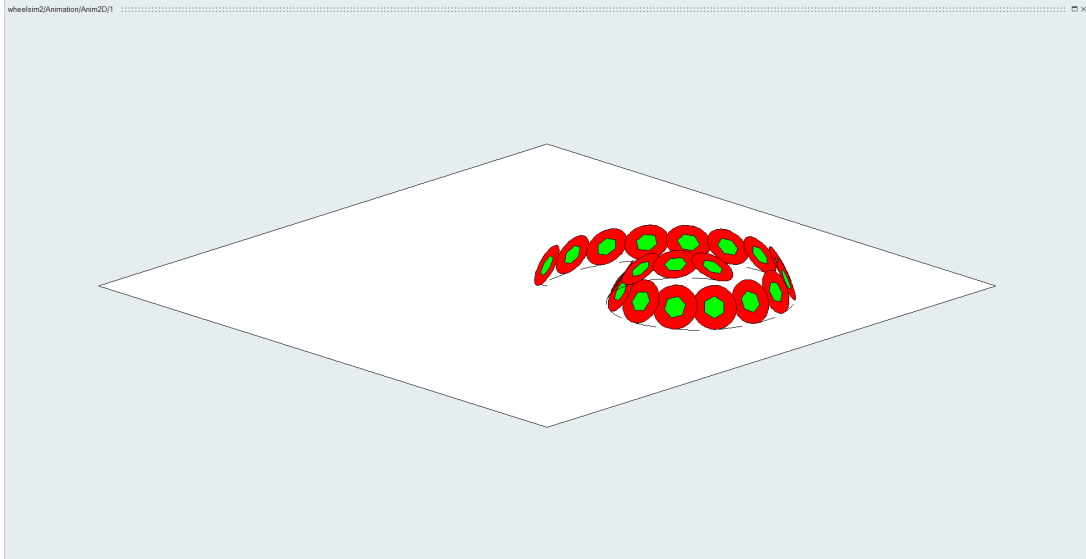


Figure 19.12: The final image of the animation.

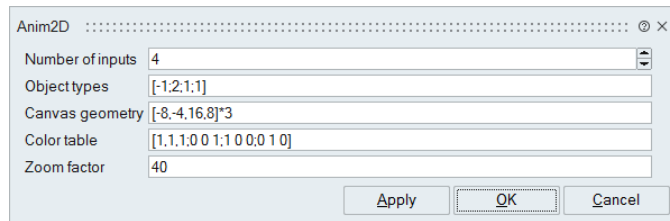


Figure 19.13: The modified dialog box of the **Anim2D** block.



## Chapter 20

# Blocks to execute OML and Python scripts interactively

The blocks **ExecOMLScript** and **ExecPythonScript** can be used in Activate models to run scripts, respectively in OML and in Python. The script associated with the block is ran by the user in the block diagram editor interactively through the block GUI. Unlike other Activate blocks, these blocks do nothing during simulation.

The main usage is to allow users to initialize the environment, for example by running a script to linearize a plant and construct a controller before running a simulation to validate the controller.

Unlike the Initialization script, which is executed at every simulation, the **OML** script in **ExecOMLScript** is executed on demand. Such scripts can also be executed directly in **OML** prior to running simulations. The advantage of using a **ExecOMLScript** block is that the model is self contained (the script is included in the model).

Two demo models are provided using the **ExecOMLScript** block, one for linearization and the other for optimization. A demo is also provided to show the usage of the **ExecPythonScript** block.

### 20.1 ExecOMLScript blocks: examples

In most cases, the script associated with the **ExecOMLScript** block in the model must be executed before running a simulation. The execution of the script creates **OML** variables that are needed by the simulator.

#### 20.1.1 Controller design based on linearization

The `inv_pendulum_lin_OML_exec.scm` model contains the classical inverted pendulum on a cart system with a linear controller modeled by a state-space block. The controller ( $A_t, B_t, C_t, D_t$ ) matrices are defined by the structure `_syslin`, if defined in the base environment. Otherwise they are defined so that the associated transfer function is zero (so no control). This is done in the top diagram Context script as follows:

```
_syslin=GetFromBase('_syslin', []);
```

```

if isempty(_syslin)
    At=[];
    Bt=zeros(0,2);
    Ct=zeros(1,0);
    Dt=zeros(1,2);
else
    At=_syslin.a;
    Bt=_syslin.b;
    Ct=_syslin.c;
    Dt=_syslin.d;
end

```

The construction of the `_syslin` structure is done by linearizing the Super Block containing the model of the cart-pendulum system and using the resulting linear system to construct an observer-based controller based on pole placement. This is done by the **OML** script associated with the **ExecOMLScript** block. By executing this script, the controller is computed, placed in the `_syslin` structure and saved in the base environment (the **OML** script runs in the base environment):

```

clear('_syslin');
% vector of input port indices considered for linearization
inps = 1;
% vector of output port indices considered for linearization
outs = [1,2];
% Modified context
ctx=struct; ctx.z0=0; ctx.th0=0; ctx.phi=0;

model=bdeGetCurrentModel;
% Selected Super block to linearize
superblock = 'pendulum';

[A,B,C,D]=vssLinearizeSuperBlock(model,superblock,inps,outs,0,ctx);
n=size(A,1);
Kc=-place(A,B,-1*ones(1,n));
Kf=-place(A',C',-2*ones(1,n)); Kf=Kf';

At=A+B*Kc+Kf*C+Kf*D*Kc; Bt=-Kf; Ct=Kc;

_syslin.a=At; _syslin.b=Bt; _syslin.c=Ct; _syslin.d=[];

```

If the model is simulated without running the script, the cart pendulum system is simulated without control and as expected the pendulum is not stabilized. On the other hand, if the script is run before, the pendulum is stabilized.

## 20.1.2 Controller design based on optimization

The `car_min_time_opt_OML_exec.scm` contains the model of a vehicle. The objective is to find the best gear ratios so that the vehicle can travel 1 Km in least amount of time. The optimization is performed over the gear rations `gear` and the gear changing times `T`.

This model has been used and explained earlier in the Optimization chapter. Here the script performing the optimization is placed inside an **ExecOMLScript** block. The script finds the optimal values of `gear` and `T`. They are used during simulation if already computed, otherwise default values are used. This is done in the Initialization script of the model as follows:

```
T=GetFromBase('T', 4*ones(4,1));  
gear=GetFromBase('gear', [3.2;1.8;1.3;1;.8]);
```

## 20.2 ExecPythonScript blocks: example

The demo model `reinforcement_learning_python_exec.scm` considers again the cart pendulum system but here the controller is designed using reinforcement learning.

The reinforcement learning process used for the controller design of this inverted pendulum on a cart system is based on the cart-pole system implemented by Rich Sutton et al. The code is inspired by the 'gym' environment `CartPole` and the corresponding example in `stable-baselines` package.<sup>1</sup> The Python code installs the required Python packages if needed.

The learning code is run by executing the Python script inside the **ExecPythonScript** block (this is done double-clicking on the block and 'Execute'-ing the Python script). The execution of the script takes a few minutes; at the end the result is saved in the model's temporary folder. At this point the model can be simulated. The controller obtained from the learning process is loaded by the **PyCustomBlock** and used during simulation to implement the feedback controller.

This implementation of the reinforcement learning requires that the model of the plant (cart-pendulum in this case) be expressed in Python. So the plant model is actually created twice (once in Activate inside the cart pendulum Super Block, and once in Python). So there is a risk of discrepancy between the models.

An alternative to the above approach is to use the Activate model to create the code used by reinforcement learning. This can be done by using the code generation capabilities of Activate to create a C code for the corresponding Super Block, and interface it in Python. In this approach not only the plant model is created only once (so simplicity and no risk of discrepancy) but the performance is also improved since the reinforcement learning program runs simulations in C code instead of Python code.

The Activate model `reinforcement_learning_python_Pcode.scm` contains an implementation of this approach. The OML script in the `ExecOmlScript` block must be executed first to generate the C code for the cart pendulum Super Block. This code is compiled and linked with Python. Then the Python Script in the Reinforcement learning block must be executed. This runs the reinforcement learning algorithm that creates the controller. Finally running the simulation of the model shows the result of the application of the controller.

---

<sup>1</sup><https://pypi.org/project/stable-baselines/>